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LINE OF SIGHT CONSTANTS FOR THE PRINCIPAL STARS.

By FRANK SCHLESINGER.

THE following formulae and tables are intended to facilitate the elimination of the Earth's orbital velocity from observations for motion in the line of sight. Let

λ, β, be the mean longitude and latitude of the observed star.

$\odot$, the Sun's mean longitude at the moment of observation.

Γ, the Sun's longitude at perigee ($281^\circ\ 20'$).

a, the semi-axis major of the Earth's orbit (149,480,000 kilometers).

e, the eccentricity of the Earth's orbit (0.0167).

$90^\circ - i$, the angle between the Earth's direction of motion and the radius vector of the orbit.

T, the Earth's sidereal period (31,558,000 mean solar seconds).

v, the *mean* velocity of the Earth in its orbit, defined more precisely below.

v_1, the Earth's velocity at the moment of observation.

x, the desired correction to the observed velocity of a star, or the projection of v_1 upon the line of sight.

From the theory of elliptical motion[1] we have

$$\tan i = \frac{e \sin(\odot - \Gamma)}{1 + e \cos(\odot - \Gamma)}$$

$$v_1 = v\,[1 + e \cos(\odot - \Gamma)] \sec i$$

[1] CHAUVENET'S *Astronomy*, 1, p. 636.

in which $$v = \frac{a}{\sqrt{1-e^2}} \cdot \frac{2\pi}{T}.$$

Projecting v_1 upon the line of sight,

$$x = -v\,[1 + e \cos(\odot - \Gamma)] \sin(\lambda - \odot + i) \cos\beta \sec i.$$

Eliminating i and simplifying we obtain

$$x = v \cos\beta \sin(\odot - \lambda) + v\,e \cos\beta \sin(\Gamma - \lambda).$$

Or, if we put

$$b = v \cos\beta$$
$$c = v\,e \cos\beta \sin(\Gamma - \lambda)$$

we have finally

$$x = b \sin(\odot - \lambda) + c.$$

The quantities b and c do not involve the longitude of the Sun and are, therefore, practically constant for any particular star. Neglecting proper motion, β cannot change more than 49″ in a century while $\Gamma-\lambda$ is increased about 20′ in the same period. The maximum value of b is about 30 kilometers per second, and c never exceeds 0.50 kilometers per second. It follows that if we tabulate the values of b and c for the present epoch they will hold during the coming century without ever introducing so large an error in x as 0.01 kilometers per second.

It is interesting to note that the first term in the above expression for x is what would have resulted from the assumption of a circular orbit for the Earth. That is, although the Earth's velocity varies in both amount and direction from that of a circular orbit, yet the combined effect is constant for any particular star. The analogue to this proposition is found in the theory of aberration.[2]

In the accompanying table the constants *log* b and c have been computed for all the brighter stars visible from northern observatories. Some variables and a few stars with large proper motions are also given. The unit of velocity is the kilometer per second, and the tables and formulae conform to the usual convention of denoting approach by the negative, and recession by the positive sign. The stars are arranged in the order of increasing right ascensions. To enable a star to be readily

[2] BESSEL, *Tabulae Regiomontanae*, XIX.

found the approximate right ascension is given at frequent intervals. The longitude for 1900 and the latitude have also been tabulated, though only the former is needed in the reductions. The longitude must be brought up to the year of observation, $1900 + t$, by adding to the tabular longitude,

$$d\lambda = 0.838' \times t.$$

A short table giving the values of $d\lambda$ for every year up to 1930 will be found at the end of the paper. For a few stars with very large proper motions, such as 1830 *Groombridge*, 61 Cygni, o^2 Eridani, μ Cassiopeiae, etc., the above expression for $d\lambda$ will not suffice when t becomes large.

The value adopted for a, the semi-axis major of the Earth's orbit, is based upon 8.80″ as the solar parallax; an error of 0.01″ in this constant corresponds to a maximum error of 0.034 kilometers per second in the correction to an observed velocity. Had a different parallax been employed the only effect upon these tables would have been the addition of a constant to each *log b*. For Γ, the Sun's longitude at perigee, the round value 281° 20′ has been employed; this is about its value for 1950, referred however to the equinox of 1900.

Much assistance in checking the computations was obtained from a table of the longitudes and latitudes of six hundred stars, published by C. Chabrol in the *Connaissance des Temps* for the year XII (1804), and referred to the equinox of 1800. This table contains nearly all the stars in the present paper, and very little labor was required to bring the data up to 1900 with sufficient accuracy. However, these results were used merely as a check, all the computations being based directly upon the right ascensions and declinations of the stars referred to the equinox and epoch of 1900. The following formulae were employed:

$$\tan \lambda = [9.96255] \tan \alpha + [9.59987] \sec \alpha \tan \delta,$$
$$b = [1.47371] \sec \lambda \cos \alpha \cos \delta,$$
$$c = [8.224]\, b \sin (281^\circ\ 20' - \lambda).$$

For c a graphical check was constructed, with *log b* and λ as the arguments.

Name	Mag.	λ 1900	β	log b	c
$0^h 3^m$					
α Andromedae ...	2.0	12°55.0′	+25°41′	1.4285	−0.45
β Cassiopeiae	2.1	33 43.2	+51 14	1.2705	− .29
γ Pegasi	2.6	7 45.7	+12 36	1.4631	− .49
ι Ceti	3.3	359 31.1	−10 01	1.4670	− .48
T Ceti	Var.	335 15.0	−20 30	1.4453	− .37
$0^h 20^m$					
ζ Cassiopeiae	4.0	33 40.7	+44 43	1.3254	− .33
π Andromedae ...	4.0	21 17.3	+27 08	1.4230	− .44
δ Andromedae ...	3.3	20 25.2	+24 21	1.4333	− .45
α Cassiopeiae	Var.	36 23.8	+46 37	1.3106	− .31
β Ceti	2.0	1 10.5	−20 47	1.4445	− .46
$0^h 40^m$					
η Cassiopeiae	3.8	38 50.1	+47 02	1.3072	− .30
γ Cassiopeiae	2.0	42 32.6	+48 48	1.2924	− .28
μ Andromedae ...	4.0	27 46.8	+29 39	1.4128	− .42
ε Piscium	4.0	16 07.8	+ 1 05	1.4736	− .50
μ Cassiopeiae	5.6	39 41.5	+43 13	1.3363	− .32
$1^h 3^m$					
η Ceti	3.1	10 21.8	−16 06	1.4563	− .48
β Andromedae ...	2.3	29 00.6	+25 57	1.4276	− .43
τ Piscium	4.0	26 55.4	+20 44	1.4446	− .45
θ Ceti	3.0	14 49.8	−15 46	1.4570	− .48
δ Cassiopeiae	2.8	46 32.0	+46 24	1.3123	− .28
$1^h 20^m$					
α Ursae min......	2.0	87 10.1	+66 06	1.0815	− .05
η Piscium	3.6	25 25.2	+ 5 22	1.4718	− .48
υ Persei	3.6	41 03.0	+35 24	1.3849	− .35
φ Persei	4.0	43 12.2	+36 50	1.3770	− .35
τ Ceti	3.3	16 27.2	−24 52	1.4315	− .45
$1^h 40^m$					
ζ Ceti	3.0	20 32.9	−20 20	1.4457	− .46
ε Cassiopeiae	3.3	53 22.4	+47 32	1.3031	− .25
α Trianguli	3.6	35 28.2	+16 48	1.4548	− .44
γ Arietis	4.0	31 47.2	+ 7 10	1.4703	− .46
ξ Piscium	4.0	26 07.3	− 7 56	1.4695	− .47
$1^h 49^m$					
β Arietis	2.8	32 34.4	+ 8 29	1.4689	− .46
50 Cassiopeiae	4.0	62 10.3	+54 23	1.2390	− .18
υ Ceti	4.0	18 01.2	−31 02	1.4066	− .42
γ Andromedae ...	2.4	42 50.0	+27 48	1.4205	− .38
α Arietis	2.0	36 15.8	+ 9 58	1.4671	− .45
$2^h 2^m$					
β Trianguli	3.0	40 57.4	+20 35	1.4451	− .41
ο Ceti	Var.	30 07.5	−15 56	1.4567	− .45
$ξ^2$ Ceti	4.0	36 04.3	− 5 52	1.4714	− .45
123 Piazzi II	6.6	37 23.2	− 8 00	1.4695	− .44
δ Ceti	4.0	36 10.3	−14 28	1.4597	− .44
$2^h 35^m$					

Name	Mag.	λ 1900	β	*log b*	*c*
$2^h\ 37^m$					
θ Persei.........	4.0	53° 16.0′	+31° 37′	1.4040	−0.32
γ Ceti	3.3	38 02.3	−12 00	1.4641	− .44
π Ceti	4.0	32 21.2	−28 15	1.4186	− .41
μ Ceti	4.0	40 32.2	− 5 34	1.4716	− .43
η Persei.........	3.6	57 18.5	+37 28	1.3733	− .28
$2^h\ 44^m$					
41 Arietis	3.8	46 48.4	+10 27	1.4665	− .40
τ Persei.........	4.0	56 31.1	+34 22	1.3904	− .29
η Eridani	3.0	37 20.9	−24 33	1.4326	− .41
α Ceti...........	2.3	42 55.4	−12 35	1.4631	− .41
γ Persei.........	3.0	58 37.6	+34 31	1.3896	− .28
$2^h\ 58^m$					
ρ Persei.........	Var.	53 30.8	+20 34	1.4451	− .35
β Persei.........	Var.	54 46.4	+20 25	1.4396	− .34
ι Persei.........	4.0	57 50.4	+30 38	1.4084	− .30
12 Eridani	3.3	33 11.0	−44 43	1.3253	− .33
α Persei.........	2.0	60 41.2	+30 07	1.4107	− .28
$3^h\ 18^m$					
ο Tauri	3.6	49 46.1	− 9 20	1.4679	− .39
ξ Tauri	3.6	50 30.8	− 8 48	1.4686	− .38
f Tauri	4.0	52 11.7	− 5 55	1.4714	− .38
ε Eridani	3.0	46 47.8	−27 44	1.4207	− .36
δ Persei.........	3.1	63 24.5	+27 18	1.4225	− .27
$3^h\ 37^m$					
ο Persei.........	4.0	59 44.8	+12 10	1.4638	− .32
ν Persei.........	4.0	62 25.8	+22 08	1.4405	− .29
δ Eridani	3.0	49 27.5	−28 43	1.4167	− .34
17 Tauri	4.1	58 00.9	+ 4 11	1.4725	− .34
η Tauri	3.0	58 35.8	+ 4 03	1.4726	− .34
$3^h\ 42^m$					
τ^6 Eridani........	4.0	45 58.0	−41 53	1.3456	− .31
27 Tauri	4.0	58 57.5	+ 3 54	1.4727	− .34
ζ Persei	3.0	61 43.7	+11 19	1.4652	− .31
ε Persei	3.3	64 16.9	+19 06	1.4491	− .29
ξ Persei	4.0	63 34.6	+14 56	1.4588	− .30
$3^h\ 53^m$					
γ Eridani........	3.0	52 27.9	−33 12	1.3963	− .31
λ Tauri	Var.	59 14.2	− 7 58	1.4695	− .33
ν Tauri	4.0	58 31.2	−14 28	1.4597	− .33
c Persei	4.0	68 06.1	+26 14	1.4265	− .25
o^2 Eridani........	4.6	58 52.4	−28 21	1.4182	− .30
$4^h\ 12^m$					
γ Tauri	4.0	64 24.3	− 5 44	1.4715	− .30
δ Tauri	4.0	65 28.3	− 3 59	1.4727	− .29
ε Tauri	3.6	67 03.9	− 2 35	1.4733	− .28
θ^2 Tauri	3.6	66 33.7	− 5 51	1.4714	− .28
α Tauri	1.0	68 23.5	− 5 28	1.4717	− .27
$4^h 31^m$					

Name	Mag.	λ 1900	β	log b	c
$4^h\ 31^m$					
ν Eridani	4.0	65°25.0′	−25°08′	1.4305	−0.26
53 Eridani	4.0	63 51.4	−36 01	1.3816	− 25
μ Eridani	3.6	67 56.2	−25 23	1.4296	− 25
π^3 Orionis	3.5	70 30.7	−15 24	1.4578	− 25
π^5 Orionis	4.0	71 05.6	−20 01	1.4466	− 24
$4^h\ 50^m$					
ι Aurigae	3.0	75 14.6	+10 27	1.4665	− 22
10 Camelopardis	4.0	79 52.4	+37 25	1.3737	− 14
ε Aurigae	Var.	77 26.7	+20 56	1.4441	− 19
ζ Aurigae	4.0	77 14.2	+18 11	1.4514	− 19
η Aurigae	3.6	78 03.0	+18 16	1.4512	− 19
$5^h\ 0^m$					
ε Leporis	3.5	70 39.3	−44 58	1.3234	− 18
β Eridani	3.0	73 52.9	−27 52	1.4201	− 20
λ Eridani	4.0	73 48.9	−31 33	1.4042	− 20
μ Leporis	3.3	73 59.6	−39 04	1.3638	− 18
α Aurigae	1.0	80 27.6	+22 52	1.4382	− 16
$5^h\ 10^m$					
β Orionis	1.0	75 26.0	−31 08	1.4061	− 19
τ Orionis	4.0	76 27.0	−29 51	1.4119	− 19
η Orionis	3.3	78 45.6	−25 33	1.4290	− 17
γ Orionis	2.0	79 33.0	−16 50	1.4547	− 18
β Tauri	2.0	81 10.7	+ 5 23	1.4718	− 17
$5^h\ 20^m$					
β Leporis	3.2	78 16.5	−43 55	1.3312	− 14
δ Orionis	Var.	80 57.9	−23 34	1.4359	− 16
α Leporis	3.0	79 59.0	−41 04	1.3510	− 14
λ Orionis	3.5	82 18.6	−13 23	1.4618	− 16
ι Orionis	3.1	81 36.1	−29 13	1.4146	− 15
$5^h\ 31^m$					
ε Orionis	2.0	82 04.0	−24 31	1.4327	− 15
ζ Tauri	3.3	83 23.3	− 2 12	1.4734	− 15
σ Orionis	3.7	82 41.9	−25 57	1.4275	− 14
ζ Orionis	2.0	83 17.1	−25 18	1.4299	− 14
γ Leporis	3.9	83 27.7	−45 49	1.3169	− 11
$5^h\ 41^m$					
ζ Leporis	3.6	84 35.4	−38 14	1.3689	− 11
κ Orionis	2.6	85 00.1	−33 05	1.3969	− 12
ν Aurigae	4.0	86 53.3	+15 42	1.4572	− 12
δ Leporis	4.0	85 45.8	−44 17	1.3285	− 10
α Orionis	Var.	87 21.5	−16 02	1.4565	− 11
$5^h\ 50^m$					
δ Aurigae	4.1	88 31.1	+30 50	1.4075	− 10
η Leporis	3.6	87 30.3	−37 37	1.3724	− 09
β Aurigae	2.0	88 30.9	+21 30	1.4423	− 10
θ Aurigae	3.0	88 32.7	+13 46	1.4611	− 11
η Geminorum	Var.	92 02.5	− 0 54	1.4736	− 07
$6^h\ 9^m$					

Name	Mag.	λ 1900	β	log b	c
$6^h\ 16^m$					
μ Geminorum	3.0	93°54.2′	− 0°50	1.4737	−0.06
β Canis Maj......	2.6	95 47.7	−41 16	1.3497	− .04
γ Geminorum	2.3	97 42.4	− 6 45	1.4707	− .03
S Monocerotis....	Var.	98 58.2	−13 11	1.4621	− .02
ε Geminorum	3.3	98 32.4	+ 2 04	1.4734	− .02
$6^h\ 39^m$					
ξ Geminorum	3.6	99 49.0	−10 07	1.4669	− .01
α Canis Maj......	1.0	102 42.3	−39 35	1.3606	+ .01
θ Geminorum	3.3	99 43.6	+11 00	1.4656	− .01
ε Canis Maj......	1.6	109 22.4	−51 22	1.2691	+ .04
ζ Geminorum	Var.	103 35.6	− 2 03	1.4734	+ .02
$7^h\ 0^m$					
δ Canis Maj......	2.0	112 00.4	−48 28	1.2952	+ .06
λ Geminorum	3.8	107 23.1	− 5 39	1.4716	+ .05
δ Geminorum	3.3	107 07.4	− 0 11	1.4737	+ .05
R Canis Maj......	Var.	113 05.8	−38 10	1.3692	+ .08
ι Geminorum	4.0	107 33.8	+ 5 45	1.4715	+ .05
$7^h\ 20^m$					
η Canis Maj......	2.4	118 09.1	−50 37	1.2761	+ .09
β Canis Min......	3.0	110 47.9	−13 30	1.4615	+ .08
α Geminorum	2.0	108 50.8	+10 06	1.4669	+ .06
α Canis Min......	1.0	114 24.4	−16 00	1.4566	+ .11
κ Geminorum	3.6	112 16.2	+ 3 04	1.4731	+ .09
$7^h\ 39^m$					
β Geminorum	1.3	111 50.2	+ 6 41	1.4708	+ .09
ι Navis	3.0	130 00.3	−43 17	1.3358	+ .17
β Cancri	3.6	122 51.8	−10 18	1.4667	+ .18
30 Monocerotis....	3.6	128 27.7	−22 27	1.4395	+ .21
ο Ursae Maj	3.3	111 35.9	+40 14	1.3565	+ .07
$8^h\ 30^m$					
δ Cancri	4.0	127 19.5	+ 0 05	1.4737	+ .22
ε Hydrae	3.3	130 57.3	−11 07	1.4655	+ .24
ζ Hydrae	3.3	133 11.0	−10 59	1.4657	+ .26
ι Ursae Maj	3.0	121 24.6	+29 35	1.4131	+ .15
α Cancri	4.0	132 14.7	− 5 05	1.4720	+ .26
$8^h\ 54^m$					
10 Ursae Maj	4.0	123 53.1	+23 43	1.4354	+ .18
κ Ursae Maj	3.3	122 32.2	+28 58	1.4157	+ .16
θ Hydrae........	4.0	138 53.3	−13 03	1.4624	+ .30
40 Lyncis	3.3	130 26.9	+17 58	1.4520	+ .23
α Hydrae........	2.0	145 53.3	−22 23	1.4397	+ .32
$9^h\ 23^m$					
h Ursae Maj	3.3	119 24.9	+45 09	1.3220	+ .11
θ Ursae Maj	3.0	125 53.0	+34 55	1.3875	+ .17
ο Leonis	3.6	142 51.3	− 3 45	1.4728	+ .33
ε Leonis	3.0	139 18.5	+ 9 43	1.4074	+ .30
ν Ursae Maj	3.6	124 52.3	+42 39	1.3403	+ .15
$9^h\ 44^m$					

Name	Mag.	λ 1900	β	log b	c
$9^h\ 47^m$					
μ Leonis	4.0	140°02.1′	+12°21′	1.4636	+0.30
η Leonis	3.3	146 30.5	+ 4 52	1.4721	+ .35
α Leonis	1.3	148 26.4	+ 0 28	1.4737	+ 37
λ Hydrae........	4.0	157 58.8	− 22 01	1.4408	+ .39
λ Ursae Maj......	3.3	138 09.1	+29 53	1.4118	+ 26
$10^h\ 11^m$					
ζ Leonis	3.0	146 10.1	+11 52	1.4643	+ .34
γ Leonis	2.0	148 12.4	+ 8 49	1.4686	+ .36
μ Ursae Maj......	3.0	139 50.2	+29 00	1.4156	+ .27
μ Hydrae........	4.0	163 38.9	− 24 40	1.4321	+ .40
ρ Leonis	4.0	154 59.5	+ 0 09	1.4737	+ .40
$10^h\ 30^m$					
U Hydrae........	Var.	164 53.8	−20 24	1.4456	+ .42
ν Hydrae........	3.3	168 58.5	−21 48	1.4415	+ .43
46 Leonis Min	4.0	149 28.1	+24 56	1.4312	+ .34
β Ursae Maj......	2.3	138 01.5	+45 07	1.3223	+ .21
α Ursae Maj......	2.0	133 47.5	+49 41	1.2847	+ .17
$11^h\ 0^m$					
ψ Ursae Maj......	3.1	147 24.6	+35 32	1.3842	+ .29
β Crateris	4.0	177 09.6	−25 38	1.4287	+ .44
δ Leonis	2.3	159 54.7	+14 20	1.4600	+ .41
θ Leonis	3.3	162 01.5	+ 9 41	1.4675	+ .43
ξ Ursae Maj.	3.8	155 56.4	+24 44	1.4319	+ .37
$11^h\ 13^m$					
ν Ursae Maj......	3.3	155 15.1	+26 09	1.4268	+ .36
δ Crateris	3.3	175 18.0	−17 35	1.4529	+ .46
ι Leonis	4.0	166 09.7	+ 6 06	1.4712	+ .45
γ Crateris	4.0	177 50.7	−19 40	1.4476	+ .46
λ Draconis	3.3	128 55.4	+57 14	1.2071	+ .13
$11^h\ 27^m$					
ξ Hydrae........	4.0	186 36.3	−31 35	1.4041	+ .42
χ Ursae Maj......	3.8	152 15.5	+41 33	1.3479	+ .29
β Leonis	2.0	170 13.8	+12 17	1.4637	+ .45
β Virginis	3.3	175 44.7	+ 0 41	1.4737	+ .48
1830 Groombridge...	6.5	165 09.0	+36 00	1.3817	+ .36
$11^h\ 48^m$					
γ Ursae Maj.....	2.3	149 03.9	+47 08	1.3064	+ .25
o Virginis	4.0	176 18.1	+ 8 32	1.4689	+ .48
ϵ Corvi	3.0	190 16.6	−19 40	1.4476	+ .47
δ Ursae Maj.....	3.4	149 39.0	+51 39	1.2664	+ .23
γ Corvi.........	2.0	189 20.2	−14 30	1.4597	+ .48
$12^h\ 12^m$					
η Virginis	3.3	183 26.2	+ 1 22	1.4736	+ .49
δ Corvi	2.3	192 03.7	−12 11	1.4638	+ .49
β Corvi	2.3	195 58.5	−18 02	1.4518	+ .47
κ Draconis	3.3	134 50,6	+61 45	1.1488	+ .13
γ Virginis	3.0	188 45.5	+ 2 48	1.4732	+ .50
$12^h\ 37^m$					

Name	Mag.	λ 1900	β	*log b*	*c*
$12^h\ 49^m$					
ε Ursae Maj.	2.0	157°30.9′	+54°19′	1.2397	+0.24
δ Virginis	3.0	190 04.3	+ 8 38	1.4688	+ .49
12 Canum ven.....	2.9	173 09.8	+40 07	1.3572	+ .36
ε Virginis	2.6	188 32.9	+16 13	1.4561	+ .48
γ Hydrae........	3.2	205 37.3	—13 44	1.4611	+ .47
$13^h\ 15^m$					
ζ Ursae Maj......	2.1	164 16.7	+56 22	1.2170	+ .25
α Virginis	1.0	202 26.8	— 2 03	1.4734	+ .49
ζ Virginis	3.3	200 44.6	+ 8 39	1.4687	+ .49
η Ursae Maj......	2.0	175 31.3	+54 23	1.2388	+ .28
η Boötis.	3.0	197 55.9	+28 06	1.4193	+ .44
$13^h\ 50^m$					
τ Virginis	4.0	206 20.9	+13 05	1.4623	+ .47
α Draconis.......	3.3	156 02.1	+66 22	1.0769	+ .16
ι Virginis	4.0	212 23.7	+ 7 14	1.4703	+ .46
α Boötis.	1.0	202 50.3	+30 48	1.4076	+ .42
λ Boötis.	4.0	185 33.6	+54 39	1.2361	+ .29
$14^h\ 20^m$					
θ Boötis.....	3.8	181 11.2	+60 07	1.1710	+ .24
ρ Boötis.....	3.6	201 23.0	+42 27	1.3416	+ .36
γ Boötis.	2.9	196 15.5	+49 33	1.2858	+ .32
ζ Boötis.....	3.3	211 38.0	+27 53	1.4201	+ .41
μ Virginis	4.0	218 43.6	+ 9 42	1.4675	+ .44
$14^h\ 40^m$					
109 Virginis	3.6	217 07.2	+17 07	1.4540	+ .43
α Librae........	2.3	223 41.3	+ 0 21	1.4737	+ .42
ξ Boötis	5.0	212 08.3	+33 47	1.3934	+ .39
β Ursae Min......	2.0	131 53.6	+72 59	0.9402	+ .07
δ Librae	Var.	223 53.2	+ 8 16	1.4692	+ .42
$14^h\ 57^m$					
β Boötis	3.0	202 50.4	+54 10	1.2412	+ .29
γ Scorpii.........	3.4	229 17.6	— 7 38	1.4698	+ .39
δ Boötis.....	3.0	211 44.6	+48 59	1.2908	+ .31
β Librae.........	2.0	227 58.6	+ 8 31	1.4689	+ .40
μ Boötis.....	3.8	211 46.5	+53 26	1.2488	+ .28
$15^h\ 21^m$					
γ Ursae Min......	3.0	140 09.6	+75 14	0.8800	+ .08
ι Draconis........	3.0	183 30.9	+71 06	0.9842	+ .16
β Coronae bor....	3.8	217 43.2	+46 04	1.3150	+ .31
θ Coronae bor....	4.0	218 02.6	+48 34	1.2944	+ .29
α Coronae bor....	2.0	220 53.1	+44 20	1.3282	+ .31
$15^h\ 35^m$					
γ Coronae bor. ...	3.8	223 28.3	+44 31	1.3268	+ .30
α Serpentis.......	2.3	230 40.2	+25 31	1.4291	+ .35
β Serpentis.......	3.3	228 32.6	+34 20	1.3905	+ .33
κ Serpentis.......	4.0	228 22.4	+37 07	1.3754	+ .32
μ Serpentis.......	3.3	234 32.6	+16 15	1.4560	+ .35
$15^h\ 45^m$					

Name	Mag.	λ 1900	β	log b	c
$15^h 45^m$					
ε Serpentis	3.3	232°55.7′	+24°01′	1.4344	+0.34
γ Serpentis	3.6	231 21.5	+35 14	1.3858	+ .31
ε Coronae bor.	4.0	227 42.9	+46 05	1.3148	+ .28
δ Scorpii	2.3	241 10 5	− 1 58	1.4735	+ .32
β Scorpii	2.0	241 47.6	+ 1 01	1.4737	+ .32
$16^h 0^m$					
θ Draconis	3.6	195 16.5	+74 27	0.9022	+ .13
φ Herculis	4.0	220 13.0	+63 47	1.1188	+ .19
δ Ophiuchi	3.0	240 54.3	+17 15	1.4537	+ .31
ε Ophiuchi	3.3	242 00.6	+16 27	1.4556	+ .30
τ Herculis	3.3	222 58.2	+65 50	1.0857	+ .17
$16^h 17^m$					
γ Herculis	3.1	237 48.9	+40 01	1.3578	+ .26
η Draconis	2.6	193 02.5	+78 27	0.7754	+ .10
α Scorpii	1.3	248 22.0	− 4 33	1.4723	+ .27
λ Ophiuchi	3.7	244 11.7	+23 35	1.4358	+ .28
β Herculis	2.3	239 41.5	+42 43	1.3398	+ .24
$16^h 30^m$					
ζ Ophiuchi	2.6	247 49.9	+11 24	1.4650	+ .27
ζ Herculis	2.6	240 04.9	+53 07	1.2520	+ .20
η Herculis	3.1	237 22.6	+60 18	1.1686	+ .17
κ Ophiuchi	3.3	250 25.9	+31 51	1.4028	+ .22
ε Herculis	3.3	246 55.7	+53 16	1.2505	+ .17
$17^h 0^m$					
η Ophiuchi	2.3	256 34.3	+ 7 12	1.4703	+ .21
ζ Draconis	3.0	181 50.6	+84 46	0.4340	+ .04
α Herculis	Var.	254 45.2	+37 18	1.3743	+ .18
δ Herculis	3.0	253 21.9	+47 42	1.3018	+ .16
U Ophiuchi	Var.	256 40.5	+24 13	1.4337	+ .19
$17^h 11^m$					
π Herculis	3.1	250 40.0	+59 34	1.1783	+ .13
θ Ophiuchi	3.4	259 59.9	− 1 50	1.4735	+ .18
β Draconis	2.6	250 33.7	+75 17	0.8783	+ .06
α Ophiuchi	2.0	261 02.8	+35 52	1.3824	+ .14
ξ Serpentis	3.6	263 09.0	+ 7 57	1.4695	+ .15
$17^h 35^m$					
ι Herculis	3.3	258 29.4	+69 17	1.0224	+ .07
β Ophiuchi	3.0	263 56.5	+27 57	1.4198	+ .13
X Sagitarii	Var.	265 50.7	− 4 24	1.4724	+ .13
μ Herculis	3.3	263 50.4	+51 08	1.2712	+ .09
γ Ophiuchi	3.6	265 14.2	+26 08	1.4269	+ .12
$17^h 50^m$					
ξ Draconis	3.3	263 20.9	+80 17	0.7006	+ .03
θ Herculis	4.0	267 04.9	+60 42	1.1633	+ .06
ν Ophiuchi	3.6	268 21.4	+13 41	1.4612	+ .11
ξ Herculis	3.6	267 47.8	+52 42	1.2561	+ .07
γ Draconis	2.3	·266 34.5	+74 56	0.8885	+ .03
$17^h 55^m$					

Name	Mag.	λ 1900	β	*log b*	*c*
$17^h\ 55^m$					
67 Ophiuchi	4.0	268°47.1′	+26°24′	1.4259	+0.10
W Sagitarii	Var.	269 42.0	— 6 08	1.4712	+ .10
γ Sagitarii	3.3	269 52.0	— 6 58	1.4705	+ .10
70 Ophiuchi.......	4.7	270 06.7	+25 59	1.4275	+ .09
72 Ophiuchi.......	3.3	270 46.0	+33 00	1.3973	+ .08
$18^h\ 3^m$					
o Herculis	3.8	271 18.1	+52 12	1.2611	+ .05
μ Sagitarii	4.0	271 49.0	+ 2 21	1.4733	+ .08
Y Sagitarii	Var.	273 40.6	+ 4 30	1.4724	+ .07
η Serpentis.......	3.0	274 18.0	+20 28	1.4454	+ .06
109 Herculis	4.0	276 23.8	+45 04	1.3227	+ .03
$18^h\ 20^m$					
χ Draconis.......	3.8	74 42.4	+83 33	0.5242	— .03
α Lyrae	1.0	283 54.9	+61 44	1.1490	— .01
110 Herculis........	4.0	283 23.3	+43 25	1.3348	— .01
β Lyrae	Var.	287 29.8	+56 00	1.2213	— .03
σ Sagitarii.......	2.3	280 59.3	— 3 26	1.4729	.00
$18^h\ 50^m$					
R Lyrae..........	Var.	293 50.7	+66 11	1.0798	— .04
ε Aquilae	4.0	286 52.3	+37 35	1.3727	— .04
γ Lyrae	3.3	290 32.1	+55 02	1.2320	— .05
ζ Aquilae	3.0	288 24.3	+36 12	1.3805	— .05
λ Aquilae	3 1	285 56.3	+17 35	1.4529	— .04
$19^h\ 2^m$					
π Sagitarii	3.1	284 51.3	+ 1 28	1.4736	— .03
δ Draconis	3.0	15 50.7	+82 53	0.5667	— .06
κ Cygni	4.0	313 32.8	+73 48	0.9190	— .07
δ Aquilae	3.3	292 14.2	+24 50	1.4316	— .09
β Cygni	3.0	299 51.9	+48 59	1.2908	— .10
$19^h\ 30^m$					
σ Draconis	5.3	29 16.9	+80 55	0.6719	— .07
γ Aquilae	3.0	299 32.8	+31 15	1.4056	— .13
δ Cygni	2.8	314 52.2	+64 25	1.1089	— .12
δ Sagittae	4.0	301 59.8	+38 55	1.3647	— .14
α Aquilae	1.3	300 21.9	+29 19	1.4142	— .14
$19^h\ 47^m$					
η Aquilae	Var.	299 02.4	+21 32	1.4423	— .14
ε Draconis	3.8	31 20.4	+79 29	0.7349	— .09
β Aquilae	4.0	301 02.0	+26 41	1.4248	— .15
γ Sagittae	3.6	305 39.2	+39 12	1.3630	— .16
θ Aquilae........	3.0	303 31.1	+18 44	1.4500	— .18
$20^h\ 10^m$					
$α^2$ Capricorni	3.3	302 27.7	+ 6 57	1.4705	— .18
β Capricorni	3.0	302 39.0	+ 4 36	1.4723	— .18
γ Cygni	2.4	323 27.8	+57 08	1.2083	— .18
θ Cephei.........	4.0	3 30.0	+73 56	0.9158	— .14
ε Delphini.......	4.0	312 40 2	+29 05	1.4152	— .23
$20^h\ 29^m$					

Name	Mag.	λ 1900	β	log b	c
$20^h\ 32^m$					
β Delphini	3.3	314°56.9′	+31°56′	1.4025	−0.23
α Delphini	3.6	315 59.3	+33 02	1.3972	− .24
α Cygni	1.6	333 57.3	+59 55	1.1738	− .20
δ Delphini	4.0	316 43.7	+31 57	1.4023	− .24
γ Delphini	4.0	317 58.9	+32 43	1.3987	− .25
$20^h\ 42^m$					
ε Cygni	2.6	326 20.6	+49 25	1.2869	− .23
ε Aquarii	3.6	310 19.6	+ 8 06	1.4694	− .24
η Cephei	3.6	3 15.0	+71 46	0.9691	− .15
ν Cygni	4.0	334 45.6	+54 55	1.2332	− .23
ξ Cygni	4.0	339 25.6	+56 35	1.2146	− .23
$21^h\ 2^m$					
61¹ Cygni	5.7	335 20.5	+51 53	1.2642	− .25
ζ Cygni	3.0	331 39.5	+43 42	1.3328	− .28
τ Cygni	4.0	337 12.7	+50 33	1.2768	− .26
α Aquilae	4.0	321 43.5	+20 08	1.4463	− .30
α Cephei	2.6	11 24.3	+68 55	1.0298	− .18
$21^h\ 20^m$					
β Aquarii	3.0	322 00.0	+ 8 38	1.4688	− .32
β Cephei	3.0	34 10.7	+71 09	0.9832	− .15
γ Capricorni	3.6	320 23.3	− 2 33	1.4733	− .31
ε Pegasi	2.3	330 29.6	+22 08	1.4405	− .35
κ Pegasi	4.0	337 32.7	+36 39	1.3781	− .33
$21^h\ 40^m$					
μ Cephei	Var.	8 20.3	+64 11	1.1126	− .22
δ Capricorni	3.0	322 08.5	− 2 35	1.4733	− .33
α Aquarii	3.0	331 57.5	+10 40	1.4661	− .38
ι Aquarii	4.0	327 19.3	− 2 04	1.4734	− .36
ι Pegasi	4.0	343 00.5	+34 16	1.3909	− .36
$22^h\ 4^m$					
θ Pegasi	3.3	335 26.0	+16 21	1.4558	− .39
ζ Cephei	3.4	12 35.3	+61 09	1.1573	− .24
γ Aquarii	3.4	335 19.0	+ 8 15	1.4692	− .40
ζ Aquarii	4.0	337 30.6	+ 8 51	1.4685	− .41
δ Cephei	Var.	16 14.0	+59 32	1.1787	− .25
$22^h\ 26^m$					
7 Lacertae	4.0	6 45.5	+53 18	1.2502	− .37
η Aquarii	3.8	339 00.4	+ 8 09	1.4693	− .42
ζ Pegasi	3.3	344 45.5	+17 41	1.4527	− .42
η Pegasi	3.0	354 19.6	+35 07	1.3865	− .39
λ Pegasi	4.0	351 39.9	+28 48	1.4164	− .41
$22^h\ 43^m$					
τ Aquarii	4.0	337 11.9	− 5 40	1.4716	− .41
μ Pegasi	4.0	352 59.6	+29 24	1.4139	− .41
ι Cephei	3.4	31 52.3	+62 37	1.1365	− .21
λ Aquarii	4.0	340 10.7	− 0 23	1.4737	− .43
δ Aquarii	3.0	337 28.6	− 8 11	1.4693	− .41
$22^h\ 50^m$					

Name	Mag.	λ 1900	β	log b	c
$22^h\ 52^m$					
α Piscis aus......	1.3	332°27.1′	—21°07′	1.4435	—0.37
ο Andromedae...	3.6	6 23.7	+43 45	1.3325	— .36
β Pegasi........	Var.	357 58.7	+31 08	1.4061	— .41
α Pegasi........	2.0	352 05.5	+19 25	1.4483	— .44
O^a Aquarii........	4.0	338 37.0	—14 29	1.4597	— .41
$23^h\ 5^m$					
3077 Bradley.......	6.0	22 16.8	+54 34	1.2369	— .28
γ Piscium.......	4.0	350 02.3	+ 7 17	1.4702	— .46
λ Andromedae...	4.0	16 54.4	+43 47	1.3322	— .36
ι Andromedae...	4.0	14 41.9	+41 02	1.3513	— .38
γ Cephei........	3.3	58 42.1	+64 39	1.1053	— .14
$23^h\ 40^m$					
ω Piscium.......	4.0	1 11.1	+ 6 22	1.4710	— .49

PRECESSION IN LONGITUDE.

1901	+0.8′	1911	+ 9.2′	1921	+17.6′
1902	+1.7	1912	+10.1	1922	+18.4
1903	+2.5	1913	+10.9	1923	+19.3
1904	+3.4	1914	+11.7	1924	+20.1
1905	+4.2	1915	+12.6	1925	+20.9
1906	+5.0	1916	+13.4	1926	+21.8
1907	+5.9	1917	+14.2	1927	+22.6
1908	+6.7	1918	+15.1	1928	+23.5
1909	+7.5	1919	+15.9	1929	+24.3
1910	+8.4	1920	+16.8	1930	+25.1

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THEORY OF THE DETERMINATION OF THE ELEMENTS OF A PARABOLIC ORBIT FROM TWO OBSERVATIONS OF APPARENT POSITION, AND ONE OF THE MOTION IN THE LINE OF SIGHT.

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WITH the great light-gathering power of modern telescopes, and the many improvements in the construction and methods of application of the spectroscope, it may be expected that measurements of relative motion in the line of sight will be made with a very high degree of precision. If such measurements of relative motion can be secured for a body moving around the Sun, together with the necessary number of apparent positions, it is evident that these data may be made the basis for the determination of the elements of its orbit. The purpose of this paper is to set forth the theory of the computation of the elements, supposing that the orbit is a parabola, and that at two epochs observations of the apparent position have been made, and that at the second epoch the motion in the line of sight has been observed.

§ 1. FORMULATION OF THE PROBLEM.

Let λ, β, and ρ represent the longitude, latitude, and distance respectively of the comet, taking the center of the Earth as origin. Let the derivatives of these quantities with respect to the time be represented by the same letters with primes. Suppose the observations are made at the two epochs, t_1 and t_2.

The observed quantities λ_1, λ_2, β_1, β_2, and ρ_2' depend upon the elements of the unknown orbit, hence we may write the following equations:

$$\left.\begin{aligned} \lambda_1 &= \theta_1 \text{ (elements)}, \\ \lambda_2 &= \theta_2 \, (\; . \; . \; . \; . \; . \;), \\ \beta_1 &= \phi_1 \, (\; . \; . \; . \; . \; . \;), \\ \beta_2 &= \phi_2 \, (\; . \; . \; . \; . \; . \;), \\ \rho_2' &= \psi_2 \, (\; . \; . \; . \; . \; . \;). \end{aligned}\right\} \quad (1)$$

These five equations involve the five elements as unknowns, and when solved, furnish the solution to the problem. They are, however, transcendental, and so exceedingly involved that a direct solution of them would present almost insurmountable difficulties. This suggests the alternative of first determining *intermediate quantities* from which the elements can be found. As such, it may be mentioned that one absolute position with reference to the Sun, and the velocity and direction of motion, are sufficient for the unique determination of the elements. If one desired to determine these intermediate quantities it would only be necessary to find *one* geocentric distance and the velocity and direction of motion. That is, there would be but *four* unknowns involved, instead of the five elements, and the problem would be reduced to the solution of *four* simultaneous equations.

As another set of intermediate quantities, the six coördinates defining two positions of the comet with respect to the Sun, and the interval of time required for the comet to move from one to the other, might be used. The interval of time is known, and the only unknowns in the two positions are the two geocentric distances. Therefore, $\lambda_1 \; . \; . \; . \; . \; \rho_2'$ may be considered as functions of the two unknowns, ρ_1 and ρ_2. In this case then, which is clearly the simplest possible, it is only necessary to find and solve two independent equations involving ρ_1, ρ_2 and known quantities.

Neglecting perturbations, the work must be consistent with and involve the following fundamental theorems of parabolic motion.

Theorem I. The motion of the comet is in a plane passing through the center of the Sun.

Theorem II. The areas swept over by the radius vector from the Sun are proportional to the intervals of time in which they are described.

Theorem III. The motion is in a parabola with the Sun at its focus.

It will be supposed that the effects of the Earth's motions upon the observed velocities in the line of sight have been

eliminated, so that ρ_2' represents the velocity in the direction of the Earth with respect to the Sun. The formulae for accomplishing this were given in the March number of the ASTROPHYSICAL JOURNAL, by Dr. Frank Schlesinger.

§ 2. THE TWO INDEPENDENT EQUATIONS INVOLVING ρ_1 AND ρ_2 AS UNKNOWNS.

Let x, y, and z represent the rectangular heliocentric coördinates, and r, l, and b the polar. Denote by s the chord joining the extremities of the radii r_1 and r_2. Let k represent the Gaussian constant. Then Euler's equation is

$$6k(t_2 - t_1) = (r_1 + r_2 + s)^{\frac{3}{2}} \mp (r_1 + r_2 - s)^{\frac{3}{2}}. \tag{2}$$

The upper sign is to be used if the heliocentric motion in the interval of time $t_2 - t_1$ is less than 180°, which will henceforth be supposed to be the case.

It will now be shown that (2) can be expressed in terms of ρ_1 and ρ_2 as unknowns. r_1, r_2, and s depend upon x_1, y_1, z_1, x_2, y_2, z_2, which are equal respectively to the differences of the corresponding geocentric coördinates of the comet and the Sun. The geocentric cöordinates of the Sun are given in the Nautical Almanacs, and the geocentric coördinates of the comet involve as unknowns ρ_1 and ρ_2 alone. Therefore Euler's formula is one equation of the type sought.

As a consequence of Theorem I we may write the following equations:

$$\left.\begin{aligned} Ax_1 + By_1 + Cz_1 &= 0, \\ Ax_2 + By_2 + Cz_2 &= 0, \\ Ax_2' + By_2' + Cz_2' &= 0. \end{aligned}\right\} \tag{3}$$

The constants A, B, and C, depend upon the position of the plane of the orbit, with respect to the plane of the ecliptic and the vernal equinox. Eliminating them we have,

$$\left.\begin{vmatrix} x_1, & y_1, & z_1, \\ x_2, & y_2, & z_2, \\ x_2', & y_2', & z_2', \end{vmatrix} = 0. \right\} \tag{4}$$

The expansion of this determinant may be written in the three ways,

$$\left.\begin{aligned} x_1(y_2z_2' - z_2y_2') + x_2(z_1y_2' - y_1z_2') + x_2'(y_1z_2 - z_1y_2) &= 0, \\ y_1(z_2x_2' - x_2z_2') + y_2(x_1z_2' - z_1x_2') + y_2'(z_1x_2 - x_1z_2) &= 0, \\ z_1(x_2y_2' - y_2x_2') + z_2(y_1x_2' - x_1y_2') + z_2'(x_1y_2 - y_1x_2) &= 0. \end{aligned}\right\} \quad (5)$$

As these equations are written they are not independent, but if by some new means the parentheses can be determined, they become independent and can be used for the elimination of any two unknowns. Let it be supposed for the moment that the parentheses have been computed. Then x_1, y_1, z_1, x_2, y_2, and z_2 depend upon ρ_1 and ρ_2 alone as unknowns, as was seen in the discussion of Euler's equation. x_2', y_2', and z_2' depend upon the direction of motion and velocity of the comet at the time t_2. They may be expressed in terms of ρ_2' and two other quantities. Eliminating these two unknown components of velocity, we have from (5) the desired relation between ρ_1 and ρ_2, which may be written,

$$F(\rho_1, \rho_2) = 0. \tag{6}$$

Therefore (2) and (6) are the equations from which ρ_1 and ρ_2 may be found.

§ 3. THE COMPUTATION OF THE PARENTHESES.

Consider the parentheses in the first equation of (5). By the law of areas $(y_2z_2' - z_2y_2')$ is twice the projection, upon the yz plane, of the area described by the radius vector in a unit of time. Denoting the angles between the plane of the orbit and the three fundamental planes by i_{xy}, i_{yz} and i_{zx} respectively, we have,

$$(y_2z_2' - z_2y_2') = k\sqrt{p}\cos i_{yz}, \tag{7}$$

where p is the parameter of the parabola. $(y_1z_2 - z_1y_2)$ is twice the projection, upon the yz plane, of the area between the radii r_1 and r_2, and the chord joining their extremities. Then we have,[1]

[1] OPPOLZER'S *Bahnbestimmung der Kometen und Planeten*, I, p. 99, or WATSON'S *Theoretical Astronomy*, p. 176.

$$(y_1 z_2 - z_1 y_2) = k\sqrt{p}\,(t_2 - t_1)\left\{1 - \frac{k^2}{6}\frac{(t_2 - t_1)^2}{r_1^3} + \cdots\right\}\cos i_{yz}. \tag{8}$$

It remains to consider the second parenthesis of (5), $(z_1 y_2' - y_1 z_2')$. Since the coördinates and velocities are functions of the time we may write,

$$\left.\begin{aligned} y_1' &= f(t_1), \\ y_2' &= f[t_1 + (t_2 - t_1)]. \end{aligned}\right\} \tag{9}$$

We shall suppose that the interval of time $t_2 - t_1$ is short, then expanding the second of (9) by Taylor's formula we have,

$$\left.\begin{aligned} y_2' &= f(t_1) + f'\,(t_1)\,(t_2 - t_1) + f''\,(t_1)\frac{(t_2 - t_1)^2}{2} + \cdots, \text{ or,} \\ y_2' &= y_1' + y_1''\,(t_2 - t_1) + y_1'''\,\frac{(t_2 - t_1)^2}{2} + \cdots. \end{aligned}\right\} \tag{10}$$

The development for z_2' and x_2' are obtained by writing in place of y, z, and x, respectively. Substituting these expressions in the parenthesis $(z_1 y_2' - y_1 z_2')$ we find,

$$\begin{aligned} (z_1 y_2' - y_1 z_2') = (z_1 y_1' - y_1 z_1') + t_2 - t_1)\,(z_1 y_1'' - y_1 z_1'') &+ \\ \frac{(t_2 - t_1)}{2}\,(z_1 y_1''' - y_1 z_1''') &+ \cdots. \end{aligned} \tag{11}$$

From the fundamental equations of acceleration $(z_1 y_1'' - y_1 z_1'')$ is identically zero. It is found by differentiating $(z_1 y_1'' - y_1 z_1'') = 0$ that

$$(z_1 y_1''' - y_1 z_1''') = -(z_1' y_1'' - y_1' z_1'') = -\frac{k^2}{r_1^3}(z_1 y_1' - y_1 z_1'). \tag{12}$$

Then (11) becomes

$$(z_1 y_2' - y_1 z_2') = (z_1 y_1' - y_1 z_1')\,\left(1 - \frac{k_2}{2}\,\frac{(t_2 - t_1)^2}{r_1^3} + \cdots\cdots\right). \tag{13}$$

As a consequence of the law of areas this reduces to

$$(z_1 y_2' - y_1 z_2') = -k\sqrt{p}\left\{1 - \frac{k^2}{2}\frac{(t_2 - t_1)^2}{r_1^3} + \cdots\right\}\cos i_{yz}. \tag{14}$$

There is the question whether in any case the terms of higher orders, which we have neglected, could become sensible. We shall suppose that the interval of time between the observations is at the most only a few days. The radius r_1 will generally be

in the neighborhood of unity. The factor $\frac{k_2}{2} = 0.000148$. The next higher term than the ones written in (14) involves, in addition to the factor k^2, the factor $\frac{dr_1}{dt}$. It is easy to see that in every case this will be an extremely small fraction. In order that it might be equal to unity the radius vector would have to change a whole astronomical unit in a mean solar day. Therefore we are safe in neglecting terms of the third and higher orders.

In computing the parentheses by (8) and (14), sufficiently accurate results for the first approximation to the orbit will be obtained, owing to the small factor $\frac{k_2}{2}$, in taking r_1 equal to unity. The parentheses may now be regarded as computed and the equations (5) become independent. All unknowns, except ρ_1 and ρ_2, must now be eliminated from them in order to find (6) explicitly.

§ 4. SOLUTION OF THE TWO EQUATIONS INVOLVING ρ_1 AND ρ_2 AS UNKNOWNS.

Substituting (7), (8) and (14) in (5) we find

$$\left.\begin{aligned}
&x_1 - x_2\left\{1 - \frac{k^2}{2}\frac{(t_2-t_1)^2}{r_1^3} + \cdots\right\} \\
&\qquad + x_2'\left\{1 - \frac{k^2}{6}\frac{(t_2-t_1)^2}{r_1^3} + \cdots\right\}(t_2 - t_1) = 0, \\
&\text{and similarly} \\
&y_1 - y_2\left\{1 - \frac{k^2}{2}\frac{(t_2-t_1)^2}{r_1^3} + \cdots\right\} \\
&\qquad + y_2'\left\{1 - \frac{k^2}{6}\frac{(t_2-t_1)^2}{r_1^3} + \cdots\right\}(t_2 - t_1) = 0, \\
&z_1 - z_2\left\{1 - \frac{k^2}{2}\frac{(t_2-t_1)^2}{r_1^3} + \cdots\right\} \\
&\qquad + z_2'\left\{1 - \frac{k^2}{6}\frac{(t_2-t_1)^2}{r_1^3} + \cdots\right\}(t_2 - t_1) = 0.
\end{aligned}\right\} \quad (15)$$

Let the latitude of the Sun be neglected, and denote the

geocentric polar coördinates by R and $\odot$. Then x, y, z, x', y', and z' may be expressed as follows:

$$\left.\begin{aligned}
x &= \rho \cos\beta \cos\lambda - R\cos\odot,\\
y &= \rho \cos\beta \sin\lambda - R\sin\odot,\\
z &= \rho \sin\beta,\\
x' &= \cos\beta\cos\lambda\,\rho' - \rho\cos\beta\sin\lambda\,\lambda' - \rho\sin\beta\cos\lambda\,\beta'\\
&\qquad - \cos\odot\, R' + R\sin\odot\,\odot',\\
y' &= \cos\beta\sin\lambda\,\rho' + \rho\cos\beta\cos\lambda\,\lambda' - \rho\sin\beta\sin\lambda\,\beta'\\
&\qquad - \sin\odot\, R' - R\cos\odot\,\odot',\\
z' &= \sin\beta\,\rho' + \rho\cos\beta\,\beta'.
\end{aligned}\right\} \quad (16)$$

Apply the proper subscripts and substitute these equations in (15). Then, for abbreviation, let

$$\left.\begin{aligned}
\mu &= -1 + \frac{k^2}{2}\frac{(t_2 - t_1)^2}{r_1^3} + \cdots\cdots,\\
\nu &= \left\{ 1 - \frac{k^2}{6}\frac{(t_2 - t_1)^2}{r_1^3} + \cdots \right\}(t_2 - t_1).\\
a_1 &= -\nu\rho_2 \cos\beta_2 \sin\lambda_2,\\
a_2 &= -\nu\rho_2 \sin\beta_2 \cos\lambda_2,\\
a_3 &= \rho_1\cos\beta_1\cos\lambda_1 - R_1\cos\odot_1 + \mu(\rho_2\cos\beta_2\cos\lambda_2 - R_2\\
&\quad \cos\odot_2) + \nu(\cos\beta_2\cos\lambda_2\,\rho_2' - \cos\odot_2\,R_2' + R_2\sin\odot_2\,\odot_2').\\
b_1 &= \nu\rho_2\cos\beta_2\cos\lambda_2,\\
b_2 &= -\nu\rho_2\sin\beta_2\sin\lambda_2,\\
b_3 &= \rho_1\cos\beta_1\sin\lambda_1 - R_1\sin\odot_1 + \mu(\rho_2\cos\beta_2\sin\lambda_2 -\\
&\qquad R_2\sin\odot_2) + \nu(\cos\beta_2\sin\lambda_2\,\rho_2' - \sin\odot_2\,R_2' -\\
&\qquad\qquad R_2\cos\odot_2\,\odot_2').\\
c_1 &= 0,\\
c_2 &= \nu\rho_2\cos\beta_2,\\
c_3 &= \rho_1\sin\beta_1 + \mu\rho_2\sin\beta_2 + \nu\sin\beta_2\,\rho_2'.
\end{aligned}\right\} \quad (17)$$

Then (15) becomes

$$\left.\begin{aligned}
a_1\lambda_2' + a_2\beta_2' + a_3 &= 0,\\
b_1\lambda_2' + b_2\beta_2' + b_3 &= 0,\\
c_1\lambda_2' + c_2\beta_2' + c_3 &= 0.
\end{aligned}\right\} \quad (18)$$

Eliminating λ_2' and β_2' we have

$$\begin{vmatrix} a_1, & a_2, & a_3\\ b_1, & b_2, & b_3\\ 0, & c_2, & c_3 \end{vmatrix} = 0. \quad (19)$$

Expanding the determinant, and carrying out the computation in the original quantities, it is found, after rejecting common factors, that

$$\left.\begin{array}{l} \mu\,\rho_2+\rho_1\,\{\sin\beta_1\sin\beta_2+\cos\beta_1\cos\beta_2\cos(\lambda_2-\lambda_1)\}+\nu\,\rho_2' \\ \quad -R_1\cos\beta_2\cos(\lambda_2-\odot_1)-\mu\,R_2\cos\beta_2\cos(\lambda_2-\odot_2) \\ \quad -\nu\cos\beta_2\,\{\,R_2'\cos(\lambda_2-\odot_2)+R_2\sin(\lambda_2-\odot_2)\,\odot_2'\,\}=0. \end{array}\right\} \quad (20)$$

Let

$$\left.\begin{array}{l} m=-\frac{\nu}{\mu}\,\rho_2'+\frac{R_1'}{\mu}\cos\beta_2\cos(\lambda_2-\odot_1)+R_2\cos\beta_2\cos(\lambda_2-\odot_2) \\ \qquad +\frac{\nu}{\mu}\cos\beta_2\,(R_2'\cos(\lambda_2-\odot_2)+R_2\sin(\lambda_2-\odot_2)\,\odot_2'\,\Big\}\,. \\ M=-\frac{1}{\mu}\Big\{\sin\beta_1\sin\beta_2+\cos\beta_1\cos\beta_2\cos(\lambda_2-\lambda_1)\Big\}\,. \end{array}\right\} \quad (21)$$

Then (20) becomes,

$$\rho_2=m+M\,\rho_1, \qquad (22)$$

where m and M are known quantities. As was shown, Euler's equation, (2), depends upon ρ_1 and ρ_2 alone as unknowns. Therefore, (2) and (22) may be used for the determination of the two geocentric distances.

The equation (20) has been purposely reduced to the same form as that which arises in the theory of parabolic orbits, based on observations of position alone. The quantities m and M are entirely different in this case, but, because of the form of the equations, the solution would not be different from that given in Oppolzer's *Bahnbestimmung der Kometen und Planeten*, I, pp. 303–307. Therefore, it is not necessary to consider it here.

When this work is carried out, two positions of the comet are known, and the elements can at once be derived by familiar processes. Thus, the formulæ and references given in this paper are sufficient for the determination of the first approximation to the parabolic elements of a comet's orbit, when the computer has at his disposal two observations of position, and one of the motion in the line of sight.

THE UNIVERSITY OF CHICAGO,
May 20, 1899.

THE INFLUENCE OF THE PURKINJE PHENOMENON ON OBSERVATIONS OF FAINT SPECTRA.

By W. W. CAMPBELL.

THOSE who are familiar with the details of the spectrum observations of the Orion Nebula will, on reflection, easily convince themselves that the Purkinje phenomenon has very little to do with estimates of the relative intensities of the three principal lines. There remains the equally important question as to whether it enters the problem to any extent whatsoever.

This phenomenon seems to be an organic color effect, arising from the observer's effort to compare, numerically, two unlike objects. Professor Runge's laboratory observations showed a small Purkinje effect when he compared spectral regions of wave-lengths $\lambda\lambda$ 4862 and 5007; but for the regions $\lambda\lambda$ 4862 and 4959, an effect which was at first apparent was later reduced to zero. Professor Runge's experiments were based upon light which was initially of "medium brightness;" but his published account shows that he was fully aware of the one important condition essential to a laboratory solution of this problem, viz., that the initial intensities of the artificial lines should equal those of the nebular lines.

Now the absolute brightness of the nebular lines is surprisingly low. [For an approximate measure of the intensity of the $H\beta$ nebular line, see my paper in the May number of this JOURNAL.] The nebular lines are so faint that many—and I think all—observers are unable to distinguish between their colors. They lie so near the limit of color perception that some observers do not see them as blue-green, but as gray. In this and similar cases, does the Purkinje effect enter at all? Great numbers of observations are made under these conditions and it is important that the question should be settled.

In 1889, Hering and Hillebrand observed that the distribution of brightness in a faint colorless spectrum, as viewed by normal eyes, is identical with the distribution of brightness in a spectrum of *every* degree of intensity, as seen by totally color-blind observers. [*Sitzungsb. d. Wien. Akad.*, 1889; *Wied. Ann.*, **62**, 17.]

About the same time, Professor A. König observed "dass die Vertheilung der Helligkeit im Spectrum in einzelnen Fällen auch dann ungeändert bleibt, wenn durch peripher oder central gelegene pathologische Processe die eigentliche Farbenempfindung völlig verloren geht und nur die Empfindungsreihe Schwarz-Grau-Weiss bestehen bleibt." [*Wied. Ann.*, **45**, 607, 1892.]

These observations demonstrated that the Purkinje phenomenon is dependent upon *color-perception;* and that its effect upon observations of the Orion Nebula made by those who see the lines as gray is absolutely zero.

There remains the case of those who are able to perceive that the lines have a blue-green tinge, but who cannot distinguish *differences* in their colors. If the Purkinje phenomenon is a differential-color effect—and so it seems to be—this case would be very closely related to the preceding one in which the effect is zero.

Even if an observer were just able to distinguish differences in the colors of the lines—which I am strongly inclined to doubt—the observations of König and others clearly indicate that the effect would be exceedingly slight, and safely negligible.

The critic who assumed that a twenty-five to thirtyfold observed variation "is nothing more nor less than the Purkinje phenomenon," wrote that "Although the lines in question differ but slightly in wave-length, their brightness is so close to the limit of visibility that great differences in relative intensity . . . . may be produced." [This JOURNAL, **7**, 238.] It appears to me that this case of lines situated in the blue-green, which differ but slightly in wave-length, and whose brightness is so close to the limit of visibility, is precisely the case where the principles of color-vision would have justified the prediction of exceedingly

slight Purkinje effects. In this connection, it may be said that the observations by Hering, Hillebrand and König, bearing on this question, are among the most important in the whole domain of Physiological Optics.

LICK OBSERVATORY,
May 12, 1899.

RESULTS OF THE PHOTOGRAPHIC OBSERVATIONS OF THE LEONIDS, NOVEMBER 14–15, 1898, AT THE YALE OBSERVATORY.

By W. L. ELKIN.

AS STATED in a previous communication (this JOURNAL, 9, 20) during twelve hours exposure on the nights of November 14 and 15, 1898, sixteen meteor trails were photographed at our two stations. Of these, eight proved to be Leonids, and eight meteors from other radiants. Seven of the Leonid trails have now been fully measured and discussed, one proving too short and at too great a distance from the radiant to be of any value, and a summary of the results is herewith presented.

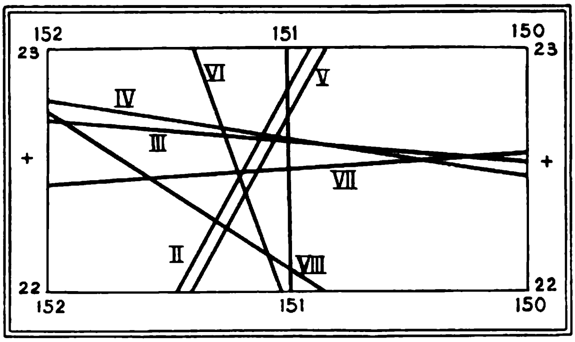
Five of the Leonid trails were secured with the apparatus at the Observatory, driven by clockwork, and the measurement of these plates presents no difficulty. The perpendicular distances of the track from all known stars within about 30′ were measured and the most probable track determined by least squares, the track being determined by the R. A. of its intersection with, and by its inclination to the equator, denoted respectively by "N" and "I."

For the plates taken at the Hamden station where the stars were allowed to trail for ten minutes, the perpendicular distances of the meteor tracks from the two ends of the star trail were measured. These two ends may be considered as two stars of the same declination as the star producing the trail, but with right ascensions differing from that of this star by t and $(10 - t)$ minutes, where t is the time between the moment of the meteor's appearance and that when the star began to trail.

The following table gives the results thus derived, the tracks being referred to the mean places of the stars for 1875.0.

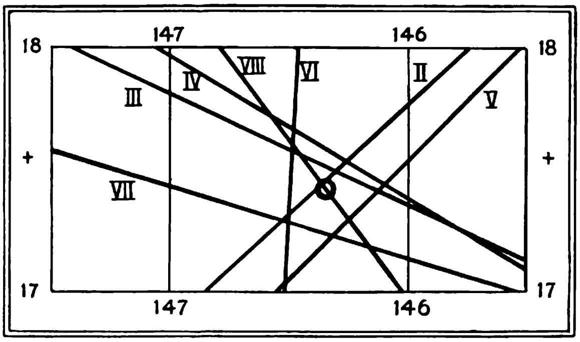
No.			Greenwich M. Time hr.	min.	N	I	Station
I.	Nov.	14	18	21.4	(Too short and distant)		Y
II.	"	14	18	15.7	341°46.3′	65°37.4′	Y
III.	"	14	20	56.9	74 26.8	23 16.4	Y
IV.	"	14	20	56.9	85 57.7	24 48.0	H
V.	"	14	20	36.8	342 17.9	65 4.8	Y
VI.	"	14	19	51.6	144 14.4	73 55.1	H
VII.	"	14	20	49.4	51 14.9	22 53.5	Y
VIII.	"	15	18	21.4	120 2.5	37 51.5	Y

A.



APPARENT RADIANT.

B



TRUE RADIANT. Long. and Lat.

I have corrected the prolongation of these tracks in the immediate neighborhood of the apparent radiant for the effect of the Earth's attraction and diurnal rotation, using for the

meteors the velocity derived from the period of* 33.25 years. The apparent tracks thus derived are shown where they intersect near the radiant in Fig. A, and the apparent radiant might be derived from them at once. But I have thought it of more interest to determine the tracks which result from a combination of these apparent tracks with the orbital motion of the Earth, and Fig. B shows these tracks in the vicinity of the true direction of the radiant in longitude and latitude. The point for which the sum of the squares of its distance from the seven tracks is a minimum is situated in longitude 146° 20.5′, latitude +17° 25.3′, Eq. 1875.0, and is shown by a circle having the size of its probable uncertainty. The longitude of the Sun for the mean of the seven times is 233° 9.4′, its radius vector logarithm 9.99505, and hence the mean orbit of the above seven meteors around the Sun, always assuming the period to be 33.25 years, is found to be:

Perihelion	1898, Nov. 10.354, M. T. Greenwich	
Long. of Perihelion -	46°45′	Eq. 1875.0
Node - - - - -	233 9	
Inclination - -	162 33	
Eccentricity - - -	0.9046	
Period - - -	33.25 years.	

The elements deduced by Oppolzer for Comet 1866 I are:

Perihelion	1867, Jan. 11.171, M. T. Berlin	
Long. of Perihelion -	42°24.0′	Eq. 1866.0
Node - - - - -	231 26.1	
Inclination - -	162 41.9	
Eccentricity - -	0.9054	
Period - - -	33.176 years.	

According to Berberich, the principal perturbations of the meteors arriving on Nov. 15, 1898 are:

In Long. of Perihelion	+110.1
Node	+ 50.2
Inclination	+ 42.7
Eccentricity	+ 0.0016

and the precession from 1866 to 1875 amounts to +7.3′, +7.4′, and +0.1′ for Perihelion, Node, and Inclination respectively, so

that we might expect changes in the direction actually found in general from Oppolzer's orbit.

Unfortunately only one of the Leonid meteors was secured at both stations, so that we can deduce but one height of apparition and disappearance. Trail III on the Observatory plates (the one shown on Plate I of Vol. IX) combined with IV gives for these two heights, 59.97 and 53.14 geog. miles, or 111.2 and 98.6 kilometers, a result in near accordance with that derived from the Perseids.

In this preliminary communication it seems hardly worth while to enter closely into the accuracy with which the radiant is derived from the photographs, but I may mention that, as can be inferred from the diagram A, the apparent radiant on Nov. 14 from trails II to VII, is located with a probable error of about $\pm 2'$ in each coördinate. The trail on Nov. 15, No. VIII, deviates considerably, but diagram B shows that this deviation is due mainly to the change of position of the Earth.

THE DIRECT CONCAVE GRATING SPECTROSCOPE.

THE ULTRA-VIOLET HYDROGEN SERIES.

THE GREAT NEBULA OF ORION.

By S. A. MITCHELL.

A NOTE has been published in this JOURNAL regarding the application of the concave grating to stellar photography.[1] The grating has been tried in several different ways, but the best results are obtained by using it as a direct grating spectroscope; that is, the light from the star falling directly on the grating, is diffracted and brought to a focus on the photographic plate.

This method was tried successfully at the Johns Hopkins University (*loc. cit.*). A small grating of one meter radius of curvature, with 15,000 lines to the inch, was employed. Although this grating had a ruled surface of only 1×2 inches, a photograph of Sirius was obtained which showed sixteen lines of the hydrogen series. This seemed so promising that a large grating was made, having a ruled surface of $2 \times 5\frac{3}{4}$ inches. This grating has a radius of curvature of one meter, and is ruled with 7219 lines to the inch. It was mounted on the $9\frac{1}{2}$-inch Hastings refractor of the Johns Hopkins University. Experiments were continued, the results obtained being very good, considering the situation of the Observatory. The telescope is on the sixth floor of the Physical Laboratory, which is continually subject to the jars, the dust, and the glare of the city.

In November 1898 an opportunity was presented, through the kindness of Professor Hale, of working at the Yerkes Observatory. The spectroscope was mounted on the 12-inch Brashear refractor, and experiments have been continued through the past winter and spring.

The mounting for the grating is exceedingly simple.

[1] POOR and MITCHELL, "The Concave Grating for Stellar Photography," this JOURNAL, 8, 157, 1898; *Monthly Notices*, March 1898.

consisting only of the ordinary grating holder with side and back adjustments, and a focusing apparatus for the plate-holder. By these means the spectrum is focused on the photographic plate, and the plate and grating made parallel, thus giving a "normal" spectrum (*loc. cit.*). The whole is inclosed in a wooden box clamped to the tube of the equatorial, which is used merely for the purpose of following the star.

The box is fastened to the tube in such a way that the ruled lines of the grating are parallel to the equator. The slight irregularities of the driving clock then do not alter the definition of the photograph. To insure good definition, it is necessary only to follow accurately in declination, and this is accomplished very easily by using the filar micrometer in connection with the equatorial. It is found[1] that for this grating the astigmatism amounts to 0.003 inches, and therefore, if the star were followed accurately, the resulting spectrum would be of this width. Consequently the spectrum was generally widened by regulating the driving clock so as to cause the star to trail in right ascension.

The above apparatus is extremely simple, and it is a remarkably easy undertaking to photograph the spectrum of a star.

Since the light from the star passes through neither objective nor prism trains, there is no absorption of the ultra-violet light, except the small selective absorption of speculum metal; and since no slit is used no light is cut off by this means. Consequently the photographic spectrum extends far into the ultra-violet. When we consider that the photograph is obtained in so simple a manner, and that the resulting spectrum is "normal," we perceive some of the advantages of the grating spectroscope.

With the large grating used, the spectra on both sides were about equally bright. The photographic extent of the first order was about 1¼ inches, that of the second order about 2½ inches, using Seed's Gilt-Edge plates. In the second order the distance from $H\beta$ to $H\gamma$ was about 0.6 inches; from $H\beta$ in the first order to $H\beta$ in the second the distance was 2.8 inches. The first and

[1] MITCHELL, "Theory of the Concave Grating," this JOURNAL, 8, 110, 1898.

second orders together had an extent of about 4 inches, and conquently, using a photographic plate 1 × 5, both spectra could be photographed on the same plate. The plates were bent as closely as possible to the proper radius (this JOURNAL, March 1898).

By adjusting grating and plate-holder so as to make them parallel, the second order spectrum was found to be "normal." These adjustments were very easily made.

Photographs were made of a large number of stars of different magnitudes, the results of a few of the plates being as follows:

Plate	Date	Star	Exposure	Width of spectrum	Results
	1898		m.	in.	
108	March 31	Vega	20	0.02	14 Hydrogen lines, K
115	April 2	Vega	10	.01	14 Hydrogen lines, K
133	April 6	Capella	40	.02	F, G, h. H. K. 125 lines between F and H. (Night very hazy)
136	April 8	Spica	40	.02	12 Hydrogen lines, K; 10 other lines
166	Dec. 11	Sirius	2	.005	14 Hydrogen lines, K
	1899				
177	Jan. 11	Capella	5	.01	Fully exposed
190	Jan. 18	Rigel	5	.01	21 Hydrogen lines, K; many other lines. (Measured for ultra-violet hydrogen series)
191	Jan. 18	Sirius	5	.01	Excellent definition. 75 lines between $H\beta$ and $H\gamma$
214	Jan. 31	Sirius	15	.04	
214	Jan. 31	Rigel	15	.02	Excellent definition. 21 Hydrogen lines
222	Feb. 6	Sirius	15	.03	
224	Feb. 6	Procyon	25	.02	
227	Feb. 7	Orion nebula	200	—	Hydrogen series as far as $H\rho$. Temperature —11° F.
230	Feb. 11	Orion nebula	210	—	Temperature —18° F.
238	March 6	ε Orionis	65	.01	
239	March 6	Regulus	34	.01	13 Hydrogen lines, K
240	March 6	ε Ursae Majoris	66	.015	14 Hydrogen lines, K
241	March 6	Spica	41	.015	
242	March 6	Vega	13	.02	15 Hydrogen lines, K
248	March 23	α Cygni	20	.02	18 Hydrogen lines; many fine lines
252	March 23	ε Ursae Majoris	30	.01	
253	March 28	Procyon	25	.02	
254	March 28	Regulus	40	.02	
259	March 31	η Ursae Majoris	80	.02	
273	April 28	α Aquilae	40	.02	Night hazy
275	May 1	Arcturus	30	.01	

Most of the photographs were taken by allowing the star to trail slightly in right ascension, thus widening the star's spectrum. Account must be taken of this fact in considering the length of exposure.

These plates show, (1) the large extent of the ultra-violet light, (2) excellent definition, (3) that the exposure necessary compares very favorably with that in other forms of spectrographs. To show the excellence of the definition, it may be stated that on photographs of Sirius about 75 lines were measured between $H\beta$ and $H\gamma$. These lines were very sharp and narrow. (The results of these measures will be communicated at a later date.)

Twenty-one lines of the ultra-violet hydrogen series were measured—the series therefore is extended as far as $H\chi$. The series was seen to extend farther than this, the head of the series being measured at λ 3646.1, but the dispersion was not sufficient to separate the single lines. Of the different white stars photographed, the hydrogen lines in Rigel were found to be the sharpest and the best for measuring. The following measures are the results from three plates of Rigel.

Measuring lines of known wave-length in different spectra, it was found that the spectra were absolutely normal, and these measures are accordingly made on that assumption.

The hydrogen series is found from the well-known law of Balmer. Using Rowland's values of the first two lines of the series—$H\alpha$ and $H\beta$, which are standards—the value of the head of the series is λ 3646.140. From the values of $H\alpha$, $H\beta$, and $H\gamma$, the value is λ 3646.138. From the first five lines of the series—$H\alpha$ to $H\epsilon$—the value of the head of the series comes out as λ 3646.143. The series was calculated from the value $\lambda = 3646.140$, and the results are given in the following table.

The first column contains the designations of the hydrogen lines, the second the values of the wave-lengths as measured from plates of Rigel, the third the values obtained from Balmer's law, the fourth column gives the difference between the measured values and those derived from the law.

ULTRA-VIOLET HYDROGEN SERIES.

Line	Measured λ	λ from Balmer's law	O—C
$H\beta$	4861.5	4861.5	0.0
$H\gamma$	4340.8	4340.7	+0.1
$H\delta$	4102.1	4101.9	+0.2
$H\epsilon$	3970.5	3970.2	+0.3
$H\zeta$	3889.2	3889.2	0.0
$H\eta$	3835.4	3835.5	—0.1
$H\theta$	3798.2	3798.1	+0.1
$H\iota$	3770.8	3770.8	0.0
$H\kappa$	3750.2	3750.3	—0.1
$H\lambda$	3734.4	3734.5	—0.1
$H\mu$	3722.0	3722.1	—0.1
$H\nu$	3712.0	3712.1	—0.1
$H\xi$	3704.1	3704.1	0.0
Ho	3697.4	3697.3	+0.1
$H\pi$	3691.5	3691.7	—0.2
$H\rho$	3686.9	3687.0	—0.1
$H\sigma$	3682.9	3683.0	—0.1
$H\tau$	3679.4	3679.5	—0.1
$H\upsilon$	3676.4	3676.5	—0.1
$H\phi$	3673.9	3673.9	0.0
$H\chi$	3671.0	3671.6	—0.6
Head of series	3646.1	3646.1	0.0

The residuals given in the fourth column speak for themselves, and show the value of the measures made with the grating used in this manner.

Probably the most interesting photographs, however, were those of the Great Nebula of Orion. Two photographs were obtained, one on February 7, the other on February 11. These are especially interesting on account of the controversy which has been going on in regard to the intensities of the different lines of the spectrum. Although Professor Keeler has definitely settled this question by means of his admirable photographs with color screens, still a few words on the present photographs may not come amiss. Photographs with a slitless spectroscope—an objective prism or a grating—show a spectrum made up of images of the object photographed. Thus, in the present photographs, the lines of the spectrum consist of a series of images of the nebula. Although the stars in the immediate neighborhood of the nebula give spectra which overlap the spectrum of the

nebula itself, still it is easy enough to distinguish the bright images of the nebula from the spectra of the stars.

There are marked differences in the appearances of the different images making up the spectrum. The plates used were Seed's Gilt Edge. The brightest image in the whole spectrum is that due to the $H\gamma$ radiation. This is a trifle brighter than the chief nebular line λ 5007 and $H\beta$, which appear on these photographs as a whole about equally bright, and of about the same intensity as the violet image λ 3727. The hydrogen lines, from $H\delta$ on, appear as a series gradually decreasing in intensity. The series has been traced as far as $H\rho$. Although the images are small—the focal length of the grating is about 20 inches—they are sufficient to show large differences in the appearances of the different lines. The violet line λ 3727 has the greatest extent. The faint outlying regions show in this line a greater intensity and a greater extent than in $H\beta$. The Huyghenian regions appear about equally intense in λ 3727 and $H\beta$, and of slightly less intensity than the same regions in $H\gamma$. The extent of $H\gamma$ is about the same as that of $H\beta$.

Of the greatest interest are the comparisons between the $H\beta$ line and the chief nebular line, there being marked differences of intensity in different regions. While in the Huyghenian region the line λ 5007 is brighter than $H\beta$, in the faint outlying portions $H\beta$ is clearly brighter than λ 5007. Taking into account the rapid falling off of the sensitiveness of Seed's plates from $H\beta$, we come to conclusions which are perfectly in accord with those of Professors Campbell and Keeler.

Portions brighter than the rest of the region, which show in these photographs as a double bright line, appear in the different images. By means of these it was possible to measure the wave-lengths with the accuracy shown in the accompanying table. In fact, in the fainter images of the spectrum, this double bright line was about all that appeared.

In the following table, in column 1 is given the line with its estimated intensity on a rather arbitrary scale, where 100 denotes the brightest line and 1 denotes a line just visible; column 2 gives the wave-lengths measured from the plate taken

February 7; column 3, those measured from the plate taken February 11. For the sake of comparison, in column 4 is given the wave-length of the hydrogen lines, and of the lines measured by Keeler (K)[1] and by Campbell (C)[2].

Possibly other lines than these given appear in the spectra, but these were the only lines measured with certainty on both plates.

WAVE-LENGTHS OF LINES IN THE GREAT NEBULA OF ORION.

Line and Intensity		February 7	February 11	Keeler and Campbell
	Chief Nebular (80)	5007.0	5006.9	5007.0 (K)
	(20)	4959.0	4959.0	4959.0 (K)
$H\beta$	(80)	4861.5	4861.5	4861.5
	(2)	4713	4713	{ 4716 (K) { 4713 (C)
	(1)	4665	4666	{ 466± (K) { 4664 (C)
	(1)?	4566	4566	....
	(3)	4471.9	4471.6	{ 4471.2 (K) { 4472 (C)
	(2)	4391	4390	4389 (C)
	(1)	4364	4364	{ 4365 (K) { 4364 (C)
$H\gamma$	(100)	4340.8	4340.7	4340.7
	(1)	4144	4144	4144 (C)
	(1)	4123	......	4121 (C)
$H\delta$	(20)	4101.6	4101.8	4101.8
	(1)	4067	4068	{ 4069 (K) { 4067 (C)
	(1)	4026	4026	{ 4026 (K) { 4026 (C)
$H\epsilon$	(10)	3970.1	3970.8	3970.2
$H\zeta$	(5)	3889.0	3889.1	3889.1
	(5)	3868.6	3868.4	{ 3868.9 (K) { 3869 (C)
$H\eta$	(3)	3835.6	3835.4	3835.5
$H\theta$	(2)	3797.9	3798.1	3798.1
$H\iota$	(2)	3771.2	3770.4	3770.8
$H\kappa$	(2)	3750.3	3750.2	3750.2
$H\lambda$	(2)	3734.5	3734.2	3734.2
	(80)	3727.5	3727.3	{ 3726.5 (K) { 3727 (C)
$H\mu$	(2)	......	3722	3721.9
$H\nu$	(1)	3712	3710	3711.8
$H\xi$	(1)	3706	3704	3704.0
Ho	(1)	3697	3698	3697.4
$H\pi$	(1)	3692	3691	3691.5
$H\rho$	(1)	3686	3687	3686.9

[1] *Astronomy and Astro-Physics*, **126**, 484, 1894.

[2] *Astronomy and Astro-Physics*, **125**, 392, 1894.

ABSOLUTE WAVE-LENGTHS.

The fundamental formula for the determination of wave-lengths (this JOURNAL **8**, 110, 1898), is:

$$\lambda = \frac{\omega}{N}(\sin \mu + \sin \gamma)$$

where λ is the wave-length, ω the grating space, N the order of the spectrum, μ and γ the angles which the line λ and the source of light respectively make with the axis of the grating, which is the axis of reference.

Absolute wave-lengths are obtained by the use of the grating in two ways, according as we make (1) $\mu = 0$, or (2) $\gamma = 0$. If we make $\mu = 0$, the line λ is then on the axis of the grating, or in other words grating and camera are parallel. The grating has been used in this manner in the above work. For variations of μ for different lines in the spectrum, such that $\cos \mu$ does not exceed unity, it is found that the spectrum is "normal" (this JOURNAL, **7**, 157, 1898). As noted above, the whole second spectrum was found to be "normal." Since we do not know for which line λ the angle $\mu = 0$, we have to find our absolute wave-lengths by taking advantage of the fact that the second order spectrum is "normal." This is done in the following manner:

As above, the first and second orders of spectra are photographed on the same plate. Take any two lines in both spectra for which we wish to know the absolute wave-lengths. Measurements on these lines will give us three quantities, viz., the distance from the first to the second order for the two lines respectively, and the distance between the two lines in the second order. From these three quantities the absolute wave-lengths may be obtained. The formulae necessary are easily derived, and the necessary calculations to obtain absolute wave-lengths are short. Theoretically this might seem to be a valuable way in finding motion in the line of sight. Practically, however, it is not. The method is a differential one, and the

small errors in setting made in the measurements are sufficient to spoil the theoretical beauty of this method.

The second method of determining absolute wave-lengths is not a differential method but a direct one. If we make $\gamma = 0$, the source of light is on the axis of the grating, and our fundamental formula reduces to:

$$\lambda = \frac{\omega}{N} \sin \mu.$$

To obtain absolute wave-lengths by this very simple formula, it is necessary only to adjust the apparatus so that the source of light is on the axis of the grating, or what comes to the same thing, the image of this source, formed at the focus, is on the axis of the grating. This adjustment is accomplished very easily by holding a candle at the radius of curvature of the grating and turning the grating until the candle and its image coincide. (This is the same adjustment as in Rowland's ordinary form of grating mounting.) If we put the center of the photographic plate on a line between the candle and the center of the grating, and make the plate parallel to the camera, the apparatus is in perfect adjustment.

This method was tried. The light from the star after just grazing the camera — so as not to cut off any light from the grating — falls on the grating. The grating is slightly tilted by the back adjustment screw. The light is brought to a focus on the photographic plate. At the center of the plate we will have the image of the star formed by direct reflection at the grating, and equidistant on either side the first order spectra of the star.

In the above formula, to find the angle μ for any line λ, we have simply to measure the distance of this line from the image of the star formed on the plate, which distance will give us directly the angle μ, since we know the focal length of the grating. If, instead of measuring the distance of the line λ from the central image, we measure it from the line λ in the other spectrum, we consequently measure double the distance, thus finding the angle μ, and so increase the accuracy of our measurements. Thus this method of finding absolute wave-lengths with

the determination of motion in the line of sight is a very simple one. The grating used in this manner does not give a normal spectrum.

Owing to the fact that the dispersion in the first order with the grating used was not sufficient for the accurate determination of motion in the line of sight, this method was not tried further than to show that the definition obtained is excellent, and that the central image is small enough to be measurable if necessary. Using a grating of larger radius of curvature we would have the double advantage of increasing the dispersion and also of being able to bend our plates more nearly to the proper radius. There would be the disadvantage of lengthening the exposure, but this will be overcome when larger gratings are ruled.

The above results were obtained by the use of a grating of $2 \times 5\frac{3}{4}$ inches, or a light gathering surface of $11\frac{1}{2}$ inches, which is less than the surface of the objective of a 4-inch telescope. Taking into account the exposure required as well as the excellent definition obtained and the accuracy of the measurements, we see the value of this form of spectroscope.

The advantages of the direct grating spectroscope may be summed up as follows :

1. Simplicity of the apparatus.

2. The ease with which a star can be accurately followed.

3. The spectrum is readily widened by regulating the driving-clock of the equatorial.

4. Great extent of the ultra-violet light.

5. Excellent definition.

6. The necessary exposure compares very favorably with that of other forms of spectroscopes.

7. The spectrum is normal.

8. Absolute wave-lengths, and consequently the motion in the line of sight, can be determined.

With a grating of say 10×15 inches, considering the exposure, the accuracy of the wave-lengths coming from good definition and a normal spectrum, and also the ease and accuracy

with which we could determine motion in the line of sight, the general results would probably be comparable in excellence with the results obtained by any spectroscope at present in use, including those attached to the Lick and Yerkes telescopes.

THE UNIVERSITY OF CHICAGO,
YERKES OBSERVATORY,
June 1899.

ON THE DISTRIBUTION OF THE ENERGY IN THE SPECTRUM OF THE BLACK BODY AT LOW TEMPERATURES.[1]

By F. PASCHEN.

MY observations on the energy spectra of different solid bodies[2] make it seem possible that the law derived by W. Wien[3] represents the emission of "the absolutely black body." In Wien's formula,

$$J = c_1 \lambda^{-5} e^{-\frac{c_2}{\lambda T}}, \tag{I}$$

where $J d\lambda$ is the energy between wave-lengths λ and $\lambda + d\lambda$ at the absolute temperature T, and c_1 and c_2 are constants, if we substitute λ^a for λ^{-5}, the value of a changing from body to body, my former observations are represented by the formulae. The value of a decreased from 6.4 to 5.2 in passing from reflecting platinum to strongly absorbing carbon.

The deviation of my former observations from the theoretical laws was such that that which was theoretically well-founded was not confirmed with certainty, while on the other hand that which was uncertain theoretically was rendered probable by the observations. The comprehensive formula of the law of emission is not supplied by theory in an unquestionable manner. If we assume the validity of the formula with the constant a derived from the experiments, then a must have the value 5, for Wien has proven that the intensity J_m of the maximum of energy varies as the fifth power of the temperature, which is the case in the assumed formula only when $a = 5$. In addition to this relation

[1] *Sitzungsberichte der Berliner Akademie;* Session of the physical-mathematical section on April 27, 1899.

[2] *Wied. Ann.*, **58**, 455, 1896; **60**, 662, 1897. (The latter will be referred to as *loc. cit.*)

[3] *Wied. Ann.*, **58**, 662, 1896.

$$\frac{J_m}{T^5} = \text{constant}, \tag{1}$$

two further relations are firmly established by Wien, or follow from his accurate derivations, viz.:

$$\lambda_m \times T = \text{constant}, \tag{2}$$

or the wave-length of the maximum of the energy curve for temperature T is inversely proportional to the temperature; and (3) the relation that the ratio of the intensity J of wave-length λ to the intensity J_m of the energy maximum of wave-length λ_m, or $\frac{J}{J_m}$, is a function of $\frac{\lambda}{\lambda_m}$, and for the energy curves at different temperatures always the same function of $\frac{\lambda}{\lambda_m}$. In a logarithmic representation all the energy curves must be congruent.

I found laws (2) and (3) to be confirmed. The experiments gave for the last relation (3) the function

$$\frac{J}{J_m} = \left\{ \frac{\lambda_m}{\lambda} e^{\frac{\lambda - \lambda_m}{\lambda}} \right\}^{a}, \tag{4}$$

which theory left indeterminate. It was only on making certain doubtful assumptions that Wien succeeded in deducing formula I, and we may regard formula (4) with a set equal to 5 as the result of Wien's research.

We therefore have to investigate whether this function (4) holds good with the value of $a = 5$, if we approach closely to the radiation of the absolutely black body. This seemed probable from my experiments, since the function was valid for radiating bodies of very different absorptive power, and since a had already taken the value 5 for the blackest body. The confirmation of relation (1), which according to theory must hold first of all, will be a crucial test whether the arrangements of the experiments sufficiently comply with the postulates of the theory; for the constant c_2 of formula I can be determined from the measures with sufficient accuracy only when this relation — in my experience the most difficult to realize — also holds good.

In attempting to answer these questions I first determined the energy spectra of cavities, the sides of which were heated by

baths. These experiments gave a somewhat closer approximation to law I than my former observations, as the relations 2, 3, and 4 appeared to be quite fully satisfied with $a = 5$, and as the observations were best represented by my previous formula with a value of about 5.2 for the exponent of λ; but it was impossible, however, to find the intensity of the maximum of energy exactly proportional to the fifth power of the temperature without overstepping the limits of error very considerably. The discrepancy might be due to the imperfect realization of the radiation of the black body, as the cavities had openings. When these were very much reduced, however, the emergent radiation still gave the discrepant result as before. The only thing in the experimental arrangements that could be held responsible for this was a variability of the absorptive power of the exposed bolometer strip with the wave-length. It therefore remained either to determine this or to so arrange the receiver of the radiation that it should constantly absorb the incident radiation as nearly completely as possible. I adopted the latter method and beg to show in what follows how far I have been able to approach to my aim. The experiments I communicate refer to the region of low temperatures and long wave-lengths. These experiments are the best adapted for judging of the blackening of the receiver, since my ordinary bolometers deviate most from my blackest ones just at this region.

ARRANGEMENT OF THE EXPERIMENTS.

The spectroscopic apparatus employed includes a fluor-spar prism loaned me by the firm of Carl Zeiss, which I had used before, and two silver concave mirrors of 35 cm focus, with precisely spherical figure, and so arranged that the astigmatism of the image was reduced as much as possible. The spectrum thus produced was so exceedingly sharp that the two broad absorption bands of aqueous vapor in the air of the room at $\lambda\, 6.0\,\mu$ and $6.5\,\mu$ were resolved into numerous sharply defined bands, while between them at $\lambda\, 6.26\,\mu$ was a place without appreciable absorption. The absorption band of carbon dioxide of the air

of the room appeared narrow like a line, but at its deepest part extinguished more than two thirds of the original energy.

All of the bolometers with which the results below were obtained were simply strips of platinum of $\frac{1}{2000}$mm thickness and most had a breadth of 0.5mm, corresponding to an angle of 5 minutes in the spectrum. The piece upon which radiation fell had as accurate a rectangular form as possible. The slit-width was altered until the energy curve of a line (or of the image of the slit when the prism was removed) became as nearly as possible an isosceles triangle, as the correction for the impurity of the spectrum due to the width of the slit can be simply calculated for this case.[1] The exposed surface of the bolometer was blackened either with lampblack or according to the Lummer-Kurlbaum[2] method with platinum black. The layer of black was given two or three times the thickness prescribed in their rule or employed in my earlier bolometers. The extremely slight thickness of the bolometer strip gives the advantage that in spite of the thick covering with black the galvanometer deflection, even with a period of six seconds, behaves just as on breaking a shunt across the branch of the bolometer if the conductors to the sensitive parts are screened by metallic diaphragms, and protected above and below from the latter for a space of some 0.5 mm.

To produce a further effect of blackening, the bolometer strip was placed, according to the principle proposed by myself (*loc. cit.* p. 722) with its middle exactly at the center of a reflecting hollow shell which had a small aperture for the admission of the radiation. Only that hemisphere of the shell was present on which the radiation reflected from the strip could fall, while the strip was so fixed that it could reflect toward all possible parts of the hemisphere. The frame of the bolometer could be moved by a micrometer until the strip covered its image. For reflecting hemispheres I used one of 45 mm diameter with poor polish and an inaccurate surface. A second one, cut exactly spherical by

[1] C. Runge, *Schlömilch's Zeitschrift für Math. und Phys.*, **42**, 205, 1897.

[2] F. Kurlbaum, *Proceedings of the Physical Society of Berlin*, p. 11. June 14, 1895.

Zeiss of Jena, of diameter 50 mm, had a splendid polish.[1] Both were of German silver. For the present research with the bolometer strips from 5 to 7 mm long and comparatively broad, it was sufficient to project the spectrum sharp in the plane of the strip. If one is to work with higher dispersion it is better to throw a sharp image of the spectrum in the plane of the slit of the hemisphere, and to make this slit equal to the image of a line. A very perfect arrangement in respect to the blackening of the strip and the projection of the spectrum is obtained if a bolometer is placed in the central plane of the hemisphere, of such width that the incident radiation does not fully cover its strip in length and breadth, and so that the radiation diffusely reflected from the strip always returns upon sensitive parts in spite of the aberrations for rays out of the center of the sphere. This also has the advantage of allowing, even with high dispersions, the use of a sensitive surface bolometer, which of course must be opaque for its whole breadth. The visual observation of the spectrum projected upon the exterior wall of the hemisphere is effected by a properly adjusted mirror. Since the bolometer strip reflected upon itself is heated by the current, and gets a great part of its radiation back again, the equality of the resistances of the bolometer will be disturbed. In order that the second strip should also get back at least a part of its radiation, I attached a plane silver mirror parallel to the central plane at the position of its image. With the strongest currents permissible, however (0.05 amperes), no such inequality of resistances arose to render the bolometer useless, so that the mirror was unnecessary.

I used for galvanometer a newly constructed instrument which is more sensitive than my former one,[2] and is rendered so far astatic that its directing force is chiefly due to the quartz fiber. Only in this way was it possible to work with so delicate a galvanometer at a place strongly affected by the earth-currents from a near by electric street railway.

[1] Viewed with a microscope magnifying about 150 times, the image of a fine thread in the center appeared as sharp as the thread itself.

[2] *Wied. Ann.*, 50, 417, 1893.

The radiating cavities were cylindric or pear shaped and of such size that the distance was from 10 to 13 cm from the rear wall to the aperture of about 1 sq. cm area. Only a part of the rear wall could send radiations through the suitably diaphragmed aperture. A larger vessel enclosed the hollow radiator, and served either to boil a liquid or to contain the vapors led in from a special vessel in which the liquid was boiled. The hollow radiator was always in the vapors. The arrangement for different temperatures was briefly as follows:

1. 100° C. The steam from water boiling in a flask was led into the enveloping vessel. Hollow radiators of copper oxide, lampblack and platinum black were investigated.

2. 190° C. Impure commercial aniline was boiled with a return condenser in the metallic enveloping vessel. The temperature of the cavity was determined each time by a thermometer certified by the *Reichsanstalt.*

3. 304° C. Arrangement the same as last, with impure commercial di-phenylamin. Cavities covered with copper oxide or lampblack.

4. 450° C. Sulphur was boiled in the jacket of a double-walled glass vessel with a condensing tube. The inner vessel constituted the cavity. Its surface, which had been roughened by etching, was covered by a thick layer of copper oxide. In several experiments a lampblack layer was put on over this, but it disappeared very quickly.

The glass vessel was closely surrounded by the metallic jacket. The flame played freely only under the boiling sulphur. With this arrangement the temperature of the interior of the cavity was from 449° to 451° C., according to the atmospheric pressure and the arrangement of the flame, being indicated by a mercury thermometer extending to 550° C. (certified by the *Reichsanstalt*) or by a thermo-element referred to this.

Differences of temperature as small as 1° C. with aniline, and 3° C. with sulphur, were determined by a thermo-element in the interior of the cavity, in some cases. The temperature was then taken at the point of the rear wall opposite the aperture. All of

the cavities could easily be provided with smaller apertures by the introduction of diaphragms. Some of the cavities in zinc were covered with platinum black by the reduction of a solution of 1 per cent. platinum chloride and 0.1 per cent. lead acetate on the zinc wall. The blackening thus obtained almost surpasses that of the electrolytic process of Lummer and Kurlbaum if the basic salts of zinc have been removed with dilute acetic acid. The cavities were placed in front of the spectroscope in such a way that the radiation from the aperture entirely filled a fixed diaphragm before the prism.

Energy curves only were observed, the sensitiveness of the bolometer in the different series being compared by the well-known shunt method of Ångström, or by comparative observations of a constant source of radiation, or by the measurement of the bolometer current. I proceeded along the spectrum to $\lambda\ 5\mu$ by steps of the width of the bolometer strip (an angle of about 5′), since the correction for impurity of the spectrum can then be very easily and accurately computed, as shown by Runge.

The first of the strong absorptions of aqueous vapor begins at 5μ[1]. At 6.26μ there is a narrow place between the two bands which shows no absorption in my spectrum, but at which the correction for impurity of spectrum reaches a considerable amount on account of the falling off of energy on the two sides. Beyond the second strong water absorption, a region begins at about 7.7μ, which exhibits as far as about 9.3μ no appreciable absorption by the air of the room, but does show absorption by the substance of the prism. In this region the four wave-lengths, 7.738μ, 8.246μ, 8.806μ and 9.324μ were investigated, account being taken of the absorption by the prism.

For the calculation of the normal energy spectrum, in which the energy contained in a constant, narrow range of wave-lengths is considered a function of the wave-length, I made use of my new determination of the dispersion of fluor-spar, in which the

[1] PASCHEN, *Wied. Ann.*, **53**, 335, 1894. Graphical representations of the energy curve covering these regions of absorption may be found in *Wied. Ann.*, **52**, Plate, Fig. 1; **51**, Plate, Fig. 2, Curve 1.

prism and grating spectra had considerably greater dispersion and sharpness than in my earlier determinations, so that the results had errors from two to three times smaller than before. My first measure[1] proved the more correct, and the accuracy of the determinations of the constants of Ketteler's dispersion formula was increased, so that the differential quotients of this formula which were used in the reduction have throughout the spectrum errors less than those of observation (at least in this research).

In addition to the corrections I have previously dealt with in detail, for width of the slit, for reflection from the surfaces of the prisms and mirror, and for the spectrum of the slide[2] in front of the slit serving as a zero point, the absorption of the prism was eliminated for the region from 7.7μ to 10μ (see below).

MEASUREMENTS.

I will cite only the result of one of the series of measures with an ordinary lampblacked bolometer, for which series the radiating cavities were arranged in the same way as for the later experiments. Other measurements with ordinary bolometers and cavities heated by baths gave similar results.

BOLOMETER STRIP WITH A THICK LAYER OF LAMPBLACK OF AN ANGULAR WIDTH IN THE SPECTRUM OF FIVE MINUTES.

Temperature		λ_m	$\lambda_m \times T$	J_m	$\frac{J_m}{T^5} \times 10^{14}$
304.0° C. =	577.0° Abs.	4.818	2780	1.546	2.421
190.5	463.5	5.964	2764	0.4083	2.105
99.9	372.9	7.393	2757	0.136	2.08

Radiating cavity of copper oxide.

The theoretical curve of formula (4) with $a = 5$ gives the above results from the observations. The product of $\lambda_m \times T$ is

[1] *Wied. Ann.*, 53, 301, 812, 1894.

[2] When this was let down an almost entirely closed cavity, containing a thermometer, presented itself to the slit. The intensity of its spectrum was computed for all the temperatures read and for all the wave-lengths, according to the principles of this research, and was then added to the observed intensity. The procedure is similar in testing the law of total radiation.

accordingly constant within the error of determination of λ_m,[1] but the intensity J_m of the wave-length λ_m of the maximum of energy, measured in arbitrary units, is not proportional to T^5 within the limits of error. Hence no certain conclusion can be drawn from these and similar measures.

BOLOMETER STRIP I, COVERED WITH A THICK LAYER OF LAMPBLACK, 5.0′ BROAD, AT THE CENTER OF THE POORLY REFLECTING HEMISPHERE.

Temperature		λ_m	$\lambda_m \times T$	J_m	$\frac{J_m}{T^5} \times 10^{14}$	
450.0° C. =	723.0° Abs.	4.009	2898	4.798	2.430	Lampblack in cavity[2]
...	...	...	...	...	...	
304.1	577.1	5.010	2891	1.547	2.416	Copper oxide in cavity
304.1	577.1	5.010	2891	1.544	2.402	
304.1	577.1	5.018	2896	1.567	2.438	
304.1	577.1	5.013	2893	1.553	2.419	Mean
...	...	...	...	...	...	
191.0	464.0	6.224	2888	0.5206	2.421	Copper oxide in cavity
191.0	464.0	6.216	2884	0.5260	2.446	
191.5	464.5	6.197	2879	0.5296	2.449	
191.2	464.2	6.212	2884	0.5254	2.439	Mean
...	...	...	...	...	...	
99.7	372.7	7.727	2880	0.176	2.45	Copper oxide in cavity
99.8	372.8	7.732	2883	0.173	2.40	
99.7	372.7	7.736	2883	0.175	2.43	
99.7	372.7	7.732	2882	0.175	2.43	Mean

The results of these measures show a distinctly better accord with the theoretical laws. Since the value of the product $\lambda_m \times T$ has been very greatly increased by the blackening of the bolometer, one cannot tell whether the arrangement already so far satisfies the theoretical postulates that this new value can be considered as valid within the errors of observation. This

[1] These errors are larger than in the later measures, as the course of the curve was not as accurately reproduced by formula (4) as with the black bolometers.

[2] In two further series, at 450° C., the layer of copper oxide in the cavity was too thin, so that it was somewhat transparent at short wave-lengths; and it was only with long waves, which glass emits strongly, that it was sufficiently black, so that I obtained larger values for $\lambda_m \times T$ (up to 2928).

value seems, moreover, to be slightly variable with the temperature. Further objections are discussed later. Therefore further measures were made in which the blackening of both bolometer and cavity was more perfect.

BOLOMETER STRIP II, 5.0′ WIDE, COVERED THICKLY WITH PLATINUM BLACK, IN THE POORER REFLECTING HEMISPHERE.

Temperature		λ_m	$\lambda_m \times T$	J_m	$\frac{J_m}{T^5} \times 10^{14}$	
450.0° C. =	723.0° Abs.	4.021	2907	3.707	1.877	Thick layer of copper oxide in cavity
450.0	723.0	4.018	2905	——	——	Same, but smaller aperture, covered with lampblack
450.6	723.6	4.007	2899	3.741	1.886	
	Mean		2904			
304.0	577.0	5.012	2892	1.200	1.869	Lampblack in cavity
191.0	464.0	6.223	2887	0.403	1.873	
[191.0	464.0	6.310	(2927)	0.393	1.826]	Glass cavity, with walls etched rough and not blackened [1]
	Mean of all		2894 [2]			

This bolometer could not bear the strength of current necessary for investigating the temperature of 100° C.

BOLOMETER STRIP III, 6.3′ WIDE, BLACKENED WITH PLATINUM, IN THE PERFECT HEMISPHERE.

Temperature		λ_m	$\lambda_m \times T$	J_m	$\frac{J_m}{T^5} \times 10^{14}$	
449.5° C. =	722.5° Abs.	4.019	2903	7.492	3.807	Copper oxide in cavity
304.1	577.1	5.009	2891	2.446	3.805	
190.7	463.7	6.227	2887	0.806	3.76	Platinum black in cavity
190.2	463.2	6.240	2891	0.806	3.78	
100.5	373.5	7.722	2884	0.279	3.83	
	Mean		2892			

[1] At large wave-lengths glass emits as strongly as lampblack. Since it is only in a slight degree transparent at 4.5μ, a glass cavity cannot be used even at 100° C. One gets too great a wave-length for the maximum of energy, and at short wave-lengths the energy curve falls off too abruptly, in so far as the bands of the spectrum of the heating vapors do not shimmer through.

[2] The mean at 450° C. was calculated once, the value 2927 not at all.

BOLOMETER STRIP IV, 5.0′ WIDE, BLACKENED WITH PLATINUM, IN THE PERFECT HEMISPHERE.

Temperature		λ_m	$\lambda_m \times T$	J_m	$\frac{J_m}{T^5} \times 10^{14}$	
450.5° C. =	723.5° Abs. (T)	3.998	2891	3.980	2.006	Lampblack in cavity
304.1	577.1	5.006	2889	1.219	2.012	
190.0	463.0	6.230	2885	0.422	1.984	Platinum black in cavity
190.7	463.7	6.230	2890	0.434	2.023	
99.8	372.8	7.742	2886	0.143	1.98	
	Mean - - -		2888			

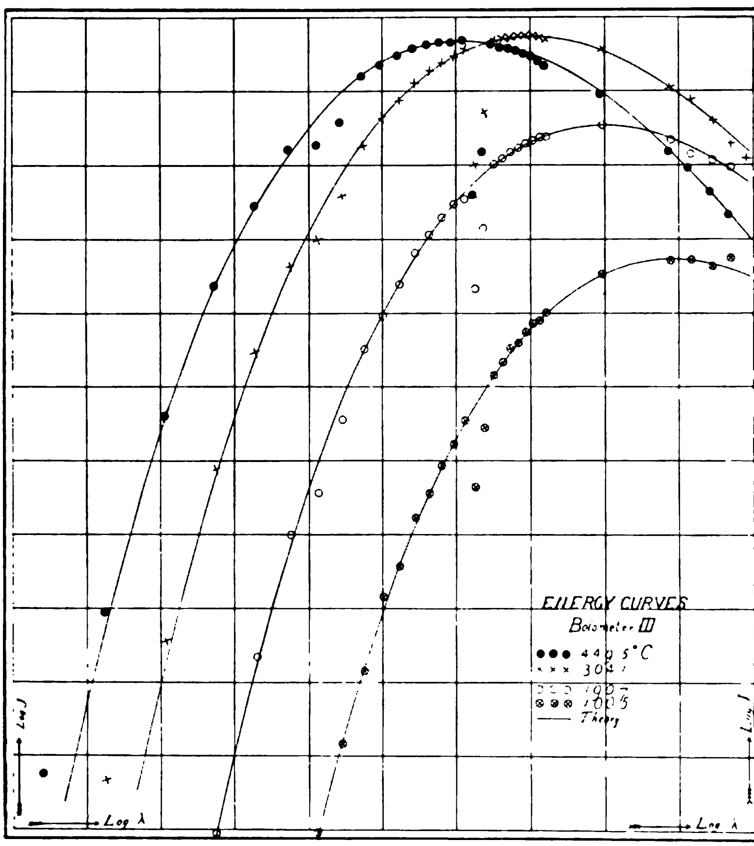
BOLOMETER STRIP V, 5.0′ WIDE, FIRST COVERED THICK WITH LAMPBLACK AND THEN WITH AN ELECTROLYTIC DEPOSIT OF PLATINUM BLACK, IN THE PERFECT HEMISPHERE.

Temperature		λ_m	$\lambda_m \times T$	J_m	$\frac{J_m}{T^5} \times 10^{14}$	
450.0° C. =	723.0° Abs. (T)	3.997	2890	4.207	2.121	Lampblack in cavity
304.1	577.1	5.012	2893	1.361	2.117	
189.4	462.4	6.249	2889	0.449	2.123	Platinum black in cavity
100.0	373.0	7.745	2889	0.153	2.12	
	Mean - - -		2890	- - -	2.120	

For judging of the accuracy of these results of the computations I would refer to the methods of reduction for the evaluation of the observed energy curves which are fully treated in my two previous papers. Here as there the most liberal use was made of the principle of the congruence of the energy curves represented logarithmically, which has been proven both theoretically and experimentally (Wien's third relation). Since formula (4) with $a = 5.00$ always reproduced the curve within the limits of error, excepting two or three unreliable extreme points at small wave-lengths, the value of λ_m and J_m could be determined with very great accuracy with a uniform regard of all the observed points. The figure gives the logarithmic representation, with $\log J +$ const. as ordinate and $\log \lambda$ as abscissa, of four energy curves obtained with bolometer III. The full line represents the curve of the formula (4) with $a = 5.000$, as it most closely fits the observations. The congruence is seen at once from the fact that this line is the same for all four curves.[1]

[1] In *loc. cit.* the energy curves were drawn one above the other in order to prove this.

Only the extreme points of short wave-lengths, where the measured energy at most amounts to $\frac{1}{100}$ of the maximum energy, deviate in the direction to cause a suspicion of diffuse light. This is entirely possible, since the maximum of energy of the observed prism-spectrum lies very near these points.



In order to exhibit the validity of law I in still another way, I have put together as "isochromatic lines" the intensities observed for the same wave-lengths from the four energy curves of the last series of measures (bolometer V), by treating $\log J$ as a function of $\frac{1}{T}$ for each wave-length. This was possible

for the reason that observed points of these series lay nearly at the same wave-lengths. An interpolation in the course of the curve would have been less convincing. Within the errors of measurement the isochromatic lines thus obtained were straight lines, and the values of c_1 and c_2 of formula I calculated from them agree, as was to be expected, with the more precise values of these constants given by the evaluation of the observations as energy curves, or hence given by the above tables.

The latter values are:

$$c_2 = \lambda_m \times T \times 5 = 2890 \times 5 = 14450$$

$$c_1 = \frac{J^m}{T^5}(\lambda^m \times T)^5 \times e^5 = 2.120 \times 10^{-14} \times 2890^5 \times 2.7183^5 = 634100.$$

The following table contains the summary of the observed values of log J included for the isochromatic lines. The calculation of the constants of the straight line for each wave-length yields the values of c_1 and c_2 given below them, the means of which (omitting the line at 1.887μ) are

$$c_2 = 14450, \qquad c_1 = 629100.$$

Since only a small number of observed points were included here, while all the observed points were taken into account in the calculation of the energy curves, we could not expect a better agreement in the value of c_1. The values derived from the energy curves are of course more accurate.

SUMMARY OF CERTAIN OBSERVED POINTS LOG J FOR ISOCHROMATIC LINES (BOLOMETER V).

T (Abs.)	$\frac{1}{T}$	λ=7.783	6.263	4.663	3.355	2.280	1.887μ
	0.00						
373.0	2681	0.184−1	0.132−1	0.846−2	0.155−2	—	—
462.4	2162	0.599−1	0.647−1	0.548−1	0.1305−1	0.043−2	
577.1	1733	0.951−1	0.0826	0.1240	0.9311−1	0.236−1	0.669−2
723.0	1383	0.2407	0.4375	0.5954	0.5775	0.1869	0.825−1
	c_2=	14440	14460	14450	14450	14450	14360 [1]
	c_1=	629400	636900	630300	629600	613700	738800 [1]

[1] These points at the extreme end of the rising branch are also here exhibited as lying too high (c_1 too large), and due to light of longer wave-length (c_2 too small.)

There is still one last consequence of the observations to be adduced in forming a judgment of the results obtained with the different arrangements of bolometers. I could not know on general principles whether a disturbing absorption of the air of the room would be present in addition to the absorption of the prism, in the region from 7.7μ to 10μ. As this subject has not been sufficiently investigated, since it was desired to include this region for the temperature 100° C., I have determined the factors for curves of higher temperature which were safely located by observations at short wave-lengths, with which factors the observed intensities beyond 7.7μ are to be multiplied in order that they should fit the theoretical curves. The factors thus determined therefore correct everything that would tend to make the observed intensities too small. I have on the other hand calculated by the law of absorption the factor which alone eliminates the absorption of the prism. It appeared that the factors for the blackest bolometers determined solely from the depression of the energy curve agreed thoroughly with the factors which corrected the absorption of the prism, so that it is improbable that there is any further absorption of any extent at the wave-lengths here observed. Lampblacked bolometer I in the poor hemisphere demands on the other hand still larger factors, which deviate increasingly with increasing wave-lengths, thus rendering necessary the conclusion that in this region this bolometer strip has less absorptive power than the others. The two following tables give information as to this:

I. DETERMINATION OF THE FACTORS FROM THE DEPRESSION OF THE ENERGY CURVE.

$\lambda =$	7.738	8.245	8.807	9.324
Bolometer I[1]	1.275	1.645	2.695	4.968
II	1.213	1.536	2.420	4.288
III	1.202	1.567	2.353	4.181
IV	1.205	1.563	2.388	4.436
V	1.217	1.555	2.449	4.454
Mean of II to V	1.209	1.555	2.403	4.340

[1] The energy curves of this bolometer are corrected with these factors.

2. CALCULATION OF THE FACTORS FROM THE ABSORPTION OF THE PRISM.

If the layer of fluor-spar of thickness l absorbs the fraction a, the emergent radiation from a prism of base B must be multiplied by the factor

$$\frac{\log (1-a)^{\frac{B}{l}}}{\left\{(1-a)^{\frac{B}{l}} - 1\right\} \log e}$$

to obtain the incident radiation, as may be seen from a simple integration based on the law of absorption. The prism had its corners chipped, and had by no means a mathematically simple shape[1]. Measurement of the base gave in the mean a value of 43.0 mm. In the following table δ is the minimum deviation of the prism, λ the corresponding wave-length, and a the amount of the absorption I observed in a clear plate of fluorite of 4.056 mm thickness. The factor computed is calculated from the absorption, the factor observed is that determined from the energy curves above.

δ	λ	a	Factor comp.	Factor obs.
25°19.1′	7.738μ	0.0365	1.211	1.209
24 29.1	8.245	0.0885	1.570	1.555
23 29.1	8.807	0.1805	2.404	2.403
22 29.1	9.324	0.3125	4.042	4.340

Not until $\lambda = 9.324\mu$ does the observed intensity have to be increased more than corresponds to the absorption of the prisms. This may be due to the absorption of the air of the room or to an erroneous determination of the absorption of the fluor-spar, but it has no bearing on our conclusions. The energy curves obtained with an ordinary lampblacked bolometer must lie too low even at short wave-lengths, representing a power of absorption of the lampblacked surface decreasing for increasing wave-lengths. The layer of lampblack (through which the radiation has to pass twice) is therefore still considerably transparent even

[1] In *Wied. Ann.*, 53, 333, 1894, I give the observations of the total loss of light, from which 5.05 per cent. is to be deducted for loss by reflection.

in a thick stratum, or else it has a rising reflecting power with increasing wave-length.[1] From several experiments the latter seems to me to be the case. Any such deviation at these long wave-lengths would be naturally expected, since the absorption of the lampblack bolometer strip in the reflecting shell can only be approximately equal to unity. Although the arrangement of bolometers II to V was different and represented an increase of the blackening (passing from the imperfect to the perfect hemisphere), nevertheless they absorbed almost equally in this spectral region, so that they seem to justify the conclusion that they all approximate "the absolutely black body" in this region so nearly that the casual deviation cannot produce appreciable error. I therefore believe that it is permissible to employ the results of the measurements with these bolometers in the determination of the constants of law I.

The constant c_2 is most accurately determined from these experiments by the fivefold value of the constant product $\lambda_m \times T$. This is in the mean:

For bolometer	II	III	IV	V	Mean of all.
	2894	2892	2888	2890	2891

The single values of a series of observations which deviate most widely from the mean of all are:

2907 (Bolometer II, 450°, cavity with layer of copper oxide);
2884 (Bolometer III, 100.5°, cavity with layer of platinum-black).

The first value is not entirely reliable, since it easily happened that some parts of the layer of copper oxide fell off, leaving the glass uncovered. This always gives occasion for an increase of the wave-length of the maximum.

I estimate the highest possible error of the mean of all as something like 3 per thousand.

Accordingly c_2 will be $= 5 \times 2891 = 14455$, with an error which at most I estimate at 40.

Since in formula I, $\frac{c_2}{\lambda T}$ represents a number, c_2 must have the

[1] K. Ångström has observed in thin layers an increasing transparency of lampblack with increasing wave-length. *Wied. Ann.*, 36, 720, 1889.

dimension: Wave-length $\times$ Temperature. Therefore

$$c_2 = 14455 \times \text{Degree Centigrade of absolute scale.}$$

The value of c_2 found by H. Wanner and myself[1] in a different region of temperature and wave-length by measurements of an entirely different kind was 14440, but it had considerably less accuracy.

The bolometric measurements at high temperatures recently published by O. Lummer and E. Pringsheim[2] yielded values of $\lambda_m \times T$ from 2837 to 2928, mean 2879, for one series, and for another series, in which the exponent 5.2 held good in the general formula I, five values between 2766 and 2986, mean 2876. From these they deduce for values of c_2, 14395 from the first series, and 14955 from the second series. Taking into account the errors possible according to the results of the observations and to the methods of reducing them, the agreement with my value seems to be good.

F. Kurlbaum[3] has measured the difference of the total radiation of a cavity coated with lampblack at 100° C. and at 0° in Watts $\times$ cm^{-2}, and has calculated that the heat equivalent of the total radiation emitted by a black body at the absolute temperature T amounts to

$$1.277 \times 10^{-12}\, T^4 \frac{\text{Gr. Cal.}}{\text{cm}^2 \times \text{sec.}}$$

or

$$5.32 \times 10^{-12} \times T^4 \text{ Watts} \times \text{cm}^{-2}.$$

By integrating formula I we get[4]

$$\int_0^\infty J\, d\lambda = 6\, \frac{c_1}{c_2^4}\, T^4 \qquad \text{(II)}$$

The constant c_1 has by formula I or formula II the dimensions

$$\text{Energy of radiation} \times (\text{Wave-length})^4.$$

Their numerical result depends, therefore, on the unit of the

[1] This JOURNAL, 9, 304, 1899.

[2] *Verhandlungen der Deutschen Physikalischen Gesellschaft*, February 3, 1899.

[3] *Wied. Ann.*, **65**, 746, 1898.

[4] *Loc. cit.*, p. 666, where for $\alpha = 5$, $\Pi\,(\alpha - 2) = 6$.

energy of radiation. In my experiments this was arbitrary and was different for the six different bolometers, so that a more precisely definite value of the quantity c_1 could not be computed from these experiments alone. In order to refer the measurements to a definite unit of radiation let us now determine the value of c_1, with which the total radiation of my source of radiation reaches the quantity determined by Kurlbaum, so that the energy of radiation is measured in

$\frac{\text{Gr. Cal.}}{\text{cm}^2 \times \text{sec.}}$ or $\frac{\text{Watts}}{\text{cm}^2}$. From the relation

$$\underbrace{6\,\frac{c_1}{c_2^4}\,T^4}_{\text{Second member of II}} = \underbrace{1.277 \times 10^{-12}\,T^4\,\frac{\text{Gr. Cal.}}{\text{cm}^2 \times \text{sec.}}}_{\text{Kurlbaum's Heat-equivalent}}$$

it follows that with the value $c_2 = 14455 \times$ Temperature degrees that

$$C_1 = 9292\,\frac{\text{Gr. Cal.}}{\text{cm}^2 \times \text{sec.}} \times \mu^4,$$

or

$$38710\,\frac{\text{Watts}}{\text{cm}^2} \times \mu^4.$$

With these values of the constants the equivalent in work for the radiation of the black.body of Temperature T, for any region of wave-length whatever, may therefore be computed by formula I.

Minor Contributions and Notes.

THE THIRD CONFERENCE OF ASTRONOMERS AND ASTROPHYSICISTS.

The committee charged with the selection of a time and place for holding a third conference of astronomers and astrophysicists met in the city of Washington on February 8, 1899, and by unanimous vote of the members present, Messrs. Newcomb, Morley, Hale, and Comstock, resolved that such a conference should be held at the Yerkes Observatory early in the following September, the precise date to be subsequently determined by Professor Hale. In accordance with this resolution, and at Professor Hale's invitation, the conference will be held at the Yerkes Observatory, Williams Bay, Wis., beginning on Wednesday, September 6, and closing on Friday, September 8.

In its plan and scope this conference will be similar to those held in 1897 and 1898 at Williams Bay and Cambridge, accounts of which have been published in this Journal and elsewhere. The committee charged with perfecting a plan for the organization of a permanent society of astronomers and astrophysicists to have charge of future conferences will present its report at this time.

A circular giving information regarding local arrangements will be issued shortly.

Geo. C. Comstock,
Secretary.

REVIEWS.

Photometrische Durchmusterung des nördlichen Himmels, enthaltend alle Sterne der B. D. bis zur Grösse 7.5. Von G. MÜLLER UND P. KEMPF.

Theil I. Zone 0° bis + 20° declination. 4to, pp. 501. Potsdam, 1894. Theil II. Zone + 20° bis + 40° declination. 4to, pp. 465, Potsdam, 1899.

PART I of this important work is contained in Vol. IX of the *Publicationen des Astrophysikalischen Observatoriums zu Potsdam.* The recent appearance of Part II, in Vol. XIII of the same series, calls for a review of both parts as a whole, though it is difficult to do justice, within the limits of a few pages, to such a thorough and comprehensive undertaking. The authors were at first inclined to confine their work to telescopic stars, beginning at the lower limits of the *Harvard Photometry* and *Uranometria Oxoniensis*, but finally determined upon the limits indicated in the titles to the volumes. The wisdom of this choice became more evident as the work progressed, since on its completion we will have the magnitudes of the northern naked eye stars and a large number of the telescopic ones, all measured in one homogeneous series with an accuracy not elsewhere attained. The importance of this work, both as a model and basis for future photometric studies, and as a foundation for researches on parallax and stellar evolution, can hardly be overestimated. Until the last twenty years stellar photometry has lagged far behind measures of position, and has hardly kept pace with spectroscopic studies. The authors express the opinion that in stellar astronomy measures of magnitude are of greater importance than measures of position, and that in the future, spectroscopic, colorimetric and photometric studies should go hand in hand.

The first important consideration in the plan of the work was the choice of that form of photometer best adapted to the purpose in view. Several pages of the introduction are devoted to a comparison

of the merits and demerits of a variety of instruments, and the reasons for the choice are given in so convincing a manner that no one can doubt their force. They pass over the astrometer of Herschel and the divided-objective prism photometer of Steinheil, as possessing only an historic interest; the latter, in spite of the valuable work done with it by Seidel, being out of the question on account of its limited scope, since the star images were not compared at the focus, but expanded into disks, thus confining its use to the brighter stars. The forms then left to consider are, the wedge photometer using the method of extinctions, and the two varieties of polarizing photometers using the method of equalities, the "Meridian Photometer" of Pickering, and the form devised by Zöllner. Taking these three in their order, the authors admit the many advantages possessed by the wedge photometer, holding as it does the first rank in simplicity of construction and convenience of use. It permits of measurements through a wide range of magnitudes; and, so far as depends on the determination of the point of extinction, the accuracy of the results is scarcely exceeded by any form of instrument. The great drawback to its use in an extended survey, like that proposed by the authors, is the impossibility of obtaining homogeneous glass of strictly neutral tint, all the specimens in use showing so strong a selective absorption as to cause large errors in the measurement of stars differing considerably in color. There is also the difficulty of determining accurately the constant of the wedge, and of eliminating the varying sky illumination from the measurements, so that the authors consider this form as not adapted to their purpose.

The two forms then left to consider were, first, the "Meridian Photometer," devised by Pickering, which brings the images of two stars to equality by a polarizing eyepiece; and second, the Zöllner form, in which the image of the object to be measured is compared with that of an artificial star, reduced by polarization to equality with the real one. The great advantage of the "Meridian Photometer" consists in the direct comparison of two equalized stars at the focus, a condition favorable to accuracy of measurement. Some of the disadvantages of this form are: the formation of two images by different objectives of possibly unequal transmissive power, which also may vary on account of dust, dampness, etc., the images not being as sharp as could be desired from one reflection, and the transmission of the rays through so many media; the loss of half the light by polarization;

the limitation of its use to the vicinity of the meridian; and the partial polarization caused by the reflection from the mirror used in front of the larger objectives. These many drawbacks led the authors to choose the Zöllner form in spite of the objections arising from the use of an artificial star whose image did not always closely resemble that of the real star, and the danger of variation in the intensity of the lamplight. Both these objections were carefully considered by the authors, who refer in this connection to the anomalous results obtained by Th. Wolff,[1] which in their opinion were caused by the use of too large a diaphragm, giving the artificial star a planetary rather than a stellar appearance, and causing differences of magnitude to be systematically measured too small. In the present work the authors avoided this source of error by a proper choice of diaphragms, and showed that as a result of this precaution it did not appear in their work, the proof being supplied by a series of observations in which this source of error was completely eliminated. The danger of variation in the intensity of the petroleum flame used for the artificial star was met by enclosing it so carefully as to completely protect it from currents of air, and, in addition, any possible effect was eliminated by measuring the same standard stars at the beginning, in the middle, and at the end of each zone. The authors consider that by the observance of these precautions the Zöllner photometer was best adapted to their purpose. The greater part of the work was done with two such instruments called *C* and *D*. The smaller one, *D*, was used with a Steinheil refractor of 13.5 cm aperture for stars fainter than the 6th magnitude. For the brighter stars, *C* was provided with objectives of 67 mm aperture (C_{I}), 36.5 mm (C_{II}), and 21.5 mm (C_{III}).

Of no less importance than the choice of instruments was the arrangement of the standard stars on which the catalogue magnitudes should depend, and no small part of the success of the work was due to the good judgment displayed in this particular. Polaris was considered as a standard, that the results might be directly comparable with the *Harvard Photometry* and the *Uranometria Oxoniensis*, but it was rejected for two reasons: its distance from most of the zone stars would introduce considerable and unknown errors from local irregularities in the transparency of the air; and also, since the larger part

[1] *Photometrische Beobachtungen an Fixsternen*, Leipzig, 1877; also, *Photometrische Beobachtungen an Fixsternen aus den Jahren 1876 bis 1883*, Berlin, 1884.

of the program stars were telescopic, the difference in magnitude between them and the standard would be greater than could be measured with accuracy. It was then determined to base the magnitudes on a list of 144 standard stars, distributed to the best advantage around the northern heavens. As the work planned was divided into four parts, including respectively the stars from Dec. 0° to + 20°, + 20° to + 40°, + 40° to + 60°, and + 60° to + 90°, the standards were distributed in the following manner: Nos. 1 to 48 encircled the northern hemisphere at Dec. about + 10° and at intervals of about 30 min. in R.A.; Nos. 49 to 96 in a similar way at + 30°, and the remaining 48 near + 60°. In each of these series the stars were alternately about 5.3 and 6.7 magnitude, the brighter being used with photometer *C* for the naked-eye stars, and the fainter with *D* for the telescopic stars. It was thus possible to compare each "zone" of program stars with two standards which closely included them in R. A. at about the same average Dec. and magnitude as the zone stars. This arrangement seems an ideal one for the work, making it possible to practically eliminate the systematic errors.

The thoroughness with which these standard stars were compared among themselves and a consistent system of magnitudes derived, excites our admiration. Each star was compared with its two neighbors in the same zone and with one brighter and one fainter star in each of the other zones, giving 6 values for each star and 432 combinations in all. The pairs were compared at nearly the same zenith distance and on only the best nights, so that the uncertainty in the correction for atmospheric extinction should be reduced to a minimum. The value of this correction (taken from Potsdam *Publications*, **3**, 285) to a measured difference of magnitude, never exceeded 0.13. The comparisons were mostly made with photometer *D*, with the aperture reduced so that the reading on the position circle of the Nicol prism for the fainter star was about 12°, therefore for the brighter star it did not exceed 40°, as no differences greater than 2.5 magnitudes occurred. The observations of the standards were made in series of 18 stars each, occupying 35 or 40 minutes, so that the observer was neither hurried nor wearied. The separate combinations were each measured eight times, four by Müller and four by Kempf. If the extremes of the values thus found differed by as much as 0.4 mag., the pair was remeasured by one observer, if by more than 0.5 mag., by both. When it is remembered that such a difference included the effect of

instrumental errors, of personal equation between the observers, and most dangerous of all, the effect of local inequalities in the atmospheric extinction, since the stars were often observed at widely varying azimuths, the precision of the results furnishes convincing evidence that these errors were well eliminated. On account of these precautions revision was necessary in only 93 cases out of 432. A difference greater than 0.3 mag. between a single measure and the mean of 8 occurred but 11 times in 3564 observations, less than 1/3 of 1 per cent., and *there was no residual greater than 0.4 mag.* The one source of possible error in this arrangement would seem to lie in the fact, already mentioned, that the corrections for extinction were taken from a mean table prepared for Potsdam, instead of being deduced from separate observations for each night. In the *Annals of Harvard College Observatory*, **23**, 91, Professor Pickering gives the separate values for the coefficient of extinction for different nights at Cambridge. The mean value is 0.25, but it varies from 0.19 to 0.90, though the statement is made that when the coefficient is above 0.50 the air is distinctly hazy. Such a large possible variation would seem to render the use of a mean value of doubtful propriety, or at least restrict the observations to the very best nights. That this latter condition is strictly fulfilled in the work under review is completely proven by the agreement of the separate values, the probable error of a single measured difference of magnitude being ± 0.073, that of the mean of 8 only ± 0.027, a degree of accuracy which attests the high rank to which the work is entitled.

The problem of deducing from these differences a consistent system of magnitude for the standard stars was solved by a series of approximations. The *B. D.* magnitudes were substituted in the six equations for each star and the means were again used as a second approximation. This process was continued till a further substitution would make no change in the result. Thus the two systems were made to exactly correspond, the average magnitude of the 144 stars being 6.02, both with the Potsdam and *B. D.* values. The individual differences varied from -0.76 mag. to $+0.89$ mag., which was considered by the authors to be due almost entirely to accidental errors in the *B. D.* estimates. The differences between the two systems, taken without regard to sign, averaged 0.27, and the sum of all the differences was 0.00. The authors are therefore led to believe that the relative brightness of the fundamental stars is determined within 0.05 mag.

An instructive result from the observations of the standard stars was the discovery that one of them, No. 56 (*B. D.* + 30° 591), was a variable of unusually slow rate of change. It was observed as a fundamental star 48 times, between 1888 February 25 and 1891 February 13, with very consistent results, the mean being 6.31 mag. It was used as a comparison star with the zones in Part II from 1890 November 22 to 1895 November 16, when its variability was first recognized, and then followed as a variable till 1898 March 20. The results are given in detail in *Astronomische Nachrichten*, **146**, 170, 1898,[1] and the following table of means is taken from Part II, p. 7:

Interval of time							Number of observations	Mag.
1888	Feb.	25	to	1889	Feb.	28	20	6.30
1889	Nov.	9	"	1890	Mar.	13	14	6.33
1890	Oct.	1	"	1891	Feb.	13	16	6.30
1893	Nov.	9	"	1894	Mar.	24	5	6.44
1895	Feb.	8	"	1895	Apr.	2	9	6.60
1895	Sept.	26	"	1896	Apr.	2	41	6.69
1896	Aug.	28	"	1897	Apr.	9	28	6.82
1897	Aug.	18	"	1898	Mar.	20	26	6.92

The zones with which this star was used were reduced with the other standard, No. 54 or 58.

Section 1 of Part I contains the separate results of the measures of each of the 432 pairs of standard stars, besides several tables giving method of reduction and the agreement of the resulting magnitudes.

Section 2 of Part I gives the observations in detail for the Zone 0° to + 20°. Every observed quantity is given, and nearly all those used in the reductions, so that the entire work can be easily checked. The arrangement is so excellent, and will be so useful as a model for future work, that it seems worth while to describe it fully. The amendments to the plan which were introduced in Section 1 of Part II will be mentioned in their proper place.

In each part of the work the program stars were divided into three groups, according to their brightness, from 2 to 4, from 4 to 6, and from 6 to 7.5 magnitudes, respectively. The first two groups were observed with photometer *C* and the fainter group with *D*. For convenience in use of the equatorially mounted refractor carrying photometer *D*, the third group was again divided into two sub-groups, according to declination, from 0° to + 10° and + 10° to + 20° in Part I,

[1] See also *A. J.*, **20**, 45.

and from $+20°$ to $+30°$, and $+30°$ to $+40°$ in Part II. Within these groups the stars were divided into zones of twelve each in Part I, which number was increased to fourteen in Part II. For each of these zones two standard stars were selected, which included it in R. A. and did not differ greatly in declination from the mean of the zone stars. The odd numbered standards, averaging 5.3 mag., were used in the measures with photometer *C*, and those with even numbers, averaging 6.7 mag., with *D*. By following this carefully considered plan, it was possible to observe the stars of a given zone at nearly the same zenith distance, generally between 40° and 50°, so that the correction to a measured difference in magnitude on account of the difference in zenith distance was usually less than 0.1 mag., and never greater than 0.2 mag.; and since the azimuths were so nearly alike the uncertainty in this correction from local irregularities in absorption were quite negligible. Thus the greatest element of uncertainty in photometric work was eliminated. Of somewhat less importance was the danger, inherent in the Zöllner photometer, of under-measuring differences in magnitude when the interval is large. This was avoided by the method of grouping the zone stars and using with each group the standards of similar brightness. In this way intervals greater than 2.5 mag. were nearly always avoided, in fact they were usually less than 1.0 mag. It is evident that the eminent success of the work is due in a great measure to the proper choice of the zone and standard stars, both as regards position and magnitude. The observing program, which was strictly adhered to, was as follows: Two standard stars, six zone stars, the same standards again, six more zone stars, and finally the two standard again. Müller and Kempf alternated as observer and recorder, and by this coöperation the time occupied on a zone was usually thirty-five minutes, giving ample time for certain identification and deliberate settings of the photometer circle in each of the four quadrants. As each series was short, the observer's eye was not wearied, and his perception of the equality of the images was not dulled at the end of the zone.

It was the original intention to observe each star four times, but it was soon found that a sufficient accuracy would be obtained by two. When these results differed by less than 0.3 mag. the mean was taken as the catalogue value. The discrepancy exceeded this limit in the case of 126 stars in Part I, and 161 in Part II, being 3.6 per cent. in each. These stars were arranged in "Revision Zones," and re-measured by both observers. The colors of the stars were taken from the *Potsdam*

Spectroscopic Durchmusterung for Part I, but for Part II independent estimates were made, dividing the interval between white and red into 18 steps. The allowable margin of divergence was placed at 4 steps, which was exceeded and revision required in case of 107 stars, or 2.4 per cent of the whole number.

The observations for Part I were made on 405 nights between 1886 October 1, and 1893 April 1, for Part II on 276 nights between 1890 September 18 and 1898 May 2. The number of program and revision zones was 601 and 59 in Part I, and 685 and 54 in Part II, resulting in catalogues of 3522 and 4416 stars, respectively. The heading of each zone gives the current number, date, photometer used, observer, list numbers of the two standard stars, and the quality of the seeing, expressed with the notation 1–4, the condition 1 meaning "unusually clear and quiet," and 4 meaning "medium." (In Part II these are explained as "excellent," and "bad," respectively.) The seeing was marked 4 in case of 5 zones in Part I, and 2 zones in Part II. In each zone the first column gives the *B. D.* number of the star, then follows the sidereal time of the observation, the four settings of the photometer circle to 0.1°, then the quantity J, which is the mean of the settings, the zenith distance to 0.1°, the quantity log $\sin^2 J$, and the same corrected for atmospheric extinction by the table in Potsdam III, 285. The extreme simplicity of the reductions arises from the light of the stars being proportional to the quantities $\sin^2 J$, and accepting, as the authors do, Pogson's ratio of magnitudes, the number whose log is 0.4, the differences in brightness are converted into magnitudes by dividing the differences in log $\sin^2 J$ by 0.4. At the foot of each zone are given the three separate means of the log $\sin^2 J$ for the standard stars observed at the beginning, middle and end, also the general mean of the three which is used in the reductions. Column 11 gives the difference between this logarithm and those for the zone stars; column 12, the difference reduced to magnitudes by dividing by 0.4, and the last column, the resulting magnitudes of the zone stars, obtained by applying these differences to the mean of the values for the two standards. Footnotes relate to the measurements of double stars, and any unusual condition of the atmosphere.

A good idea of the consistency and accuracy of the results can be gathered from a comparison of the three sets of measurements of the fundamental stars with each zone. Any change in the intensity of the lamp flame, or dulling of the observer's eye, would appear in a regular

progression in the residuals obtained by subtracting the general from the three separate means. This was done in Part I with the following results:

	Number of residuals Positive	Negative
Beginning of the zone - - -	319	303
Middle of the zone - - -	292	330
End of the zone - - - -	323	299

The algebraic sum and average value of the residuals were as follows:

	Sum of residuals	Average value
Beginning - - - -	+2.83	+0.005
Middle - - - -	−3.77	−0.006
End - - - - -	+1.05	+0.002

It appears from these figures that no progressive errors of the kind indicated were present in the work. The authors do not give the absolute values of these residuals, but a partial idea of their magnitude was gained from 36 zones, equally distributed over Parts I and II. The mean value was 0.031 mag. and 0.043 mag., and the largest individual differences 0.089 mag. and 0.138 mag. for the two parts respectively, the degree of accuracy shown being in good agreement with the results given by the authors. The probable error of the measurement of a pair of standard stars was ± 0.046 for photometer *C* and ± 0.038 for *D;* that of a single standard star was ± 0.065 and ± 0.053 for the same instruments. A comparison was also made of the magnitude differences of the pairs of standard stars from the zone work, with the values obtained from the original comparisons of these stars among themselves. The results are given in detail for each of the 96 stars in Parts I and II, and separately for the two photometers. The mean difference found by subtracting the latter from the former, for *C* and *D* in their order, was + 0.010 and — 0.010 in Part I and — 0.003 and + 0.005 in Part II. The absolute values without regard to sign averaged 0.056 and 0.059 in the two parts. Finally the difference between the observers in the sense *M*—*K* was + 0.022 for *C* and + 0.035 for *D* in Part I, and — 0.02 for each in Part II. It seems to the reviewer that a consideration of these extremely consistent results will do much to give Stellar Photometry a place among the exact sciences, to which, until lately, it could lay no claim.

The Catalogues themselves contain 50 stars on a page. The first five columns are alike in Parts I and II, giving the current number, the *B.*

D. number, the R. A. and Dec. reduced to 1900, and the *B. D.* magnitude. In Part I there are six more columns, containing the separate magnitude results of Müller and Kempf, the numbers of the zones from which they were taken, unless the star was observed more than twice, the color, the adopted magnitude, being the mean of *M* and *K*, and finally Bayer's letter or Flamsteed's number of the star, or for doubles, Struve's number. In place of these six columns there are four columns in Part II, giving respectively the zones in which the star occurs unless it was observed more than four times, the color, the adopted mean magnitude, and the designation from Bayer, Flamsteed or Struve.

The "Conclusions," filling about twenty pages at the end of each part, contain several very important discussions which, however, do not readily lend themselves to condensation for the purpose of review. Only brief mention can be made of their contents. The first is a comparison of the different photometers, leading to very interesting but not entirely satisfactory conclusions. In Part I there were 240 stars measured both with *C* (aperture 67 mm) and *D* (aperture 135 mm). In Part II, 143 stars were thus compared. This was done for the purpose of testing both the homogeneousness of the work and the presence of the tendency, peculiar to the Zöllner photometer, towards measuring differences of magnitude too small. The result in Part I was that the difference between the two instruments was vanishingly small and exhibited no progression. In Part II the difference *D–C* decreased from + 0.12 for stars of magnitude 5.27 to — 0.01 for magnitude 6.11. The divergence was in the direction to be expected. A similar test of 51 stars with photometers C_{II} and C_{III} (apertures 36.5 and 21.5 mm) showed differences varying from — 0.04 to + 0.03, the progression being in a direction opposite to that in the former test. The authors conclude that the peculiar tendency of this form of photometer manifests itself only with the larger apertures in which the images of the brighter stars are diffuse.

The comparison between observers was extremely satisfactory. In Part I the difference *M–K* varied from — 0.14 for stars brighter than 3.0 to + 0.11 for those fainter than 8.0, and excluding the extremes of magnitude the divergence was only from — 0.05 to + 0.06. In Part II the agreement was much closer, the differences lay between — 0.03 and + 0.03, the slight progression manifest in Part I entirely disappearing. The absolute values of the difference in both parts

was 0.12, giving for the probable error of a single observation ± 0.057, that of a catalogue magnitude from the mean of two measures, ± 0.040, which speaks volumes for the adaptability of the instrument and the skill of the observers. The above comparison is carried out separately for stars of the different colors, the results showing the personal equation between the two to be very small.

A comparison of greatest importance is given between the Potsdam and Bonn *Durchmusterungs*. The principal results appear in the following table, which is extracted from Table VI, page 454, Part II.

	Part I		Part II	
Magnitudes	Number of stars	*P.-B. D.*	Number of stars	*P.-B. D.*
< 3.0	9	+ 0.55 m	12	+ 0.34 m
3.0 to 3.4	21	+ 0.42	24	+ 0.22
3.5 3.9	26	+ 0.36	15	+ 0.22
4.0 4.4	42	+ 0.19	46	+ 0.25
4.5 4.9	45	+ 0.08	64	+ 0.13
5.0 5.4	108	+ 0.04	126	+ 0.05
5.5 5.9	167	— 0.06	164	— 0.01
6.0 6.4	327	+ 0.01	391	+ 0.09
6.5 6.9	658	+ 0.06	912	+ 0.19
7.0 7.4	1252	+ 0.02	1628	+ 0.26
7.5	680	— 0.03	904	+ 0.27

For the naked-eye stars there is a fair agreement between Parts I and II, showing that a difference of 1.0 mag., *B. D.* is equivalent to 0.823 mag. and 0.905 mag. in the Potsdam system and giving as the log of the *B. D.* magnitude ratio 0.329 and 0.362 in the two parts respectively. For the telescopic stars the divergence is much greater, the resulting logs of the ratio being 0.400 and 0.457. Fortified by other photometric researches the authors consider these differences to be due to a lack of homogeneousness in the *B. D.* estimates.

The final comparison is that between the Potsdam results and those of the three other photometric catalogues, the *Harvard Photometry* (*H. C. O. Annals*, Vol. XIV), called here "Pickering I," the Photometric Revision of the *Durchmusterung* (*H. C. O. Annals*, Vol. XXIV), called "Pickering II," and the *Uranometria Nova Oxoniensis*, called "Pritchard." Part I contains a close criticism of the three catalogues, and Part II gives the results of a comparison of the systems, arranged according to the color and magnitudes of the stars. The mean results

are given here, condensed from Table VIII, page 459, the differences being taken in the sense Potsdam — the others :

	Part I		Part II	
	Number of stars	Diff.	Number of stars	Diff.
Pickering I	791	+ 0.17 m	799	+ 0.18 m
Pickering II	801	+ 0.13	984	+ 0.16
Pritchard	691	+ 0.13	686	+ 0.14

No progression with the magnitude is shown except in the comparison with Pickering II, where it is indicated that 1.0 mag. in the Potsdam system is equivalent to 1.062 mag. in the system of the "Photometric Revision." But when arranged according to color the case is entirely different, so much that if two stars, one white and the other yellow, were measured equal at Cambridge or Oxford, the yellow star would appear 0.3 mag. or 0.4 mag. brighter than the white in the Potsdam measures. While accounting for part of the difference by the "Purkinje" phenomenon, the authors do not consider the question as finally settled.

The appearance of the remaining parts of the work will be awaited with great interest, and one cannot but echo the hope expressed by the late lamented Lindemann, that when completed the catalogue may be issued separate from the zone observations.

J. A. Parkhurst.

Kollektivmaasslehre von Gustav Theodor Fechner, im Auftrage der königl. Sächsischen Gesellschaft der Wissenschaften herausgegeben von Gottl. Friedr. Lipps. Leipzig, 1897. (VIII + 483 pages, 8vo.)

Gauss's well-known formulae for the discussion of errors of observation constitute the so called method of least squares. The chief use of this method is to furnish some criterion for the divergencies of observations that ought not to be regarded as due to ordinary accident. These when sufficiently numerous lead to careful search for their proximate cause, and in some cases to the discovery of previously unknown causes or laws. Gauss's reasoning is not, however, as many seem to think, deductive and necessarily valid ; it rests on two assumptions ; first, that the arithmetic mean of empiric data is the most probable value ; second, that a positive error and a negative error of equal

numerical magnitude are equally probable. These assumptions Fechner calls in question, and attempts a recasting of the theory.

Not alone observations of physical or astronomical magnitudes, but statistics of all sorts are to be treated. For simplicity it is presumed that the data refer to things nearly enough alike to be classified together. For such sets of objects the name *Kollektiv-Gegenstände* is introduced. If the Gaussian hypothesis were applicable, the measurements of a sufficiently numerous set ought to lie half above, half below the arithmetical mean, *i. e.* the distribution would be symmetrical, and increasing the number ought to bring about a closer approximation to symmetry. Now in some sets of data Fechner found such symmetry lacking, and the asymmetry grew more evident with increasing number of data. For such cases, and he believes them to be more common than the symmetrical kind, Fechner proposes to regard as the most probable value *a priori* that which may be called the *densest* value, that value which corresponds to a maximum ordinate in the curve of frequency. Variations are commonly assumed to be small as compared with the magnitudes measured; but for the case of large variations his proposal is to use the logarithms instead of the natural numbers. On this basis is derived his "Generalized Gaussian Law," applicable to all sets of data where the variations are properly considered fortuitous, even those essentially asymmetrical and showing wide variation, if the curve of frequency presents only a single maximum.

Among the consequences of the generalized law perhaps the most interesting is the relation between number and magnitude of positive and negative variations from the density center. In the symmetrical case, the number of positive variations and their sum would exactly equal those of negative variations; in the general case this equality is replaced by proportionality, and the number of variations must be squared :

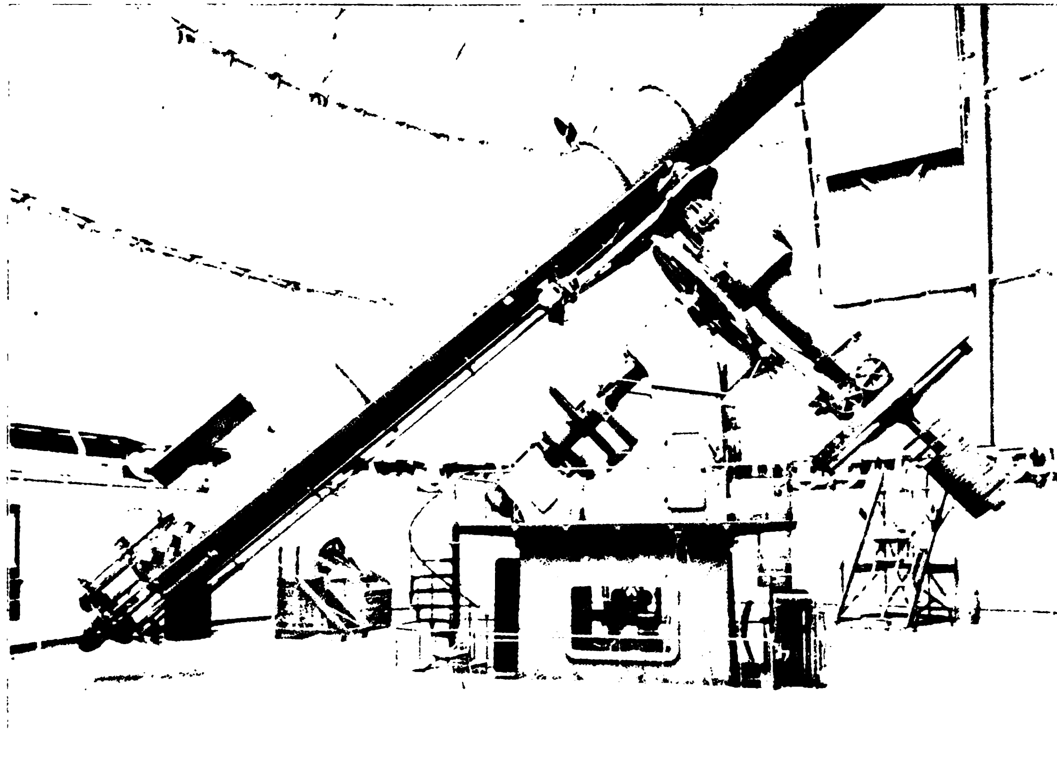
$$m'^2 : m_1^2 :: \Sigma\delta' : \Sigma\delta_1.$$

This property of the density-center may be adopted as the mathematical definition of the term, as Fechner remarks (p. 70). Otherwise the center of density must be found either roughly by platting or more exactly by the use of a series to represent the frequency-function, whose maximum will then be determined by the usual method. Explicit rules are given for all the operations described, and practical illustrations fully worked out.

As indicating the broad empirical basis of this work we note the range of statistical material employed, (pp. 28–29): Saxon recruits (over 16000), skull measurements (500), weights of internal organs of the human body, 350 heads of rye, records of daily temperature and rainfall, dimensions of *cartes de visite*, and of frames in several galleries of paintings. Among many valuable additions by the editor is a discussion based on data from the Strassburg Observatory, showing that errors of observation do not require the hypothesis of essential asymmetry (p. 456). The fact that most summer days are cooler than their average, winter days warmer, confirms the general belief. Interesting facts and suggestions are sprinkled liberally through the book, which is written, moreover, in a style as free as possible from mathematical technicalities. For one wishing to form a hasty acquaintance with the work, the first six chapters (98 pages) are especially to be recommended.

HENRY S. WHITE.

PLATE I.



The Forty-Inch Telescope and Stellar Spectrograph.

THE

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ON THE SPECTRA OF KRYPTON.

By C. RUNGE.

SPECTROSCOPICALLY krypton bears a close analogy to argon. Like argon it emits two different line spectra, one with Leyden jar and spark gap in the secondary circuit of an induction coil, the other without Leyden jar and spark gap. As in the case of argon this latter spectrum consists on the whole of less refrangible lines, and I believe, if it were possible to fill a vacuum tube with pure krypton, the color of the tube would change from yellow to blue, when the Leyden jar and the spark gap are interposed. But as far as I know there are no means as yet of getting rid of the admixture of argon. I prepared the gas after the prescriptions of W. Ramsay and Morris Travers.[1] Professor E. Warburg kindly let me have about ¾ liter of liquid air condensed in the Linde machine of the Physical Institute of Berlin. It is well known that liquid air may be kept for a considerable time in an open silvered vacuum vessel,[2] by which the influx of heat and consequently the evaporation of the liquid are greatly reduced. In an open vessel of this kind I brought the liquid

[1] W. RAMSAY and MORRIS TRAVERS, *Proc. Roy. Soc.*, **63**, 405, 1898.

[2] J. DEWAR, *Proc. Chemical Society*, Jan. 14, 1896.

air in my hand from Berlin to Hannover. It lasted four days before all was evaporated. The last fifteen liters were caught in inverted flasks over water. They contained about eighty-five per cent. of oxygen. A few liters of this supply were set apart and from the rest the oxygen was removed partly by metallic copper and partly by pyrogallic acid. I have to thank Professor Eschweiler for helping me in some of these operations. After removing the oxygen, the nitrogen had to be got rid of. Before proceeding to do this I wanted to see what the spark spectrum looked like. I took the very last that had evaporated, removed the oxygen and through the remaining sixty cubic centimeters of gas I passed a spark. The gas exploded violently. I repeated the experiment with the next but last that had evaporated and the gas again exploded, shattering the flask as in the first case. I repeated it a third time with a stronger flask. This time the flask withstood the explosion, but the large beaker glass containing a weak solution of potash into which the neck of the inverted flask dipped, was broken by the pressure propagated through the solution, although there was a considerable amount of free surface all around the neck of the flask. I then tried to explode the gas again over mercury at a reduced pressure, in order to measure the amount of contraction after explosion. But I did not succeed in making the gas explode again. The explosive ingredient, therefore, must have a smaller tension then either krypton or argon, as its percentage is increased relatively to the percentage of krypton and argon by the process of evaporation. But I am at a loss to explain of what it consists. There is an observation of Theodore de Saussure (*Ann. de Chim. et Phys.*, **44**, 52 and 53), who found that 2000 parts of air, from which all carbonic acid has been removed, when exploded with pure hydrogen contain one part of carbonic acid. There is another observation by Boussingault (*Ann. de Chim. et Phys.*, **57**, 148) that the air contains hydrogen, and lately Armand Gautier (*C. R.*, **127**, 693, 1898) has confirmed these observations as regards the air near human habitations, while he found that pure sea air contains free hydrogen to the extent of about 15 cubic

cm in 100 liters. So far as I can see, however, these facts alone do not explain the explosion.

The remaining gas was now mixed with the supply rich in oxygen that had been set apart and sparked for several days over a weak solution of potash. The last sparking was done with a surplus of oxygen until no further appreciable contraction took place and no traces of nitrogen lines were to be seen in the spectrum of the spark. After removing the oxygen by means of pyrogallic acid about 30 cubic centimeters of the gas remained. At atmospheric pressure the condensed spark between platinum electrodes showed besides some platinum lines a great number of argon lines. In the less refrangible part the principal lines of the red spectrum of argon were to be seen besides the principal lines of the blue spectrum of argon and traces of the green and yellow krypton lines. In the more refrangible part photographs of the spectrum show the "white" spectrum of argon[1] and the stronger lines of the second of the two krypton spectra described below. The argon lines are mostly widened and rather diffuse. In the vacuum tube without Leyden jar and with not too low pressure the krypton lines come out bright. At the same time the carbon bands are very conspicuous. I believe their origin is the same gas that Ramsay and Morris Travers have called metargon. A. Schuster has already called attention to the fact, that the spectrum of metargon as described by Ramsay and Morris Travers seems to be identical with the spectrum of carbon.[2] It is indeed remarkable, as Ramsay and Morris Travers have pointed out, that if metargon is a compound of carbon, it should not be absorbed by sparking it with oxygen over a solution of potash. I have convinced myself, however, that the bands in my vacuum tube coincided accurately with the carbon bands, although the gas was also sparked with oxygen over a solution of potash. In the following list I give in the first column the wave-lengths of a number of edges determined from the neighboring argon lines

[1] See J. M. Eder and E. Valenta, *Denkschriften der Wiener Akad.*, 1896.

[2] A. Schuster, *Nature*, 58, 199, 255, 269.

and in the second column the determinations of H. Kayser and myself of the edges of the carbon bands in the spectrum of the electric arc.

Vacuum tube edges of bands	Kayser and Runge edges of carbon bands electric arc
4684.92	4684.99
4697.63	4697.62
4715.37	4715.36
4737.20	4737.23
5129.29	5129.44
5165.27	5165.38
5540.38	5540.92
5585.32	5585.56
5635.37	5635.49

The differences may well be due to errors of observation, as it is more difficult to determine the wave-length of an edge than the center of a symmetrical line. I think there can be no doubt that these bands are the same as those in the electric arc. As regards the intensities of the lines composing the bands I have, however, observed a considerable difference between the bands in the vacuum tube and in the electric arc. In the green band 5165 of the electric arc there are a series of weak triplets between strong close doublets (see the photograph given in Kayser and Runge's article, *Abhandlungen der Berliner Akademie*, 1889). In the vacuum tube the triplets are much stronger and the doublets much weaker than in the electric arc. The dispersion of the short focus concave grating with which I have been working is not great enough to study these differences satisfactorily and I have therefore not followed out the subject at present. I do not think it possible that the carbon bands are due to impurities introduced after sparking the gas. For impurities caused in the mercury pump never, as far as I know, produce the carbon bands, but invariably produce the so-called bands of carbon monoxide, which were not to be seen in my vacuum tube. I think it most likely that there is a combination of argon and carbon that is able to resist the sparking with oxygen. I noticed that the carbon bands are to be seen on photographs

that F. Paschen and myself have taken of the red spectrum of argon. Here also the gas was sparked with oxygen over a solution of potash, nevertheless the carbon was not removed. The cyanogen bands also made their appearance in the krypton vacuum tube and all the lines of the red spectrum of argon. Besides I noticed some other bands of which I do not know the origin. They have nothing to do with krypton as they were also observed in argon tubes. At first some of the nitrogen bands were to be seen; but they disappeared after I had run the tube for some time. With low pressure the carbon bands are greatly reduced in intensity; the krypton lines are also weakened and the lines of the blue spectrum of argon make their appearance. With a Leyden jar and a spark gap the spectrum of krypton changes as well as the spectrum of argon. The new lines are mostly in the blue part of the spectrum.

SPECTRUM OF KRYPTON EMITTED BY A VACUUM TUBE WITHOUT LEYDEN JAR AND SPARK GAP.

Wave-length	Intensity[1]	Number of determinations	Mean error	Lines mentioned by Ramsay and Morris Travers	Remarks
4274.09	4	7	0.02	......	
4318.70	2	5	0.03	4317	
4319.760	4	7	0.013		
4362.76	2	5	0.02	......	
4376.24	3	5	0.02	4387	
4400.05	1	2	0.03	......	
4454.07	4	8	0.03	......	
4463.82	5	8	0.03	4461	
4502.43	4	8	0.03	......	
4624.46	1	4	0.04	......	
4671.42	2	3	0.03	4671	
....	..	..	....	4736	There are some krypton lines in this neighborhood when a Leyden jar and spark gap are used
....	..	..	....	4807	
....	..	..	....	4830	
....	..	..	....	4834	
5562.363	4	7	0.020	5560.6	
5570.417	8	8	0.015	5568.8	
....	..	..	....	5829	Not seen
5871.071	8	6	0.018	5867.7	
....	..	..	....	6011	Not seen
7587.48	3	3	0.06	......	
7601.47	4	3	0.11	......	

[1] The scale of intensity is weakest : 1 ; strongest : 10.

SPECTRUM OF KRYPTON EMITTED BY A VACUUM TUBE WITH LEYDEN JAR AND SPARK GAP.

Wave-length	Intensity	Number of determinations	Mean error	Remarks
3654.11	3	3	0.03	
3686.26	1	3	0.035	Diffuse
3741.85	3	3	0.02	
3778.29	4	2	0.04	
3783.40	4	2	0.03	
3912.36	1	2	0.10	Diffuse
3920.59	1	2	0.10	Diffuse
4057.16	2	2	0.035	
4065.19	3	3	0.05	Diffuse. Argon ?
4088.53	6	3	0.07	
4145.27	3	3	0.035	Possibly sulphur 4145.266 (Eder and Valenta)
4293.10	5	3	0.02	
4318.22	2	3	0.07	
4355.62	5	4	0.04	
4436.96	2	4	0.05	
4464.11	1	3	0.04	Diffuse
4577.31	4	4	0.02	
4615.48	4	4	0.02	
4619.30	5	4	0.03	
4634.07	4	4	0.02	
4680.67	3	4	0.05	
4694.82	2	3	0.05	
4702.73	1	2	0.02	
4739.13	5	4	0.035	
4762.66	2	2	0.06	
4825.38	1	2	0.15	
4832.22	2	2	0.03	
4844.58	1	2	0.035	
5208.57	1	2	0.04	
5292.37	2	2	0.05	
5419.38	2	2	0.025	

To select the lines due to krypton I compared the photographs with photographs taken some years ago by Paschen and myself of the spectrum of argon. On each of the two plates to be compared I removed the gelatine on one side of a straight line, cutting the lines of the spectrum at right angles. The plates were then laid together in such a manner that the parts where the emulsion was left were in contact along the cut but on different sides. In this way the plates can be examined under the microscope and the lines that exist only on one of the plates are detected. None of the lines were measured visually. The

photographs cover the region from $\lambda = 2400$ A. U. to $\lambda = 8000$ A. U. The wave-lengths were interpolated, using the argon lines, some mercury lines and the D lines as standards. For the wave-lengths of the argon lines H. Kayser's[1] measurements have been used and some unpublished measurements that F. Paschen and myself have made by comparison with iron lines. In the least refrangible part I used my own measurements[2] of the argon lines, which are also based on Kayser's argon lines of the second order. The photographs were taken with a Rowland concave grating of one meter radius. The mounting has been described in the paper on the series spectra of oxygen, sulphur and selenium.[3]

These lists are, I presume, very far from complete. As long as krypton is so strongly diluted with argon the weaker krypton lines are likely to escape notice, or may even not appear at all.

The analogy of the first spectrum of krypton to the red spectrum of argon is further borne out by the fact that the wave-numbers of several pairs of lines show equal differences. For simplicity's sake I have in the following table not corrected to *vacuo*, as the correction does not affect the difference of wave-numbers to any appreciable extent.

λ	$1/\lambda$	Difference
4274.09	23396.79	945.42
4454.07	22451.37	
4318.70	23155.12	944.89
4502.43	22210.23	
5562.363	17977.97	945.30
5871.071	17032.67	
	Mean	945.20

The deviations from the mean are : 0.22, 0.31, 0.10, which correspond to the differences in wave-length : 0.04, 0.06, 0.03. These differences are well within the limits of error.

[1] H. KAYSER, this JOURNAL, 4, 1896.

[2] C. RUNGE, this JOURNAL, 9, 281, 1899.

[3] C. RUNGE and F. PASCHEN, this JOURNAL, 8, 70, 1898.

RADIATION FROM A PERFECT RADIATOR

By W. E. Wilson.

ALTHOUGH Kirchhoff introduced the conception of a perfectly "black" body in the deductions of his well-known law connecting the emissive and absorptive power of a body in regard to radiant heat, he seems not to have investigated the subject experimentally, but points out that the law of radiation for a truly "black body" must necessarily be of a simple character.[1]

During a conversation with Mr. Lanchester in the autumn of 1895, he pointed out to me that he thought if we took a hot enclosure into which there was only a small aperture and measured the radiation passing out through this aperture that the internal walls of the enclosure would behave as a perfect radiator, whether they were a bright metallic surface or coated with lampblack or any other substance.

About the same time Ch. E. St. John also pointed out that in a heated enclosure, such as a fire-clay furnace, metals raised to a red heat appeared of almost equal brightness whether their surfaces were polished or blackened with oxide.

As all investigations up to this time on the laws of radiation were made with the assumption that lampblack was a perfectly black body and therefore a perfect radiator, it seemed of interest to compare the radiation from it with that coming from a hot enclosure with a small aperture, and which would evidently behave as a perfect radiator.

We procured a half gallon tin *T*, and soldered it by the neck into a large biscuit box *B*. Some water was placed in the biscuit box and kept boiling with a Bunsen burner so that the tin enclosure was completely surrounded with steam at 100° C.

A Boys' radio-micrometer *R* was mounted in front of the aperture and suitable screens *S* were interposed so as to cut off all radiation except that coming from the enclosure through the

[1] G. KIRCHHOFF, *Pogg. Ann.*, 109, 292, 1860.

aperture. The outside of the biscuit box near the aperture was coated with lampblack, and by slightly moving the box we could allow this blackened surface to radiate to the radio-micrometer instead of the enclosure.

The temperature of this blackened surface of the biscuit box must have been very nearly the same as that of the enclosure, but

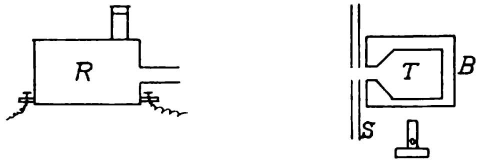


FIG. 1.

we were astonished to find that if we represented the radiation from the enclosure as 100, the radiation from an equal area of the blackened surface was only about 60.

The result of this rough experiment was so interesting that I determined to investigate the law connecting the *total* radiation and temperature of such a theoretically perfect radiator.

An enclosure was formed of a large plumbago crucible with a cover of the same material. This stood in a Fletcher's gas furnace and could be raised to any desired temperature.

A hole was bored through the walls of the furnace and also through one side of the crucible. This hole was lined with a porcelain tube through which could be seen the interior of the crucible.

A second porcelain tube also passed into the crucible and was used to carry a thermo-electric junction, made of pure platinum and platinum-rhodium. The current from this was measured by a D'Arsonval galvanometer of low resistance, and its calibrating curve, which was practically a straight line, was obtained by inserting the junction in steam at 100° C., in pure lead freezing, and in pure gold freezing. The radio-micrometer was used to measure the radiation coming from the enclosure, but as this instrument was so sensitive as to give a considerable deflection before the crucible was even red hot, some means had to be devised to

reduce the sensibility of the instrument by a known amount as the temperature of the crucible was raised.

Instead of allowing the radiation to fall directly on the radio-micrometer it was received by a concave silver-on-glass mirror, and this formed an image of the aperture of the hot enclosure on the thermo-couple of the radio-micrometer. In front of this mirror were placed a set of stops of known area, and by changing them the intensity of the image of the aperture and thus the deflections of the radio-micrometer could be altered.

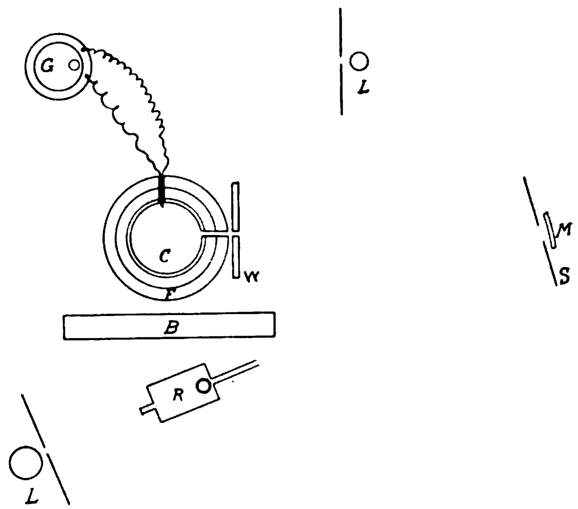


FIG. 2.

A hole was bored right through the radio-micrometer and provided with a low power positive eyepiece; by looking through this I could see an image of the aperture and also the thermo-couple hanging in front of it. By this means I could be sure that the image of the hot aperture always completely covered the thermo-couple of the radio-micrometer.

In front of the hot aperture was placed a copper screen through which a current of cold water was kept flowing. This screen was provided with a hole about 2 mm in diameter through which the radiation passed from the enclosure to the mirror and

then to the radio-micrometer. The hole was of such a size that the inside of the porcelain tube could not be seen from the radio-micrometer, but only a small area of the inside of the hot enclosure.

Observations were made by raising the temperature of the enclosure to about 1200° C.; the gas was then cut off, and the furnace and crucible were allowed to cool very slowly. Readings were then taken at frequent intervals of the deflections of the radio-micrometer, and simultaneously the temperature of the enclosure.

TABLE I.

T° absolute	Radiation	T° absolute	Radiation
1,317	1,300	787	155
1,266	1,080	761	133
1,206	920	751	127
1,193	824	726	112
1,161	736	708	99
1,105	560	700	94
1,075	460	676	81
1,050	456	663	74
1,022	400	645	65
1,007	436	630	60
988	360	614	52
963	350	593	46
945	310	582	43
921	284	551	33
901	262	541	32
875	230	522	24
855	208	491	19
826	194	466	16
805	172		

In Table I the values of a set of readings thus obtained are given, and in Fig. 3 the logarithms of these values are plotted. The straight line drawn on the chart represents a fourth power curve, and it will be seen that the observations lie very close to it, and thus seem to confirm Stefan's[1] law of radiation where $R = a\,T^4$.

In another set of readings the observations did not lie quite so close to this curve, but seemed to conform better with $R = a\,T^{4.3}$.

[1] I. STEFAN, *Wien. Ber.* (2),, **79**, 391–428, 1879.

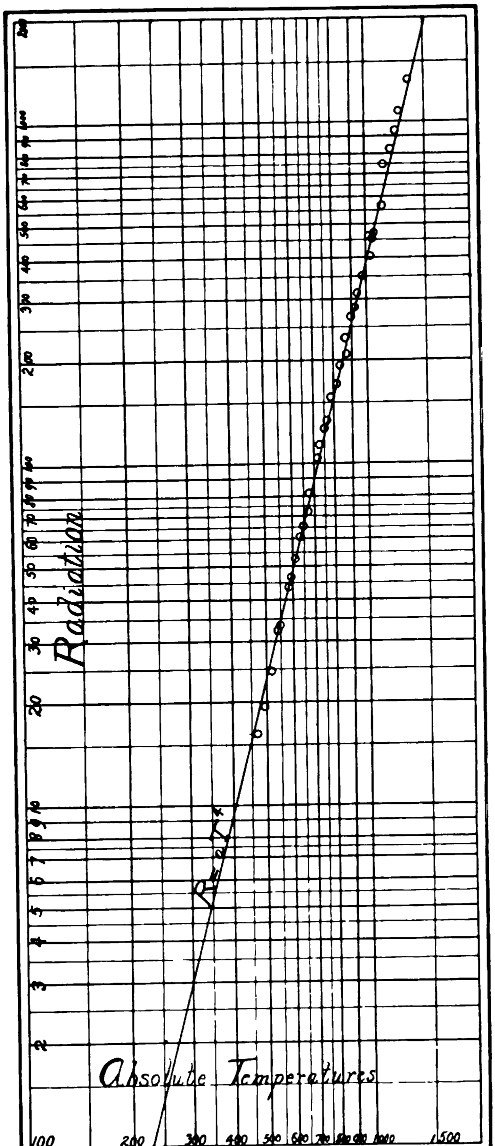

Radiation
$R = aT^4$
Absolute Temperatures
100
200
300
400
500
600
700
800
900
1000
1500

FIG. 3.

While this investigation was being carried on, Lummer and Pringsheim[1] were also working at the same subject, and their very carefully carried out experiments seem to confirm mine, and give also nearly a fourth power law.

It is only by knowing the true law of radiation that we can possibly measure directly the temperature of the Sun, and therefore any advancement in our knowledge on this subject is of the greatest importance.

In our investigation here in 1893 on the Effective Temperature of the Sun[2] we had first to study the law of radiation from a platinum strip which was raised to any desired temperature by an electric current, and the radiation from which then balanced the radiation coming from the Sun; the balancing instrument being a duplex Boys' radio-micrometer especially designed for this work.

In order to cut off all radiation from the incandescent platinum strip except that coming from a known area, the strip was covered with a water-jacket with a small hole through which the radiation passed into the radio-micrometer. The interior walls of this jacket were highly polished and plated with gold.

Since this investigation was made it has been pointed out that if we have a hot body inside an enclosure the inside walls of which are perfect reflectors, and allow heat to pass out through an aperture in the walls, the hot body will behave as if it was a perfect radiator. Now we assumed that the radiation from our platinum strip was only $\frac{35}{100}$ of that from a perfect radiator,[3] whereas our strip as mentioned was probably behaving very nearly as a perfect radiator. The fact that the law of radiation which we then found from this strip was a fourth power one, and the same as I have since found from a perfect radiator, seems also to indicate that the strip was behaving as a perfectly black body.

[1] O. LUMMER und E. PRINGSHEIM, "Die Strahlung eines 'schwarzen' Körpers zwischen 100° und 1300° C." *Wied. Ann.*, **54**, 1897.

[2] WILSON and GRAY, "Effective Temperature of the Sun." *Phil. Trans. Royal Society*, **185**, 1894.

[3] SCHLEIERMACHER, *Wied. Ann.*, **26**, 287, 1885; ROSETTI. *Phil. Mag.*, **8**, 445, 1879.

If this surmise is correct we must clearly multiply the value we obtained of the solar temperature by $\sqrt[4]{\frac{100}{35}} = 1.30$. Therefore the effective solar temperature would be 8700° C. × 1.30 = 11300° C. An experimental investigation is now being made to clear up this point.

June 1, 1899.

ON THE SPECTRA OF STARS OF SECCHI'S FOURTH TYPE. I.

By GEORGE E. HALE and FERDINAND ELLERMAN.

THE rapid progress of stellar spectroscopy during recent years, which has been due almost entirely to the development of photographic methods, has naturally followed three principal lines:

1. The determination of the general characteristics of stellar spectra, permitting a classification of stars on the basis of spectral types.

2. The measurement of the wave-lengths of dark and bright lines, for the purpose of identifying the substances present in stellar atmospheres.

3. The measurement of the displacement of stellar lines with reference to the lines of an artificial comparison spectrum, giving a means of determining the velocity of stars in the line of sight.

The measurement of the radial motion of stars, first attempted by Huggins with the inadequate instruments of a third of a century ago, has already reached an advanced stage of development. This is largely due to the work of Vogel, who first employed photographic methods in this field. The photographs taken at Potsdam for the purpose of Vogel's investigations have served, in the hands of Scheiner, for the accurate determination of the wave-lengths of numerous stellar lines. With the exception of this research, however, but little systematic work has been done in this direction.[1] An extensive field of investigation, hitherto almost unexplored, here lies open to students of astrophysics. Fortunately many of the brighter stars are within the reach of moderate apertures, and may be investigated with but small

[1] Professor Keeler is engaged in an important investigation of the spectra of stars of Secchi's third type.

additions to the instrumental equipment of the average observatory.

The study of the general characteristics of stellar spectra has been greatly facilitated by the use of the objective prism, which has been applied with such striking success by Pickering at the Cambridge and Arequipa stations of the Harvard College Observatory. The recent work of Vogel and Wilsing with the small spectrograph of the Potsdam Observatory, and the photographs of stellar spectra made in England and at the Cape by Huggins and Lockyer and McClean afford additional material for an extensive study of stellar development.

An examination of these results will show, however, that our knowledge of the spectra of stars of Secchi's fourth type (Vogel's III*b*) has advanced but little since the publication in 1884 of Dunér's memoir "Sur les Étoiles à Spectres de la Troisème Classe." Two causes sufficiently explain this fact. In the first place none of the stars of the fourth type are brighter than the 5.5 magnitude; and further, the spectra are so faint in the more refrangible region that but little is recorded on ordinary plates used with an objective prism. Thus the great store of negatives belonging to the Harvard College Observatory, so rich in other respects, contains few data available for the study of the spectra of these stars.

In planning the work of the Yerkes Observatory it was felt that the 40-inch refractor should preferably be employed in fields of investigation where its great light-gathering power would be likely to prove of most service. The objective of this instrument, which is corrected for visual observations, is especially adapted for the examination of the less refrangible regions of stellar spectra. The present investigation, which relates particularly to the visible spectra of faint red stars, was accordingly undertaken in November 1897.

RESULTS OBTAINED BY PREVIOUS OBSERVERS.

In his early survey of stellar spectra Secchi divided the stars into three classes, of which the third was given up to red stars.

Most of the stars of this character examined were naturally of the true third type; but one, *Lalande* 12561, is of the fourth, though it was classed by Secchi in the first part of his memoir, *Sugli spettri prismatici delle stelle fisse* (1867) with α Herculis, in the following words (*Catalogo*, p. 14.)

In conclusion, this is of the type of α Herculis, but with the dark zones lacking, while its own zones are so broad that some of them embrace two of those of α Herculis.

In the second part of the memoir it appears that the distinctive characteristics of fourth type spectra were recognized in the course of a survey of some twenty red stars from Schjellerup's catalogue. In describing the spectrum of 152 *Schjellerup* as characteristic of the class Secchi remarks (p. 9):

This type is composed of but three principal zones; a bright one in the green, a fainter one in the blue, and a pretty bright one in the red. This latter zone is frequently subdivided into other lesser zones.

This type differs essentially from the third, not only by the division of the zones, which have twice the breadth, but also because the zones have the greater luminous intensity on the opposite side; *i. e.*, in the fourth type the light increases in intensity from the red toward the violet, while in the third type the reverse is true. Thus if the third type were represented by a system of columns, the fourth type would be represented by cavities, supposing the illuminating light to be directed from the same side.

These stars also contain bright lines like those of the metals, and it is singular that these occur at the brightest extremity of the colored zones.

Few objects of this class were known to Secchi, but many were discovered in the subsequent observations of Vogel, D'Arrest, Pickering, and Dunér. The memoir published by Dunér in 1884, "Sur les Étoiles à Spectres de la Troisème Classe,"[1] together with Vogel's observations made with the 27-inch refractor of the Vienna Observatory,[2] afford the best data for a study of the spectra. A complete list of all fourth type stars hitherto discovered has recently been compiled by Espin,[3] who has himself made many additions to the number.

[1] *Svenska Vetenskaps-Akademiens Handlingar*, Vol. XXI, No. 2.

[2] *Publicationen der Astrophysikalisches Observatorium zu Potsdam*, Vol. IV, Part 1.

[3] *M. N.*, **58**, 443, June 1898.

WAVE-LENGTHS DETERMINED BY VOGEL.

Object	Schj. 152 (Vienna)	Schj. 152 (Vienna)	Schj. 152 (Bothkamp)	DM. + 34 4500 (Vienna)	Schj. 273 (Bothkamp)	Schj. 78 (Bothkamp)	Schj. 51 (Bothkamp)	Mean
Beginning of spectrum.			660					660
Dark band					656			656
Dark band			622		622	623		622
Dark band					6065			6065
Line in a band.........	5891		5893	5889	589	590		5893
End of band..........	5848							5848
Line.................	5741		5758	5750	578	5755		5757
Line beginning a band.	5621	5625	5628	5620	564	564	5640	5631
Line.................			552		552			552
Line.................			544					544
Group of lines			528	527	529			528
Line beginning a band.	5159	5163	5156	5161	516	515	5165	5159
Line.................	5132							5132
Beginning of band	4716		4735	4744	472	473		4729
Band			437					437
End of spectrum			430					430

Dunér's measures of fourth type (III*b*) spectra, as tabulated on p. 122 of his memoir, are given below.

WAVE-LENGTHS DETERMINED BY DUNÉR.

Object	19 Piscium	132 Schj.	152 Schj.	132 Schj.	152 Schj.	Wave-length
Band 2...........	621					621
Band 3...........	6048					6048
Band 4 (max.).....	5895	5884		5895	5910	5898
Band 5...........	5760	5757	5747	5762	5761	5760
Band 6 (beg.).....		5640	5624	5633	5634	5633
Band 7...........	551					551
Band 6 (end)......					545	545
Band 8.....	5285				5280	5283
Band 9 (beg.)......		5167	5159	5160	5164	5163
Band 9 (end)......					496	496
Band 10 (beg.)....		4714	4720	4729	4739	4727
Band 10 (end).....	463					463
End of spectrum...			437			437

The combined results of the two observers, compared with Kayser and Runge's wave-lengths of the hydrocarbon bands, are contained in the following table:[1]

[1] SCHEINER'S *Astronomical Spectroscopy*, Frost's translation, p. 314.

COMPARISON OF WAVE-LENGTHS.

Object	Vogel	Dunér	Mean	C_mH_n
Spectrum begins	660		660	
Dark band	656		656	
Dark band	622	621	6215	
Dark band	6066	6049	6058	6060. Middle of red band
Line in a band	5894	5899	5897	
End of a band	5849		5849	
Line	5758	5761	5760	
Line beginning a band	5632	5634	5633	5635.43. Beginning of yellow band
Line	552	551	5515	
Line	544	545	5445	
System of lines	528	5284	5282	
Line beginning a band	5160	5164	5162	5165.30. Beginning of green band
		496	496	
Line	5133		5133	
Beginning of a band	4730	4728	4729	4737.18. Beginning of blue band
		463	463	
Band	437	437	437	4381.93. Beginning of fifth band
End of spectrum	430		430	

As the hydrocarbons are all reduced to acetylene (C_2H_2) at high temperatures, and are characterized by a common spectrum which perhaps belongs to this substance, Scheiner remarks:

We may, therefore, go a step farther and consider that in the stars of class III*b* carbon and hydrogen are united in the form of acetylene, which is the first combination of these two elements which would ensue as the temperature fell.

Of the dark lines measured by Vogel and Dunér 5897 is considered by Scheiner to coincide with D and 5282 with the E group in the solar spectrum. The other lines could not be identified. He points out, however, that strong lines have been recorded at the following wave-lengths in the spectrum of *a* Orionis:

Vogel	Huggins	Dunér
5143	5143	5146
5451	5449	5449
5529	5526	
5760	5750	

Scheiner states that under the circumstances the agreement may be considered satisfactory, and therefore does not think it "hazardous to assert that metallic lines of type III*b* (IV) resemble those of type III*a* (III) so that the differences in these two subtypes are only due to the different chemical combinations present in their atmosphere."

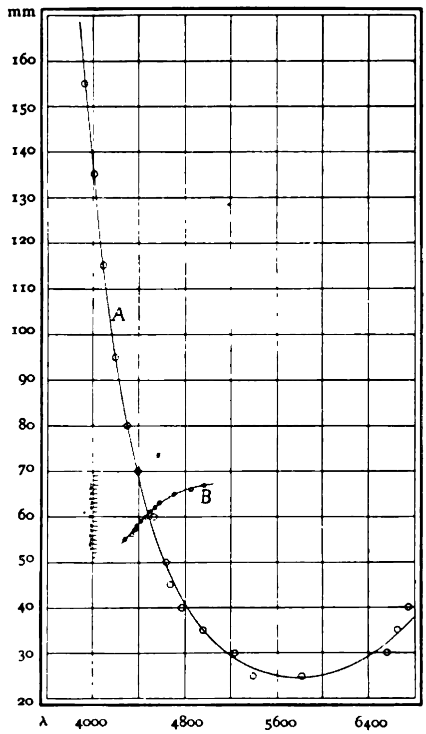
A discussion of Dunér's views regarding the development of fourth type stars, which are entitled to the greatest weight, will be deferred until after the numerical results obtained in our investigation have been given.

Fifty-five stars of the fourth type are catalogued in Dunér's memoir. Thanks to the photographic work of the Harvard College Observatory and the visual observations of Espin, this number has been increased to 242. Out of this number there are but three stars in the northern hemisphere and four in the southern that are brighter than the sixth magnitude. Of the stars which have been observed photometrically Espin finds that between the magnitude 6.1 and 7 there is a total of 23 stars; between 7.1 and 8, 39; between 8.1 and 9, 76; below 9, 80. In addition to the faintness of these objects the fact that the most characteristic portions of their spectra are in the red, yellow, and green tends to increase the difficulty of studying them photographically. It is evident that instruments especially adapted for the investigation of faint objects must be employed for the purpose.

DESCRIPTION OF THE INSTRUMENTS.

The principal instruments used in this investigation were the 40-inch refractor of the Yerkes Observatory and a stellar spectrograph containing one or three prisms. The object-glass of the telescope, which was made by Alvan Clark & Sons, has a clear aperture of 40 inches (102 cm). Its focal length for distinct vision, as determined from photographs of star trails, is 1936±cm. The focal length has been measured at various temperatures, and has been found to decrease 17.5 mm for a fall in temperature of 45°C. The form of the color curve correspond-

ing to various temperatures has been determined by Professor Frost, Professor Wadsworth, and one of the present writers. A full discussion of these results will soon be published by Professor Frost; but for the purposes of this paper, the color curve corresponding to a temperature of —18°C., and illustrated in the



COLOR CURVE OF FORTY-INCH OBJECTIVE.

accompanying figure, will suffice. It will be seen that the curve is flat enough to permit fairly satisfactory photographs of the spectral region lying between λ 5000 and λ 6500 to be taken in a single exposure. In the work on the yellow and green regions of the spectrum the slit of the spectrograph has ordinarily been set in the focus of the 40-inch objective, which corresponds to λ 5000. The spectra of fourth type stars generally increase in

brightness from the head of the yellow carbon band toward a maximum in the green. On account of the loss due to the form of the color curve and the fall in the curve of sensitiveness of ordinary isochromatic plates in the neighborhood of the *b* group, the intensity of the photographed spectra is more nearly uniform in the green than it should be. It must be borne in mind, therefore, in examining the photographs of fourth type spectra reproduced in these papers, that the green region should ordinarily have a considerably greater relative brightness than the plates give it. The same may be said of the less refrangible half of the bright zone in the yellow.

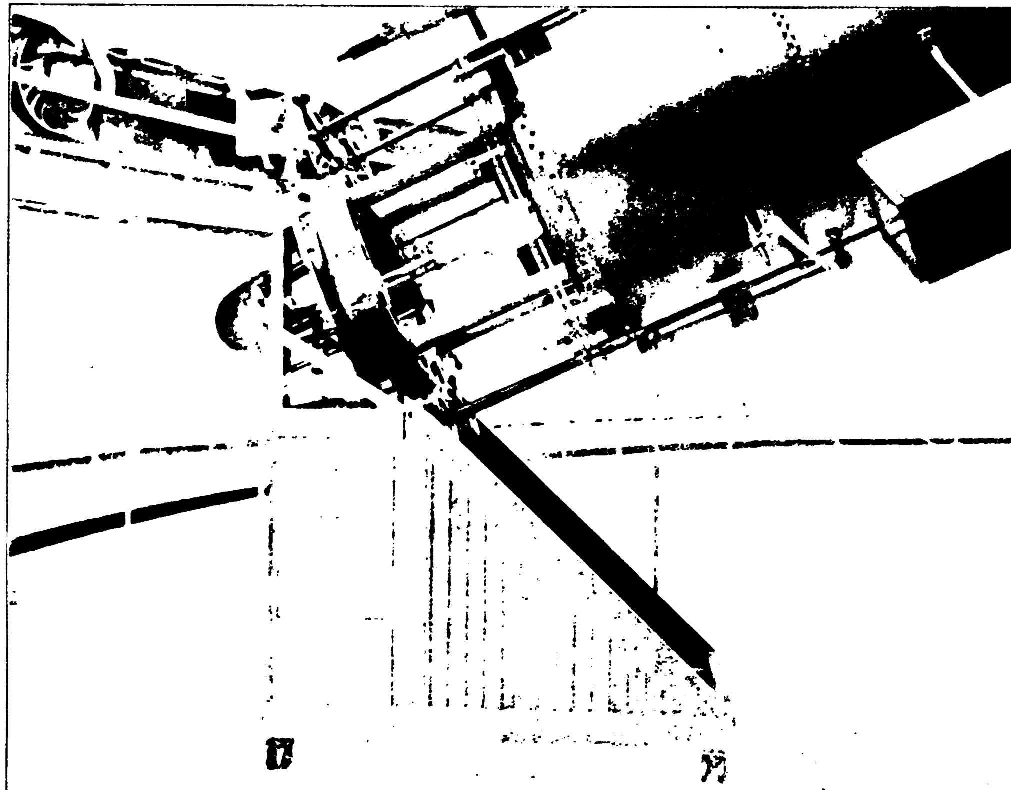
Photographs of the more refrangible portions of the spectra of some of the brighter fourth type stars were taken before a correcting lens had been obtained, but on account of the steepness of the color curve in the blue, only a limited region could be satisfactorily photographed in a single exposure. The correcting lens, which has been made by Brashear after curves calculated by Professor Wadsworth from Keeler's formulae,[1] is so effective that it permits the entire blue portion of fourth type spectra to be photographed on a single plate. It consists of a compound lens of 32 mm aperture, supported in the cone of rays from the 40-inch objective at a distance of 30 cm from the slit. When the lens is in place (it is carried by an adapter accurately centered in the tailpiece of the telescope) the divergence of the cone of rays after passing through it is 1 : 15.8, while the ratio of aperture to focal length of the collimator objective is 1 : 16.3. The introduction of the correcting lens causes the focal plane corresponding to λ 4500 to move about 60 mm toward the 40-inch objective. At the same time the color curve is made much flatter in the blue region, as is indicated by the curve (*B*). The lens not only increases the extent of the spectrum that can be photographed on a single plate, but also materially reduces the exposure time by rendering the task of guiding less difficult. The small star image seen when the correcting lens is in place can be kept on the slit much more easily than the expanded disk obtained without the lens.

[1] See this JOURNAL, 1, 101, February 1895.

The spectrograph, a photograph of which is reproduced in Plate III, was built by Brashear for the 40-inch telescope. As originally constructed, this instrument was in almost every respect similar to the spectrograph designed by Keeler for the Allegheny Observatory. As the result of a series of experiments made with the spectrograph attached to the 12-inch refractor of the Kenwood Observatory, and further work with the instrument in conjunction with the 40-inch refractor of this Observatory, the spectrograph has been reconstructed in our instrument shop after designs by Professor Wadsworth. The supporting drum supplied by Warner & Swasey to carry the spectrograph, which was made interchangeable with the tailpiece of the telescope, is no longer used. Instead, the instrument is now clamped to a large ring which was designed to carry the solar spectroscope and spectroheliograph. This ring is supported by four heavy tubes, which can be racked into the body of the telescope by means of a hand crank connected with a worm gear. In this way the spectrograph can be moved along the optical axis of the telescope for focusing the star on the slit, and the large ring can be racked up out of the way when the telescope is wanted for micrometric work. The heavy brass casting which supports the collimator of the spectrograph is held (by means of centering screws) in the center of a ribbed iron casting 85 cm in diameter, which is fitted by a cone bearing to the ring carried by the telescope. This casting has a gear cut on its circumference, which permits the spectrograph to be rotated by either one of two pinions attached to the ring. When set at the desired position angle it is clamped in place by suitable bolts.

The collimator objective has an aperture of 31 mm and a focal length of 507 mm. The collimator tube is supported within an outer fixed tube, and has a range of motion of 130 mm along the axis of the telescope. In general this adjustment is not used, the collimator being kept at a certain scale reading, and the slit brought into the focal plane by moving the entire spectrograph in the manner indicated above.

PLATE III.



The prism train ordinarily employed contains three 60° flint prisms of index $n = 1.6960$. With this train the deviation for $H\gamma$ is about 180°. The prisms are mounted on leveling screws, and are held in place on their supports by adjustable springs pressing against their upper surfaces. They are connected by a minimum deviation device designed and made by Brashear. After being set for a certain region, they are always clamped firmly in place by means of screws which pass through the top of the prism box and press against the top of the prism supports. The prisms have a distinctly yellowish color, and undoubtedly exercise considerable absorption in the blue and violet. The exact amount of this absorption has not yet been determined. The temperature of the air in the prism box is measured by a thermometer, the bulb of which is nearly in contact with the back surface of the second prism.

Three cameras of different focal lengths have been employed at various times. The first, which was supplied by Brashear with the spectrograph, has an aperture of 31 mm and a focal length of 508 mm. The second, which was made in our instrument shop, is used with an achromatic objective of 31 mm aperture and 253 mm focal length, also supplied by Brashear for visual use with a short observing telescope. As it does not give a sufficiently flat field for the best photographic results, this objective has been replaced by a photographic doublet of 37 mm aperture and 271 mm focal length. This performs admirably, giving a flat field over a long range of spectrum. In all cases the best results have been obtained when an objective corrected for the *visual* rays has been employed in the collimator.

Many of the earlier photographs of the fainter stars were made with the dispersion of a single heavy flint prism and the long (508 mm) camera. It was subsequently found, however, that much better results could be obtained, with shorter exposures, by using three prisms and a short camera. All negatives taken recently have been made in this way. A supplementary camera with photographic doublet of 40 mm aperture and about 15 cm focal length has been provided for use on the fainter stars.

The slit jaws are of polished speculum metal, as first used by Huggins. Arrangements had been provided for observing the star reflected from the first prism face, as is done at Potsdam, but a comparison of the two modes of guiding showed the reflecting slit method to be more delicate and more satisfactory in other respects. The light reflected back from the inclined slit-jaws meets a right-angle prism supported a short distance in front of the slit and just outside the cone of rays from the objective. After passing through the prism the rays are rendered parallel by a lens, and are brought to the observer's eye after two more reflections. The eyepiece is placed at a convenient point just above the prism box. During the exposure the observer keeps the star image on the intersection of the slit and a fine line engraved across it at right angles, by means of the electric slow motions of the 40-inch telescope. Stars as faint as the eighth magnitude can be satisfactorily followed in this way.

The width of the spectrum can be limited, if necessary, by adjustable jaws behind the slit. The width of the slit itself is read on the divided head of a micrometer screw, provided with right and left-handed threads, which move the jaws outward from the axis of collimation.

The comparison spark apparatus is so mounted that the (iron) electrodes can be swung into position immediately in front of the slit, at a distance of 23 cm from it. On the same support with the electrodes is a lens of 19 mm aperture, having the iron poles in one conjugate focus and the slit in the other. The angular aperture of the lens as seen from the slit is 1 : 7.4. As the ratio of aperture to focal length of the collimator objective is 1 : 16.3, the entire lens is always filled with light from the spark. Before making the exposure for the star a metal disk containing a suitable aperture is turned into position before the slit. The form of the aperture is such that a narrow occulting bar covers the middle of the slit, where the star's image is to fall, while light from the spark is permitted to pass on both sides of the center. The spark, which is produced by an App's induction coil with condenser in the secondary circuit, is given an

exposure of seven or eight seconds. The spark apparatus is then turned out of the way, the disk rotated to a point where the middle of the slit is exposed, the length of the slit reduced to correspond with the width of the occulting bar, and the desired exposure is given to the star. At the end of the exposure the middle of the slit is again covered with the bar and the light of the spark admitted as before. Thus if a change of temperature occurs during the exposure to the star, the spark lines photographed at the end will be shifted with respect to those first recorded. If the change of temperature is fairly uniform and the star lines are not shifted by motion in the line of sight, the center of the widened iron lines should coincide with the center of the corresponding star lines.

During the long exposures which have been necessary for the fainter stars the change of temperature in the prism box, which sometimes amounts to as much as 4° C., has in certain cases seriously affected the definition of the spectra. In work of this kind it is desirable to adopt some means of counteracting such variations, which not only decrease the sharpness of the spectra, but tend to introduce errors into the determinations of wave-length. With short exposures, however, the temperature change is not sufficient to affect the results.

As Plate III indicates, the spectrograph, when not in use, stands on a carriage mounted on rollers. The operation of attaching it to the telescope is a very simple one, and can easily be effected in about ten minutes. During the operation the telescope is securely anchored to the rising floor (which stands at its highest level) by means of a heavy steel bar.

When combined with a single prism (see Plate IV) the camera stands at such an angle that it is difficult to prevent flexure during long exposures. For this reason the 508 mm camera is no longer used in this way. As has been stated above, much better spectra, on a larger scale, can be obtained with shorter exposures with three prisms and the 271 mm camera. This short camera, and the still smaller one provided for very faint stars, are so well supported that they give no evidence of flexure when

used with a single prism. At present three prisms are almost invariably employed, and in this case no difficulty from flexure is experienced with any of the cameras.

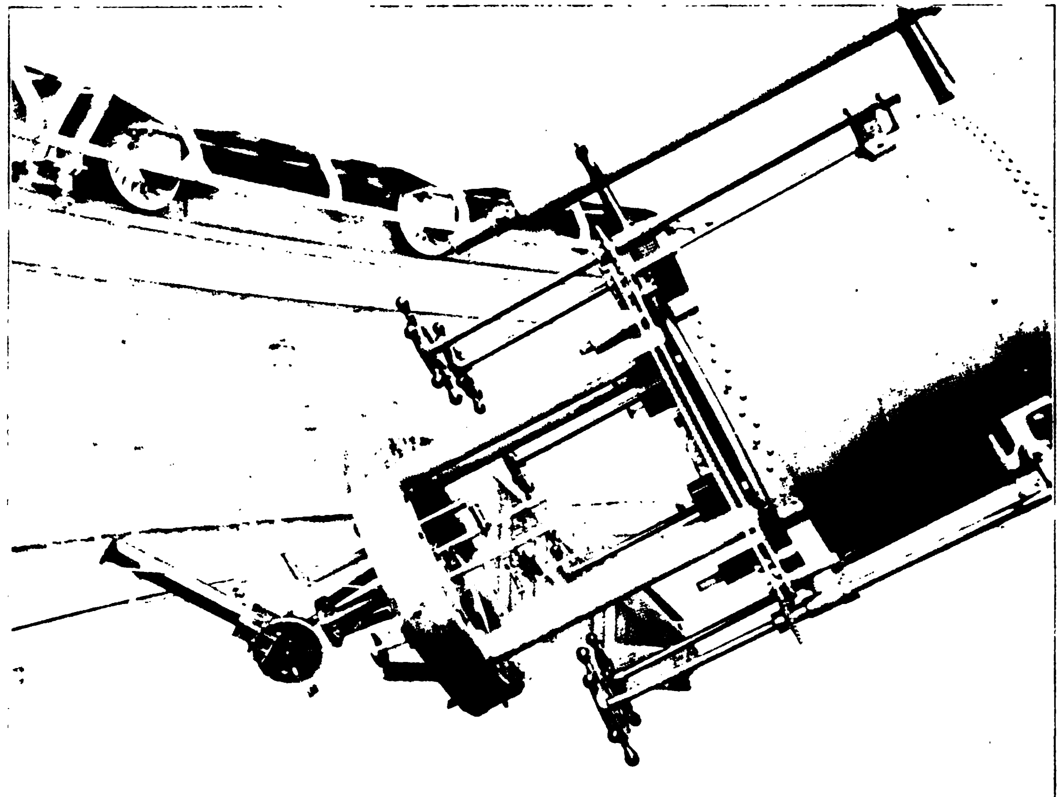
Although the diameter of the first diffraction ring in the star image given by the 40-inch objective is 0.227″ (for $\lambda = 4600$),[1] it is impracticable to use a slit of this width in ordinary spectrographic work. If the objective were quite free from chromatic aberration, and the seeing perfect, the case would be entirely different from what it is in practice. Under ordinary conditions the minimum slit-width varies directly as the focal length of the telescope employed. For the brighter stars, when it is not desired to photograph a considerable range of spectrum, slit-widths ranging from 0.01 mm to 0.04 mm may be used to advantage with an instrument having the great focal length of the 40-inch telescope. In Professor Frost's investigations of stellar motions in the line of sight such widths are actually employed. But in our work on the faint stars of the fourth type experience has shown that the best results follow from the use of slit-widths as great as 0.1 mm. As the camera lens commonly preferred has a focal length whose ratio to the focal length of the collimator objective is 1 : 1.87, it is evident that the breadth of the spectrum and also the width of the lines are reduced in this ratio. With a slit-width of 0.15 mm, and a dispersion of three 60° prisms, the yellow and green regions of the spectrum of 280 *Schjellerup* (mag. 7.8) required an exposure of nine hours.[2]

Our study of the subject, and the experience resulting from the use of the 40-inch telescope for spectrographic investigations, have tended to confirm all that has been said by others regarding the advantages of a long collimator combined with a relatively short camera. In conjunction with Professor Frost we have designed a new spectrograph for the large telescope, in which it is purposed to use a collimator objective of from 30 to 40 inches

[1] The components of κ Pegasi have been seen and measured with the 40-inch telescope when their distance was about 0.1″.

[2] This photograph has been reproduced in Yerkes Observatory *Bulletin* No. 7. With the same optical combination, and a slit-width of 0.075 mm, the green bands in the spectrum of α Orionis were photographed in twenty seconds.

PLATE IV.



One Prism Spectrograph attached to Forty-Inch Telescope.

focal length. There is every reason to expect that with this instrument better photographs than any we have hitherto obtained can be made with shorter exposures.

The width of the spectra photographed with the three-prism spectrograph and 271 mm camera is ordinarily about 0.18 mm. There would be no disadvantage in having the width less than this, but with the present ratio of camera to collimator it is difficult to make it much less when the exposure is continued several hours. The great length of the telescope tube causes it to sway more or less under certain conditions, though the adjustable canvas screen over the observing slit usually excludes the wind pretty effectually. The three prisms have a (visual) resolving power of about 33000 for λ 4860, but with the slit-widths employed in the present investigation only a small fraction of this is realized. In the region near λ4400 it is possible to separate on the photograph lines 0.8 tenth-meter apart, while at λ5600 lines 1.3 tenth-meters apart have been resolved. The scale of the negatives made with a dispersion of three prisms and the 271 mm camera is such that at λ4400 1 mm = 18.5 tenth-meters; at λ5350 1 mm = 49.6 tenth-meters.

It will be seen that the circumstances were not especially favorable for the accurate measurement of radial velocities, and when the work was undertaken it was not proposed to attempt such determinations for the faint stars under investigation. Nevertheless all precautions were taken to avoid systematic errors and the approximate velocities of a few of the brighter stars of the fourth type have been determined. The problem is hardly comparable with that of measuring the radial velocities of stars of other types as bright as the fourth magnitude, for which the exposure times are short, the displacements due to temperature variations negligible, and the lines accurately identified. The instrumental adjustments were always made with the great care requisite in velocity measurements, and the measured and computed velocities of planets were compared from time to time. Some of these check results will be given in subsequent papers of this series.

MEASUREMENT AND REDUCTION OF THE PHOTOGRAPHS.

The photographs are measured on a Zeiss comparator, which has proved itself to be admirably adapted for work of this character. The essential parts of the instrument are two microscopes, rigidly supported at a fixed distance apart, and provided with micrometer eyepieces. The plate to be measured is fastened by spring clips to a sliding stage, to which the scale, engraved on silver, is also attached. The microscope used for viewing the plate is so arranged that the magnifying power can easily be varied from 13 to 25 diameters. In practice the lowest power is generally employed. The scale is divided to fifths of a millimeter, and thousandths of a millimeter can be read directly on the micrometer head, which is divided into 100 parts. Satisfactory means have been provided for illuminating both plate and scale. An examination of the scale made at the *Reichsanstalt* at Charlottenburg shows that its absolute errors probably do not exceed 2 μ. The errors of each division, considering the scale to be one of equal parts, are being determined here by Gill's method. A silver scale, inlaid in steel and divided to fifths of a millimeter, has been made in our instrument shop for the purpose of this comparison. The micrometer of the scale microscope has been tested for errors of run, and the corresponding corrections found to be negligible in measurements of these plates. A single narrow line, extending across the field of the viewing microscope, is ordinarily employed for the settings, though a double line can be used if desired. A slow-motion screw, which clamps to the stage, greatly facilitates the work of setting on the stellar lines.

The comparison spectrum, as already stated, is that of an induction spark taken in air between iron electrodes. Under these circumstances the air lines come out very strongly, but in most instances their hazy character unfits them for use as standards. In some portions of the spectrum, however, it has been necessary to employ the sharper air lines, because of the absence of iron lines. As no published wave-lengths of the air lines could be found that were accurate enough for the purpose

Dr. Frank Schlesinger[1] was requested to photograph the iron spark spectrum with the concave grating of ten feet focus for the purpose of measuring the wave-lengths of both the air and iron lines. The results for iron were in good agreement with those obtained by Kayser and Runge.

Much time was devoted to an examination of various methods of reduction, with the purpose of selecting one well adapted for this particular work. The first determinations of wave-length were made graphically, from curves plotted on squared paper. The large scale of the curves involved the necessity of making them in sections, and difficulty was experienced in drawing curves which satisfactorily represented the observations. Accordingly the curves were drawn on a large blackboard, and much better results obtained. Although fairly good wave-length determinations were made in this way the method was not considered entirely satisfactory.

In order to obtain good results with a short curve an interpolating machine was devised, and constructed in our instrument shop. It consists of a cast-iron plate, with plane upper surface, on which a piece of thin sheet celluloid, large enough to include the curve, is fastened by metal clips. The plate is clamped to the bed of a Brown & Sharpe milling machine, on which it can be moved in two directions at right angles by means of screws with micrometer heads graduated to thousandths of an inch. The coördinates (scale reading and wave-length) of the standard lines are thus measured off, and the points plotted by means of a sharp steel pin held in a fixed position by a support firmly attached to the overhanging arm of the milling machine. After the points have been plotted a curve is drawn through them by means of the device shown in Plate V. A flexible strip of tempered steel, of U section, is mounted above the iron plate in such a way that it can be bent to any desired curvature by means of nine screws. A movable table, bearing a compound microscope with adjustable cross-hairs, and a steel graving tool with chisel edge, is brought against the steel strip. The graving tool

[1] At that time Volunteer Research Assistant at this Observatory.

having been drawn up out of action, the table is moved along in contact with the strip, which is adjusted by the screws until the residuals (the distances between the plotted points and the intersection of the cross-hairs) have been made as small as possible. The carriage is then turned through 180°, and a curve traced through the points by the aid of the graving tool, which has been so adjusted as to pass over the path previously traversed by the intersection of the cross-hairs. During this process the apparatus remains on the bed of the milling machine. In order to take out the wave-length corresponding to a given scale reading it is now only necessary to replace the steel pin with which the points were plotted by a second microscope, the cross-hairs of which have been made to coincide with a point marked by the pin (Fig. 2). When the scale coördinate has been set at the given value, the other screw is turned until the intersection of the cross-hairs coincides with the curve. The reading on the micrometer head then gives the desired wave-length. Errors in the screws of the milling machine can easily be determined by observations of a standard scale clamped to the bed. This method of determining wave-lengths proved to be satisfactory, and good results were obtained by its aid.

The publication of Hartmann's valuable interpolation formula[1]

$$\lambda = \lambda_0 + \frac{c}{(n - n_0)^a},$$

placed us in possession of a still more satisfactory means of reduction. In its approximate form

$$\lambda = \lambda_0 + \frac{c}{n - n_0}, \tag{1}$$

where n is the scale reading of the line, and c, λ_0, and n_0 are constants, this formula satisfactorily represents the spectral region λ 5150–5900. Experience showed, however, that when the constants are determined from measures of only three standard lines, the sum of the squares of the residuals corresponding to other standard lines, whose wave-lengths have been computed from the

[1] This JOURNAL, 8, 218, November 1898.

PLATE V.

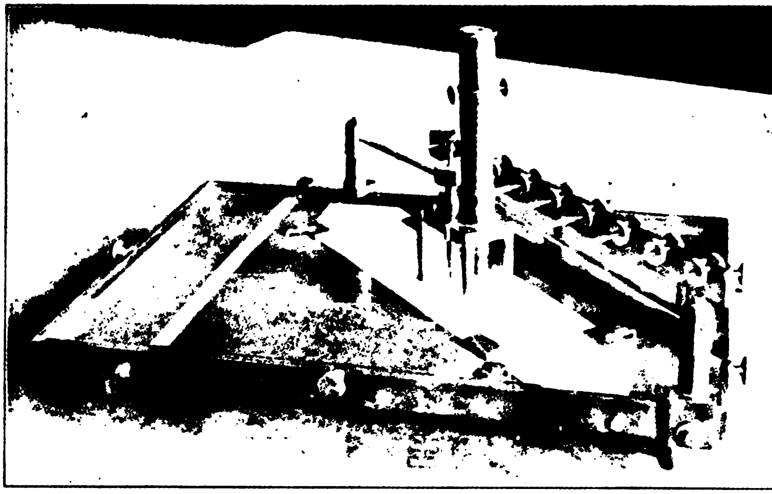


FIG. 1.—INTERPOLATING MACHINE ARRANGED FOR CURVE TRACING.

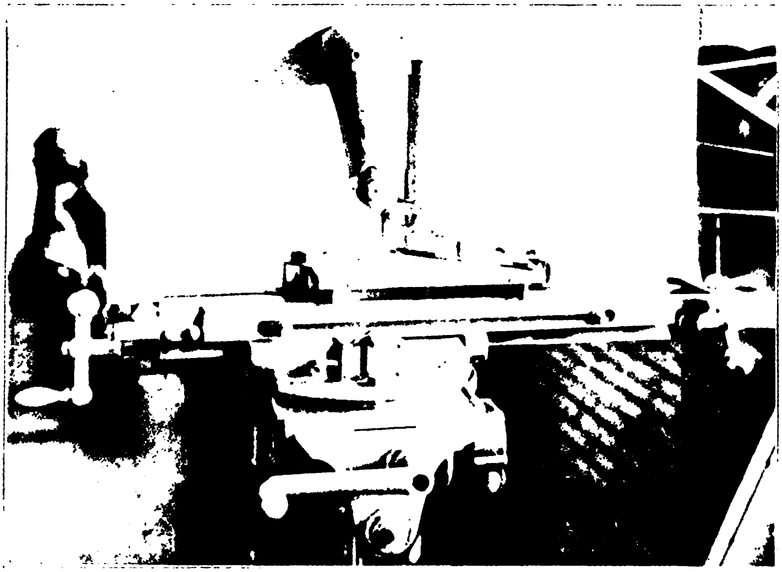


FIG. 2.—INTERPOLATING MACHINE ON BED OF MILLING MACHINE.

resulting expression, is sometimes much too large. This was found to be due, not to the inadequacy of the formula, but to the appearance of some of the standard lines on the stellar negatives. Any cause which produces lack of symmetry in a line tends to introduce an error into the determination of n. It was therefore decided to use a larger number of standard lines, and to make the reductions by the method of least squares.

From (1) we obtain observation equations of the form

$$\left.\begin{aligned} n_1 &= n_0 + \frac{c}{\lambda_1 - \lambda_0} \\ n_2 &= n_0 + \frac{c}{\lambda_2 - \lambda_0} \\ &\cdot \quad \cdot \quad \cdot \quad \cdot \\ &\cdot \quad \cdot \quad \cdot \quad \cdot \\ n_m &= n_0 + \frac{c}{\lambda_m - \lambda_0} \end{aligned}\right\} \tag{2}$$

Substituting in (1) values of n and λ corresponding to three standard lines, and solving, we obtain approximate values of the three constants, which will be called n_0', λ_0', and c'. The most probable values of n_0, λ_0 and c are $n_0 = n_0' + z_1$, $\lambda_0 = \lambda_0' + z_2$, $c = c' + z_3$, where z_1, z_2, z_3 are the corrections which are to be determined.

Substituting these quantities in the first observation equation and expanding by Taylor's theorem, we obtain

$$z_1 + \frac{c'}{(\lambda_1 - \lambda_0')^2} z_2 + \frac{1}{\lambda_1 - \lambda_0'} z_3 + m_1 = v_1 \tag{3}$$

where $m_1 = \left(n_0' + \dfrac{c'}{\lambda_1 - \lambda_0'}\right) - n_1$. The coefficients of z_1, z_2, and z_3 are obtained by differentiating the first equation in (2) with respect to each of the variables, and substituting n_0', λ_0' and c' for n_0, λ_0 and c. Terms containing powers of z_1, z_2, z_3 higher than the first have been neglected.

If we now represent the coefficients of z_2 and z_3 by b_1 and c_1, and treat the other equations in (2) in the same way, our observation equations become

$$\left.\begin{array}{l} z_1 + b_1 z_2 + c_1 z_3 + m_1 = v_1 \\ z_1 + b_2 z_2 + c_2 z_3 + m_2 = v_2 \\ \quad - \quad - \quad - \quad - \\ \quad \cdot \quad \cdot \quad \cdot \quad \cdot \\ z_1 + b_m z_2 + c_m z_3 + m_m = v_m \end{array}\right\} \tag{4}$$

Solution of the normal equations, which are formed from equations (4) in the ordinary way, gives the desired corrections z_1, z_2, z_3.

This method has proved very satisfactory in practice, and is employed in all of our wave-length determinations. The least squares solutions are greatly facilitated by the use of a *Brunsviga* calculating machine.[1]

In the measurement of the radial velocities of the brighter stars of the fourth type our endeavor has been to reduce each

[1] The method is similar to that given by Hartmann in a recent memoir (Potsdam Publications, Vol. XII, Appendix), which has reached us since the above was put in type. The following table of residuals, computed for nine standard iron lines on one of our negatives, gives a comparison of the results obtained with the approximate (H_1) and the rigorous (H_2) formulae, and illustrates the advantage of employing the method of least squares in both cases (H_1 corr. and H_2 corr.).

Line	Residuals			
	H_1	H_1 Corr.	H_2	H_2 Corr.
	Tenth-meters	Tenth-meters	Tenth-meters	Tenth-meters
10*	0.00	+0.05	+0.01	+0.12
12	− .25	− .19	− .26	− .15
15	− .19	− .11	− .20	− .10
17*	.00	+ .09	+ .01	+ .10
21	+ .09	+ .20	+ .10	+ .20
24	+ .04	+ .16	+ .06	+ .16
27	− .24	− .08	− .20	− .09
28	− .48	− .32	− .44	− .35
29*	.00	+ .19	.00	+ .10
[v v]	.396	.269	.355	.263
Constants				
n_0	−8333	−8339.8	− 4372	−4368.7
λ_0	3065.75	3065.99	2828.57	2828.55
c	113,116,053	113,115,612	550,241,300	550,178,130

*Lines used in determining constants for H_1 and H_2.

plate independently of all others, and to avoid doubtful assumptions regarding the identification of stellar lines. The peculiar type of spectrum with which we are dealing makes this last point especially important. Comparisons of fourth type spectra with those of the third type[1] bring out very clearly the presence of strong iron lines, which are also conspicuous in the solar spectrum. These lines have been employed in the radial velocity determinations. Some of them are not as susceptible of accurate measurement as other lines in their neighborhood, but it was thought better to employ them than to make unwarranted assumptions as to the origin of the latter lines.

The value of rV_s, the velocity in kilometers per second corresponding to a displacement of the lines equal to one division of the micrometer, is easily calculated.

$$V_s = \frac{\lambda}{299860}$$

gives the velocity in kilometers per second corresponding to a displacement of one tenth-meter.

From Hartmann's formula we have

$$n = n_0 + \frac{c}{\lambda - \lambda_0}.$$

Differentiating, we obtain

$$\frac{d\lambda}{dn} = \frac{(\lambda - \lambda_0)^2}{c}.$$

If dn = unity, the value of one division in tenth-meters is

$$d\lambda = \frac{(\lambda - \lambda_0)^2}{c}.$$

The values of the constants λ_0 and c are computed in the manner already described.

In the reductions we have used Campbell's expression[2]

$$v_d = -0.47 \sin t \cos \delta \cos \phi$$

for the correction due to the Earth's diurnal rotation, and Schlesinger's recently published formula[3]

$$[3.5392] \, (\Delta X \cos \alpha \cos \delta + \Delta Y \sin \alpha \cos \delta + \Delta Z \sin \delta)$$

[1] See Yerkes Observatory *Bulletin* No. 9.

[2] SCHEINER'S *Astronomical Spectroscopy*, Frost's translation, 341.

[3] This JOURNAL, **9**, 159, March 1899.

for the correction due to the motion of the Earth in its orbit. Here ΔX, ΔY, ΔZ, are the components of the Earth's motion parallel to the line of equinoxes, parallel to the plane of the equator, and perpendicular to the plane of the equator respectively. They are tabulated in the Berlin *Jahrbuch* for every twelve hours in the year, and the values corresponding to the time of the observation can conveniently be taken out with the aid of the interpolation table given in Schlesinger's article. This method has the advantage of rendering unnecessary the calculation of the latitude and longitude of the star.

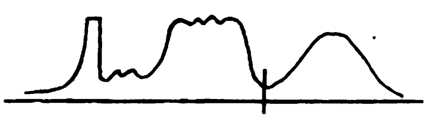
Although Ditscheiner's formula[1]

$$x = -\frac{(n^2 - l)\sin\frac{A}{2}}{nf\sqrt{l - n^2\sin^2\frac{A}{2}}}\, z^2$$

has been found to give the correction due to curvature of the spectral lines with a considerable degree of accuracy, it is often best in practice to determine the correction by actual measurement on a solar plate taken under similar conditions. Corrections for curvature have been applied in all cases where the accuracy of the wave-length determinations warranted.

BRIGHT LINES IN FOURTH TYPE SPECTRA.

We have already quoted Secchi's statement regarding the presence of bright lines in spectra of the fourth type. It is difficult to determine, however, from the conflicting evidence



found in Secchi's publications, whether he really saw the bright lines whose existence is shown by our photographs. The intensity curve of the spectrum of 78 *Schjellerup*,[2] which is reproduced herewith, favors the supposition that some of the lines were

[1] See Frost's *Scheiner*, p. 15.

[2] *Memoria Seconda*, p. 40. The red end of the spectrum is at the left.

actually seen, but if this was the case it is hard to understand why the most conspicuous of the bright lines, which falls near the head of the carbon absorption band in the yellow, was accorded so low an intensity, not exceeding that of its fainter companion toward the violet. Again, the illustration of the spectrum of the same star, published by Secchi in the second edition of *Le Soleil* (Plate M), contains no bright lines, while the drawing of the spectrum of 152 *Schjellerup* in the same plate shows several bright lines, though the most conspicuous of all are lacking. Secchi states that, in his opinion, the general appearance of the spectrum is that of a direct spectrum given by a gaseous body, rather than that of an absorption spectrum[1] though he reached no very definite conclusions regarding the physical condition of these stars.

In his memoir,[2] after pointing out the inconsistency of Secchi's statements, Dunér expresses the belief that the spectrum is simply one due to absorption. It will be seen later, however, that he has since observed one or two of the most conspicuous bright lines, which were not recorded by Secchi.

Our earliest photographs of these spectra, which were made in January 1898, seemed to leave little room for doubt as to the presence of bright lines. In view of Dunér's opinion, which was supported by the results of Vogel's observations, it was nevertheless deemed necessary to undertake a series of tests for the purpose of meeting any doubts that might arise.

The problem is to distinguish between true bright lines and bright spaces in a continuous spectrum bounded by dark lines or bands.

At the outset let us direct our attention to the apparent bright line at λ 5592 (Plate VI), returning later to a consideration of other similar lines.

1. This line is much brighter than the spectrum on either side of it. An exposure of four minutes is sufficient to photograph the line with a dispersion of three prisms, while equal

[1] *Le Soleil*, II, 458.

[2] *Loc. cit.*, 10.

density of the contiguous spectrum cannot be obtained under the same conditions with an exposure of less than about 12 to 15 minutes. If the line is supposed to be due to the continuous spectrum, it must be assumed that the carbon absorption band is interrupted at this point.

2. If the line were merely a section of the continuous spectrum, bounded by portions of the carbon absorption band, it should become less conspicuous as the dispersion is increased. Our experiments show the reverse to be true. Dispersions varying from that of a small direct-vision spectroscope to that of three heavy flint prisms were tried both visually and photographically. In all cases it was found that the contrast between the line and the contiguous spectrum increased rather than diminished with the dispersion.

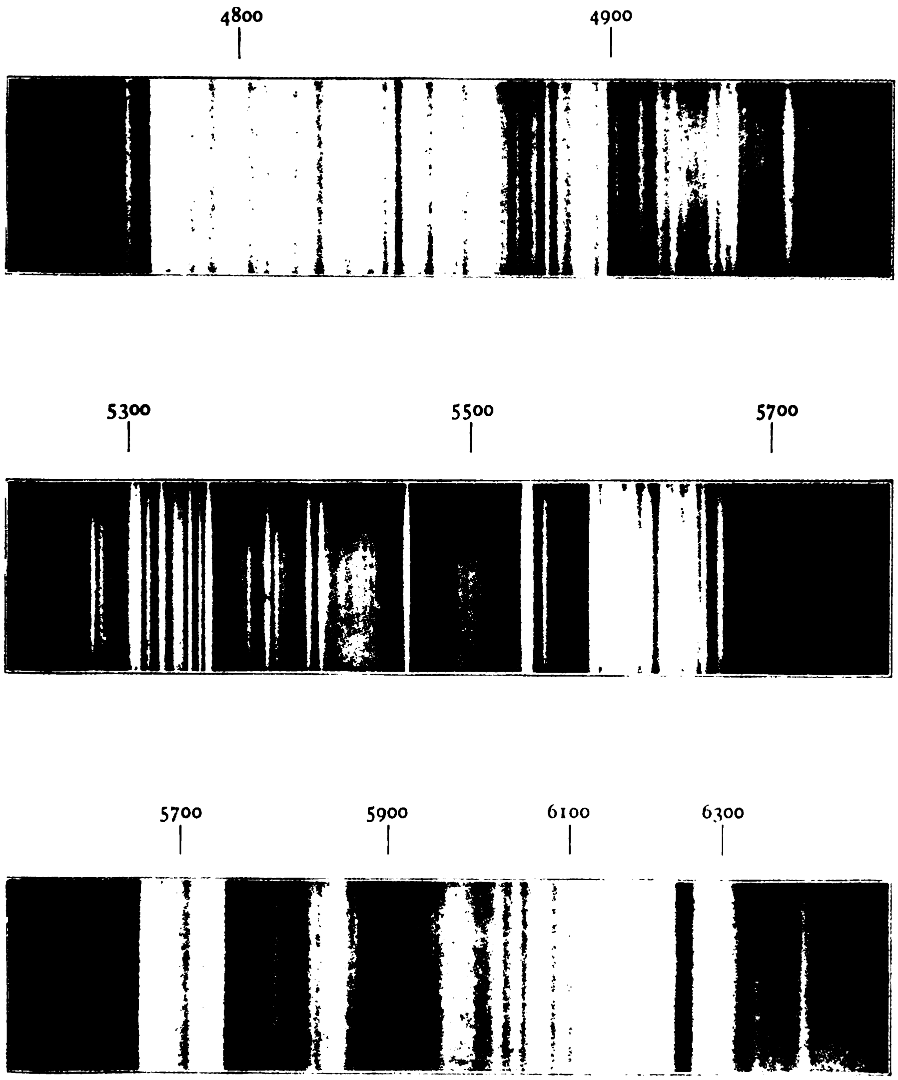
3. A bright section of the continuous spectrum would appear with less contrast as the slit was widened. Photographs taken with increasing slit-widths show no decrease in contrast, some of the best results having been obtained with the widest slits.

The conclusions based upon a study of the photographs are confirmed by visual observations. The spectra of 132 *Schjellerup* and 152 *Schjellerup* have been observed on several occasions with the three-prism spectroscope attached to the 40-inch telescope. The observing telescope employed has a focal length of 253 mm, and the eyepiece gave a magnification of 13 diameters. The bright line at λ 5592 was easily seen, as well as a number of other bright lines in the red, yellow, green, and blue.

At our request the spectrum of 152 *Schjellerup* has been examined with the 36-inch refractor of the Lick Observatory by Professors Keeler and Campbell. A dispersion of three prisms was employed. Professor Keeler describes the observations as follows:

"I compared the spectrum with Vogel's drawing in Potsdam Publications, Vol. IV. The drawing seemed to be merely a rough indication of what the spectrum actually is. What we saw was much more like your photograph. It is curious that Vogel did not see the bright line λ 550 $\pm$, as it is a conspicuous feature of

PLATE VI.



Bright Lines in the Spectrum of 152 Schjellerup.

the spectrum with the 36-inch. The bright block λ553–λ584 seems to be a complex of bright and dark lines or bands, and the dark band as shown in the drawing (λ573) is relatively too conspicuous. Vogel's dark band at λ525 is made up of lines of which there are many in the neighborhood. There is a strong line at or near D. We tried to identify it with the Na line in a spirit lamp, but the telescope was jumping in a high wind, and the comparison did not amount to much. There were many dark lines in the red.

"To my mind, there is little doubt that the spectrum of this star contains bright lines."

In an article published in a recent number of this JOURNAL (March 1899), Dunér describes his own observations of bright lines in the spectra of several stars of the fourth type, which he has been able to see with the aid of the 36 cm Steinheil refractor of the Upsala Observatory, in spite of their invisibility with the smaller instrument at Lund.

It therefore seems safe to affirm that the spectrum of 152 *Schjellerup*, as well as the spectra of many other stars of the fourth type, contain several bright lines. The elevenfold enlargements from our negatives, which are reproduced in Plate VI, will serve to give an idea of the appearance of some of these lines in the spectrum of the star just named.[1] Some of the brightest of the lines are in the region between *Ha* and *D*, which is not shown to advantage in the plate, owing to the fact that the only photograph of this part of the spectrum hitherto made was taken with a rather inefficient optical combination (one prism and 508 mm camera). The photographs of the more refrangible regions were made with three prisms and the 271 mm camera, and are much more satisfactory. The less refrangible part of the bright zone in the yellow should be much brighter than it appears in the plate, the photograph having suffered from the rapid drop in the sensitiveness of isochromatic plates just at this

[1] These photographs have not been retouched, but the contrast of the one covering region λ 5300—5700 was increased photographically in enlarging the original negative.

point. Many of the apparent bright lines in the more refrangible part of the zone we have taken to be true bright lines, after much study of the photographs. In addition to the bright lines which fall upon the yellow carbon band, there is a fine group in the green. There is room for doubt as to the true character of some of the apparent bright lines in the blue, and, in certain cases, both here and in the less refrangible region, we have thought it best to measure both the dark and bright spaces, leaving the matter to be decided in the light of future work.

Although we do not desire, at the present time, to enter into a discussion of the physical condition of fourth type stars, a single inference may be drawn from the appearance of such a line as that at λ 5592 in the spectrum of 152 *Schjellerup*. It would seem probable that the substance to which this line is due must exist in the star's atmosphere at a level above that of the carbon or hydrocarbon vapor which produces the heavy absorption bands. The case is apparently analogous to that of the Sun's atmosphere, where the hydrogen and calcium extend above the layer of carbon vapor. As one of us has recently shown, this layer is extremely thin, and lies at the base of the chromosphere.[1]

THE UNIVERSITY OF CHICAGO,
Yerkes Observatory, May 4, 1899.

[1] GEORGE E. HALE, "On the Presence of Carbon in the Chromosphere," this JOURNAL, 6, 412, December 1897.

ON THE INTERPRETATION OF THE TYPICAL SPECTRUM OF THE NEW STARS.[1]

By J. WILSING.

THE most striking phenomenon in the spectrum of Nova Aurigae (1892) was the occurrence of pairs of lines, consisting each of a bright line with an absorption line at its more refrangible edge. In view of the extensive and successful applications of Doppler's principle to astronomical problems which were being made just at that time, it was natural to explain the relative displacement by corresponding motions in the line of sight, and all of the hypotheses proposed at that time as to the nature of the new stars were based on the application of Doppler's principle. Great difficulties were presented, however, by the resulting extraordinary relative velocities in the line of sight of 1000 to 1500 kilometers, and the suspicion already existing, as to whether Doppler's principle was admissible in explaining the double spectrum, continually increased as no appreciable changes in the relative position of the double lines could be perceived during several months; this was especially the case when in the spectrum of Nova Normae (1893) the components of the pairs of bright and dark lines were also displaced in the same direction as in case of Nova Aurigae. Since, further, two other objects, β Lyrae and *P* Cygni (the Nova of 1600), also exhibited the typical spectrum of the temporary stars, in the first case mixed in with displacements of the lines which could be accounted for on Doppler's principle by orbital motion, an accidental occurrence of displacements in the same direction seemed highly improbable, and attempts accordingly had to be made to find another explanation.

Observations of different sorts on changes in the wave-lengths of spectral lines which are not to be attributed to motions are

[1] *Sitzungsberichte der k. Akademie der Wissenschaften zu Berlin.* Session of the physical-mathematical section on May 4, 1899.

available. First in the case of fluorescing bodies changes of wave-length occur, the bright bands being for the most part less refrangible than the corresponding absorption band, and being displaced with different concentration. According to E. Wiedemann and G. C. Schmidt[1] this is a rule for all luminescence phenomena in which the light is developed at low temperatures.

But double spectra can also be produced within the range of validity of Kirchhoff's law by superposition of different masses of hot gas. According to Paschen's[2] bolometric measures the emission bands of carbonic acid are displaced toward the red with rising temperature. If the hot gases were placed between the spectroscope slit and a source of light giving a continuous spectrum of less intensity, the cooler carbonic acid, mixed with the air, absorbed, on the way from the slit to the bolometer, a group of rays which lay on the more refrangible edge of the emission band of the heated carbonic acid.

The investigations of W. J. Humphreys and J. F. Mohler[3] are of special importance in the physical explanation of double spectra. They were the first to notice slight displacements of lines in the visual and ultra-violet part of the arc-spectrum of the metals as they raised to twelve atmospheres the pressure in the enclosure about the arc. The lines always moved toward the less refrangible end of the spectrum with increasing pressure, and it was found that the amount of the displacement was in general proportional to the pressure, and to the wave-length of the line, but varied for the lines of different metals, and for different series of lines of the same metal. The increase of wave-length for a pressure of twelve atmospheres indeed hardly reached 0.05 tenth-meters, so that the employment of very

[1] "Zur Mechanik des Leuchtens," *Wied. Ann.*, **37**, 177; "Fluorescenz des Natrium- und Kaliumdampfes," *Wied. Ann.*, **57**, —; "Ueber Lichtemission organischer Substanzen in gasförmigem, flüssigem und festem Zustand," *Wied. Ann.*, **56**, 18.

[2] "Ueber die Emission der Gase," *Wied. Ann.*, **51**, 1; "Bolometrische Arbeiten," *Wied. Ann.*, **53**, 287.

[3] "Effect of pressure on wave-lengths of lines in the arc-spectra of certain elements," this JOURNAL, **3**, **4**, **5**, **6**.

powerful dispersion was necessary to establish so slight a displacement.

Larger displacements, similarly in the direction of increasing wave-lengths, were found by Eder and Valenta[1] in the spectrum of argon and of sulphur by raising the pressure and employing the spark. The shifts of the sulphur lines amounted here to about 0.5 t.m., and those of the argon lines in some cases even reached 1 tenth-meter, being accompanied by much broadening and diffuseness.

The small displacements (averaging 0.4 t.m.) observed by Ebert[2] in the flame spectra of the easily volatile metallic salts on increasing the amount of vapor are attributed by him to an unsymmetrical broadening of the lines toward the red. Gouy[3] similarly noticed shifts of certain definite lines in the spectrum of sodium and sulphur which he vaporized in the arc. But here also the possibility of a rise of pressure in consequence of violent vaporization cannot be excluded.

The observations last cited suggest the direction which must be taken in the experiments for producing shifts of lines without motion in the sight-line, and ultimately for producing double spectra. It therefore becomes the problem to increase the magnitude of the displacement so that it may be of the same order as that in the double spectrum of Nova Aurigae, in which the relative displacement of the bright and dark lines amounted to from 10 to 20 tenth-meters.[4]

According to the observations of Humphreys and Mohler this should be attained by continued increase of pressure up to several hundred atmospheres. In seeking to avoid the experimental difficulties connected with so great a rise of pressure the

[1] "Spectralanalytische Untersuchung des Argons." "Die Spectren des Schwefels." *Denkschriften der k. Akad. der Wiss. zu Wien.*, 64, 67.

[2] "Die Methode der hohen Interferenzen in ihrer Verwendbarkeit für Zwecke der quantitativen Spectralanalyse." *Wied. Ann.*, 34.

[3] "Sur l'élargissement des raies spectrales des métaux." *C. R.*, 108.

[4] H. C. VOGEL, Uebe den neuen Stern im Fuhrmann. *Abhandlungen der k. Akad. der Wiss. zu Berlin 1893.*

idea occurred to me of investigating the spectra of spark discharges in liquids in which it is known that very high tensions arise.

Masson[1] early investigated the spectra of the discharge from condensers and of the arc in various liquids, but he found bright lines only in the arc with the use of metallic electrodes. Later Planté, Righi, Slouguinoff, and others studied the optical phenomena attending the discharges in liquids, but without going into the subject of the spectrum. But in the spectrum of the arc passing between platinum and silver electrodes in solutions of salts of sodium and lithium, Colley[2] found, beside the hydrogen lines, several lines of these metals, and also a series of platinum lines. Similarly Liveing and Dewar[3] observed lines due to the platinum electrodes in the spectrum of the spark discharge in liquid gases and air. A more accurate investigation of the lines in respect to position and appearance has, however, not yet been carried out.

I employed in my experiments a large inductorium, in the secondary circuit of which a spark-gap was inserted before the electrodes in the usual way in addition to the battery. With the passage of each spark a blinding discharge took place between the electrodes in the water, giving a very intense continuous spectrum crossed by faint lines. As the brightness of the continuous background and the flickering due to the irregularity of the discharge hindered the direct visual measurement of the lines, I photographed the discharge spectra in water and air on the same plate with a spectrograph, thus rendering possible a convenient and accurate determination of the relative position of the systems of lines of the two spectra. The length of the prismatic spectrum between λ 4800 and λ 4600 was about 50 mm, and the wave-lengths of sharp lines could be determined within a few hundredths of a tenth-meter. Beside this, Professor

[1] "Études de photométrie électrique." *Ann. de Chim. et de Phys.*, 31, 1851.

[2] *Journal de Physique*, Ser. 1, 9.

[3] "Preliminary note on the spectrum of the electric discharge in liquid oxygen, air, and nitrogen." *Phil. Mag.*, 38, 1894.

Lohse and Dr. Hartmann were kind enough to make me several plates with a grating spectrograph of high dispersion, and with a large prism spectrograph.

I have investigated the spectrum of the discharge in water of iron, nickel, platinum, copper, tin, zinc, cadmium, lead, and silver. In the spectrum of iron numerous pairs, consisting of a bright and a dark line, appear, the bright component being displaced considerably toward the less refrangible end of the spectrum, while the absorption lines suffer an appreciable displacement toward the red in only a few instances. Fine maxima of intensity can also be recognized in a few of the bright lines. There are, moreover, isolated bright lines which are similarly displaced appreciably toward the red. The lines are quite faint and hazy on the less refrangible edge. In the following table column 1 contains the wave-lengths of the lines whose displacement was measured; in column 2, E and A denote, respectively, emission and absorption lines; the displacements measured on different plates are given in the following columns, plus denoting displacements toward the red, and minus toward the violet; the last column contains remarks as to the appearance of the lines.

The increase of wave-length is largest for the lines which are bounded by absorption lines on the violet side. But here the measurement probably gives too large values to the displacement,[1] for if two strata of vapor are present in the neighborhood of the electrodes, the inner and hotter of which gives broadened and displaced lines, while the outer and cooler gives a normal spectrum with narrow lines, then the portion of the bright lines lying toward the violet must be neutralized by the absorption of the cooler vapor, and there remains only the portion on the less refrangible side of the absorption line caused by the broadening. A setting on the middle of the bright line therefore necessarily gives too large a displacement. Direct observation shows, however, that very often the emission and

[1] EDER und VALENTA, "Ueber den Funkenspectrum des Calciums und Lithiums und seine Verbreiterungs- und Umkehrungserscheinungen." *Denkschriften der Kais. Akad. Wien.*, **67**, 8, 1898.

IRON.

Kayser and Runge	Kind of line	Plate 1	Plate 2	Plate 3	Remarks
Tenth-meters		Tenth-meters	Tenth-meters	Tenth-meters	
3737.27	A	+0.03			
	E	+1.12			
3749.61	A	—0.09			
	E	+0.81			
3765.66	E	+0.13			
3767.31	A	—0.05			
	E	+0.17			
3797.65	E	+0.17	+0.28		
3813.12	A	—0.20			
	E	+0.07			
3815.97	A	—0.07	—0.22		
	E	+0.51	+1.12		Very distinct
3820.56	A	—0.20		+0.10	
	E	+0.93		+0.90	Very distinct
3827.96	A	—0.10	—0.16	0.00	
	E	+0.92	+0.106	+0.76	
3834.37	A	—0.07			
	E	+0.36			
3841.19	A	—0.08		0.00	
	E	+0.43	+0.58	+0.54	Distinct
3843.40	E	0.00		+0.08	
3846.96	E	+0.24			
3850.11	E	+0.32			
3860.03	A	—0.07			
	E	+0.95			
3865.65	E	+0.32			
3888.63	E	+0.40		+0.38	
3903.06	E	+0.34			
4071.79	A			+0.11	Sharply bounded; distinctly displaced
4107.58	E			+0.20	
4109.88	E			+0.22	
4118.62	E			+0.22	Very distinct; sharply bounded
4132.15	E			+0.33	
4181.85	E			+0.22	Weak, but distinct
4199.19	E			+0.16	Weak, but distinct
4260.64	E	+1.00			Very distinct
4271.30	A	—0.05			
4271.93	E	+0.76			Sharply bounded maximum of intensity
4307.96	A	—0.05			
	E	+1.12			Sharply bounded maximum of intensity
4383.70	A	0.00			
	E	+1.33			

NICKEL.

Hasselberg	Kind of line	Plate 1	Plate 2	Remarks
Tenth-meters		Tenth-met'rs	Tenth-met'rs	
3807.30	A	+0.01	—0.06	Faint
	E	+0.36	+0.43	
3858.40	A	+0.01	—0.07	Very distinct
	E	+0.39	+0.38	Edges of a strong band, diffuse on the red side
		+2.09	+1.70	
4401.70	E	+0.19	0.00	Edges of a diffuse band
		+4.08	+3.53	
4459.21	E	+0.19		Edges of a sharply-bounded band
		+5.82		

COPPER.

Kayser and Runge	Kind of line	Plate 1	Remarks
Tenth-meters		Tenth-met'rs	
4275.32	E	—0.05	Edges of a strong band sharply bounded on both sides
		+3.83	
4378.40	E	—0.10	Edges of a strong band sharply bounded on both sides
		+2.94	
4539.98	E	—0.43	Edges rather weak, more sharply bounded toward violet
		+5.10	
4587.19	E	—0.44	Edges more sharply bounded toward violet
		+5.01	
4651.31	E	—0.01	Edges of a band sharply bounded on both sides
		+3.93	

ZINC.

Kayser and Runge	Kind of line	Plate 1	Remarks
Tenth-meters		Tenth-met'rs	
4680.38	E	+0.64	Edges of a bright band, sharper toward violet
		+7.31	
4722.26	E	+1.43	Edges of a band, sharp toward violet, very diffuse toward red
		+13.20	
4810.71	A	—0.05	Edges of a strong absorption line
		+1.71	
		+1.71	Edges of a bright band, diffuse toward red
		+8.08	

TIN.

Kayser and Runge	Kind of line	Plate 1	Plate 2	Remarks
Tenth-meters		Tenth-met'rs	Tenth-met'rs	
3801.16	A	—0.07	+0.01	Sharply bounded
	E	+0.50	+0.93	Edges of a very faint band
		+5.52	+4.14	
4524.92	E	—0.26	—0.10	Very faint band, not sharp
		+4.84	+5.78	

CADMIUM.

Kayser and Runge	Kind of line	Plate 1	Plate 2	Remarks
Tenth-meters		Tenth-met'rs	Tenth-met'rs	
4413.23	E	—1.40	—0.99	Edges of a diffuse band
		+1.17	+0.88	
4678.37	E	—0.10	—0.10	Edges of a band sharp toward violet,
		+8.19	+11.01	diffuse toward red
4800.09	E	+1.52	+1.49	Edges of a band sharp toward violet,
		+8.40	+8.17	diffuse toward red

absorption spectra change places with each other, varying according to the intensity of the discharge and of the development of vapors at the electrodes, so that the two spectra overlie each other in the image.

In the spectrum of nickel a very distinct double line appears at λ 3858.40; the bright lines are broader and are more displaced toward the red than in the iron spectrum.

Aside from a slight diffuseness no appreciable displacements could be recognized in the spectrum of the discharge between platinum electrodes and water.

But the bright lines in the spectrum of copper become broad bands, and in the case of tin, zinc, and cadmium gain still more and are so greatly displaced that they can be easily perceived without magnification. It should be particularly mentioned that the more refrangible edge of the individual bands is so strongly shifted toward the red, that it lies entirely beyond the corresponding line in the normal spectrum. Narrow maxima of

intensity sometimes appear within the bright bands. The bands are sharply bounded toward the violet, but more or less diffuse toward the red. Still there is on the less refrangible side, especially in the bands of the copper spectrum, a distinct boundary where the intensity on the otherwise uniformly bright band rapidly falls off. Sharply bounded absorption lines, such as in iron and zinc, were not present in the portion investigated of the spectrum of copper and cadmium, but in the zinc spectrum there is a strong absorption line with an appreciable displacement toward the red. It is remarkable that the strong cadmium line at λ 4413.23 is not displaced, being only symmetrically broadened.

Nothing could be seen of the fine lines in the silver spectrum on the discharge in water, the spectrum being entirely continuous.

In the spectrum of magnesium there appeared in place of the triplets

$$\lambda\,3829.51,\ \lambda\,3832.46,\ \lambda\,3838.44$$

some very diffuse absorption lines without appreciable displacement; at 4481.4 an extremely broad and dim emission band could be perceived on some of the plates without using any magnifying power, the middle of which, however, coincides with the corresponding line in the normal spectrum. For magnesium I have also investigated the less refrangible portion of the spectrum. On direct examination of the spectrum the *b* lines appeared sometimes dark and sometimes they disappeared entirely and were replaced by a very broad and bright band, diffuse toward the red, the violet edge of which is more refrangible than b_4. This band, which also appears simultaneously with the absorption lines, can readily be perceived on the spectrograms, and, since it shows no trace of resolution into single lines, probably is not identical with the bands at λ 518 $\mu\mu$ and 521 $\mu\mu$, observed by Liveing and Dewar[1] in the magnesium spectrum when in the presence of hydrogen, since the latter bands are resolvable and shaded off toward the violet. Moreover,

[1] "Investigations on the spectrum of magnesium." *Proc. R. S.*, 32 and 34.

the more refrangible bands of the magnesium-hydrogen spectrum at λ 480 and 485 $\mu\mu$ are lacking on the plates. But there is a fine absorption line present at λ 521 $\mu\mu$ which coincides with the edge of a magnesium-hydrogen band. The bands of magnesium oxide at λ 4995.6 and 5006.4 are faintly indicated.

In the spectrum of lead a broad, faint absorption line is visible at λ 4058.0; at λ 4245.3 and λ 4386.4 there also occur extremely broad, faint and diffuse bright bands which extend from the place of the line in the normal spectrum toward the red.

The magnitude of the displacement and the broadening of the metallic lines is, indeed, of a similar order in case of the different plates, but noticeable differences nevertheless occur, which are in part to be attributed to the different duration of exposure and development of plates, but chiefly to the varying intensity of the discharge, which changes with the strength of the current and the distance of the electrodes. Before deciding the question of what influence the molecular forces acting between the particles of vapor and liquid would have on the appearance of the spectrum it is necessary to compare the discharge in different liquids. I have investigated the spectra of the discharges between iron electrodes in water and in alcohol only, and have found no appreciable differences; on employing oils the liquid was quickly clouded by decomposition and union with the substance of the electrodes, so that it is not possible to obtain a plate of the spectra without special arrangements.

A comparison of the displacements of the lines for the different metals shows, in agreement with the following measures of Humphreys and Mohler, that they are considerably greater in the spectrum of tin, zinc, and particularly cadmium, than for iron and platinum.

DISPLACEMENTS AT A PRESSURE OF 12 ATMOSPHERES.[1]

Platinum, 0.020	Copper, 0.033	Zinc, 0.057
Iron, 0.025	Tin, 0.055	Cadmium, 0.080
Nickel, 0.028		

If we assume with Humphreys and Mohler a proportionality between pressure and displacement, then the pressure which the volatile gases undergo on the discharge in water must amount to several hundred atmospheres.

In the metallic spectra obtained in the manner described above there now occur displacements of lines and double lines which are in every respect similar to those in the spectra of Nova Aurigae. In the spectrum of that star the middle of the much broadened bright lines, sharply bounded toward the violet and diffuse toward the red, were considerably displaced toward the less refrangible end of the spectrum. Occasionally quite sharp maxima of intensity appeared in the bright lines in the star's spectrum, just as was observed in the artificial spectra in some cases. We can therefore imagine the star's spectrum to have originated in the superposition of the absorption spectrum, as it indicates a slight vapor pressure, the dark lines being partially brightened by the bright and much broadened and displaced lines, and thus undergoing an apparent displacement toward the violet.

The fact that in the spectrum of the Nova the duplicity was especially marked in the case of the hydrogen lines is not contradictory to the assumption that the luminous gases in the photosphere were under considerable pressure. Since a long series of experiments, of which I shall report in full in another place, gave the result that the hydrogen spectrum will become continuous by broadening of the lines with increase of pressure, when at the same time the potential and the temperature of the discharge increases, as is the case in Geissler tubes, with a fixed distance of the electrodes. If, however, the induction current passes through the tube without a jar by a sufficient decrease of

[1] This JOURNAL, 3, 135, 1896.

the distance of the electrodes, there appears between the electrodes white phosphorescence which, even at an atmospheric pressure, displays the lines of the hydrogen spectrum with the same degree of sharpness as with the pressure of a few millimeters. The assumption that the hydrogen lines undergo similar displacements to the metallic and argon lines on increase of pressure might therefore be permissible, and I hope soon to be able to make the necessary experiments on this subject. In view of the tendency of the hydrogen lines to broaden with increasing pressure we must assume that the temperature in the photosphere of the Nova was relatively slight.

With the great accuracy which the determination of the displacements of lines in stellar spectra for purposes of measuring the velocity has reached, the changes of wave-lengths by pressure can no longer be neglected. In a comparison of the wave-lengths of Rowland's solar spectrum with the wave-lengths of the corresponding metallic lines it is true that Jewell[1] has found only a relative displacement of the two systems of from 0.01 to 0.02 tenth-meters, which corresponds to a pressure of a very few atmospheres, and the pressure in the photosphere of the stars in general can hardly be assumed to be higher. If, however, quantities of this order are to be taken into account in measurements of velocity whose accuracy is within a kilometer, then it is only necessary to determine the difference of the displacements dependent alone upon the pressure in the photosphere, for two lines whose displacement for an increase of pressure of one atmosphere are known. If we denote by p the pressure, by v and V the radial velocity of the star in the line of sight and the velocity of light, by a and a_1 two constants, and by l_1 and l_2 the measured displacements, we shall have

$$p\, a_1 + \frac{\lambda_1 v}{V} = l_1 ,$$

and

$$p\, a_2 + \frac{\lambda_2 v}{V} = l_2 ,$$

[1] "The Coincidence of Solar and Metallic Lines." This JOURNAL, 3, 1896.

whence
$$v = V\frac{a_2 l_1 - a_1 l_2}{a_2 \lambda_1 - a_1 \lambda_2}.$$

In order to be able to calculate the velocity v of the motion in the line of sight it is necessary only to know the ratio $\frac{a_1}{a_2}$. This ratio could be directly determined for lines of different metals by measurements of the displacements which these lines undergo in the spectrum of the discharge in liquids, if alloys of the metals were employed as electrodes, since in this case the pressure would probably be the same for both metals.

MINOR CONTRIBUTIONS AND NOTES

THE NEW ALGOL VARIABLE IN CYGNUS.[1] $+45°$ 3062.

AN announcement is made in the *Astronomische Nachrichten*, **149**, 271, that the star $+45°$ 3062, R.A. $= 20^h\ 2.4^m$, Dec. $= +45°\ 53'$ (1855), mag. 8.6, is a variable star of the Algol type. Mme. L. Ceraski, of Moscow, found it abnormally faint on a photographic plate taken on May 20, 1898, and M. S. Blajko, after observing it visually for a long time, found it again at minimum on May 7, 1899. An examination was accordingly made of the Draper Memorial photographs to determine the nature of the variation. The region was covered by 195 plates, 170 of which showed the star at its full brightness, including 28 taken in 1890, 18 in 1891, 22 in 1892, 17 in 1893, 13 in 1894, 17 in 1895, 21 in 1896, 17 in 1897, 12 in 1898, and 5 in 1899. Besides these, twenty plates show the star when it was below its normal brightness. From a discussion of these plates it appears that the minima they indicate, as well as the two minima found at Moscow, may be closely represented by the formula *J.D.* $2{,}411{,}343.605^d + 4.57294^d\ E$. The period, therefore, is $4^d\ 13^h\ 45^m\ 2^s$, with an uncertainty which probably does not exceed one or two seconds. The variation in brightness of this star amounts to about three magnitudes, and, therefore, exceeds that of any Algol star hitherto discovered. Like all other Algol stars, its spectrum is of the first type.

The announcement of the discovery of this variable reached this Observatory on June 1. On June 3, the elements and ephemeris had been determined just in time to prepare for the minimum of that night. Accordingly, the star was followed all night by Professor Wendell, assisted by Mr. Leon Campbell, and 272 settings were made with the photometer attached to the 15-inch equatorial. From these it appears that at 16.0^h G. M. T., it was 0.20 magnitude brighter than the comparison star, $+45°$ 3067, while at 19.9^h G. M. T., when observations were stopped by the dawn, it was 2.25 magnitudes fainter than the same comparison star, although it was still 1.5^h before the predicted minimum. Observations by Argelander's method were also made all night

[1] *Harvard College Observatory Circular* No. 44.

by Mr. Wm. M. Reed, with the 6-inch equatorial. Meanwhile, thirty photographic images were obtained by Mr. H. R. Colson, assisted by Mr. E. R. Cram.

The minima so far observed are given in chronological order in the following table, including, on June 3 and 8, only the photographs taken with the 8-inch Draper telescope. The value of E is given in the first column. The second column gives the designation of the plate, A denoting the 24-inch Bruce telescope, B the 8-inch Bache telescope, and I the 8-inch Draper telescope. B 1719, I 231, I 907, I 1303, I 3719, I 7744, and I 11504 are spectrum plates. The date on which the photograph was taken, the Greenwich Mean Time, the Julian Day omitting the three left hand figures, 241, and the fraction of a day following Greenwich Mean Noon, and the length of exposure, are given in the next four columns. The photographic magnitude is given in the seventh column, the computed time of minimum in the eighth, and the observed minus the computed in the ninth column. The tenth column gives a correction for the magnitude of the star derived from the figures given in the seventh and ninth columns. The sign is indeterminate, and corresponds with the assumption that the period is uniform. The corrected residuals are given in the eleventh column. No correction has been applied for the light equation. The error from this cause is small, since the star is not very far from the pole of the ecliptic. The last column gives the error in the observed magnitude assuming the computed magnitude to be correct.

OBSERVED MINIMA.

E	Plate	Date	G.M.T.	J. D.	Ex.	Mag.	Com	O-C	Corr.	O-C	Err.
		y m d	h m	d	m			d	d	d	
−176	B 1719	1887 9 23	13 10	0538.549	70	< 9.1	0538.768	−.219	......	......	.....
0	I 231	1889 12 6	11 28	1343.478	113	9.83	1343.605	−.127	+.132	+.005	−.03
+ 25	I 907	1890 3 30	20 49	1457.867	87	< 9.9	1457.928	−.061	......	......	.....
42	I 1303	1890 6 16	17 41	1535.737	75	< 9.9	1535.668	+.069	.....	......	.....
52	I 1540	1890 8 1	14 38	1581.610	13	9.17	1581.398	+.212	−.224	−.012	−.06
59	I 1777	1890 9 2	14 48	1613.617	13	9.27	1613.408	+.209	−.203	+.006	+.03
121	I 3719	1891 6 12	18 57	1896.790	60	9.90	1896.931	−.141	+.126	−.015	+.21
244	I 7744	1892 12 26	12 44	2459.531	59	9.89	2459.402	+.129	−.126	+.003	+.05
297	I 9300	1893 8 25	13 53	2701.578	11	9.32	2701.768	−.190	+.191	+.001	.00
351	A 582	1894 4 29	20 10	2948.840	12	9.92	2948.707	+.133	−.125	+.008	+.11
384	I 11504	1894 9 27	13 37	3099.567	70	< 10.2	3099.614	−.047	......	......	.....
387	I 11585	1894 10 11	13 1	3113.542	11	9.17	3113.333	+.209	−.224	−.015	−.08
471	I 13759	1895 10 30	13 47	3497.574	10	10.21	3497.460	+.114	−.113	+.001	+.06
503	I 14711	1896 3 24	19 30	3643.812	26	11.78	3643.794	+.018	−.031	−.013	−.01
517	I 15182	1896 5 27	16 43	3707.697	12	10.11	3707.815	−.118	+.116	−.002	+.03
522	I 15328	1896 6 19	17 30	3730.729	11	11.68	3730.680	+.049	−.047	+.002	.00

E	Plate	Date	G.M.T.	J. D.	Ex.	Mag.	Comp.	O-C	Corr.	O-C'	Err.
		y m d	h m	d	m		d	d	d	d	
+543	I 16009	1896 9 23	16 19	3826.680	13	11.88	3826.711	—.031	+.010	—.021	+.09
555	I 16569	1896 11 17	10 39	3881.444	11	9.61	3881.587	—.143	+.149	+.006	—.07
632	I 19445	1897 11 4	13 10	4233.549	11	9.72	4233.703	—.154	+.139	—.015	+.16
675	......	1898 5 20	.....	4430.4	..	.....	4430.339	+.061	......	......	.....
749	I 22770	1899 4 23	20 14	4768.843	23	10.23	4768.737	+.106	—.112	—.005	—.15
752	......	1899 5 7	10 54	4782.446	..	.....	4782.456	—.010	......	......	.....
758	I 22981	1899 6 3	15 55	4809.663	10	9.22	4809.894	—.231	+.222	—.009	+.08
758	I 22982	1899 6 3	16 57	4809.706	11	9.60	4809.894	—.188	+.151	—.037	+.01
758	I 22986	1899 6 3	19 40	4809.819	12	11.38	4809.894	—.075	+.067	—.008	+.20
758	I 22987	1899 6 3	20 6	4809.837	4	11.54	4809.894	—.057	+.057	.000	.00
758	I 22988	1899 6 3	20 13	4809.842	1	$<$10.8	4809.894	—.052	......	......	.....
759	I 22995	1899 6 8	14 00	4814.583	13	9.95	4814.466	+.117	—.122	—.005	—.17
759	I 22996	1899 6 8	14 14	4814.593	10	9.85	4814.466	+.127	—.129	—.002	—.06
759	I 22997	1899 6 8	14 29	4814.603	15	9.88	4814.466	+.137	—.127	+.010	+.14
759	I 22998	1899 6 8	14 43	4814.613	10	9.58	4814.466	+.147	—.153	—.006	—.06
759	I 22999	1899 6 8	14 53	4814.620	10	9.55	4814.466	+.154	—.156	—.007	—.01
759	I 23000	1899 6 8	15 9	4814.631	19	9.55	4814.466	+.165	—.156	+.009	+.07
759	I 23003	1899 6 8	16 32	4814.689	14	9.22	4814 466	+.223	—.213	+.010	+.06

E 675. This is the date on which the plate was taken from which Madame Ceraski discovered the variable. The time is not given, but, owing to the northern latitude of Moscow, it has been assumed to be near midnight. As the brightness is not stated, no correction for magnitude can be applied.

E 752. Found by M. Blajko from visual observations.

E 758. The last plate, I 22988, was taken in strong twilight so that the Pole Star was barely visible. The plate was not fogged, but the star had become too faint to be photographed.

On five plates, taken at 0519.592, 1720.483, 1935.849, 3058.784, and 3859.525, the variable appears to be about two tenths of a magnitude below its maximum brightness, 8.96, but the phase shows that it was not at minimum. On a few plates the variable appears a little brighter than normal, but these small variations are probably due to photographic effects, such as distance from center of plate, or difference in color, which affects chart images differently from spectra.

The average value of the residuals in the last column is $\pm$ 0.07. It will, therefore, be seen that the formula given above serves to compute the magnitudes for the last ten years with such accuracy that they differ from the measured values on the average by less than a tenth of a magnitude. Even these small differences could doubtless be diminished by applying the correction for the light equation, by correcting

the light curve, since the positive residuals slightly exceed the negative, and by remeasuring the more discordant plates. In each of the five cases in which the star is not seen, indicated by the sign < followed by the magnitude of the faintest star visible on the plate, computation shows that the variable must have been fainter than this magnitude. An ephemeris for the remainder of the year is given below.

EPHEMERIS OF HELIOCENTRIC MINIMA.

E	J. D.	Min. 1899				E	J. D.	Min. 1899				E	J. D.	Min. 1899			
		m	d	h	m			m	d	h	m			m	d	h	m
758	4809.89352	6	3	21	26	774	4883.06056	8	16	1	28	790	4956.22760	10	28	5	28
759	4814.46646	6	8	11	11	775	4887.63350	8	20	15	12	791	4960.80054	11	1	19	13
760	4819.03940	6	13	0	56	776	4892.20644	8	25	4	57	792	4965.37348	11	6	8	57
761	4823.61234	6	17	14	41	777	4896.77938	8	29	18	42	793	4969.94642	11	10	22	42
762	4828.18528	6	22	4	26	778	4901.35232	9	3	8	27	794	4974.51936	11	15	12	27
763	4832.75822	6	26	18	11	779	4905.92526	9	7	22	12	795	4979.09230	11	20	2	12
764	4837.33116	7	1	7	56	780	4910.49820	9	12	11	58	796	4983.66524	11	24	15	57
765	4841.90410	7	5	21	42	781	4915.07114	9	17	1	42	797	4988.23818	11	29	5	43
766	4846.47704	7	10	11	27	782	4919.64408	9	21	15	27	798	4992.81112	12	3	19	28
767	4851.04998	7	15	1	12	783	4924.21702	9	26	5	12	799	4997.38406	12	8	9	13
768	4855.62292	7	19	14	57	784	4928.78996	9	30	18	57	800	5001.95700	12	12	22	58
769	4860.19586	7	24	4	42	785	4933.36290	10	5	8	43	801	5006.52994	12	17	12	43
770	4864.76880	7	28	18	27	786	4937.93584	10	9	22	28	802	5011.10288	12	22	2	28
771	4869.34174	8	2	8	12	787	4942.50878	10	14	12	13	803	5015.67582	12	26	16	13
772	4873.91468	8	6	21	57	788	4947.08172	10	19	1	58	804	5020.24876	12	31	5	58
773	4878.48762	8	11	11	43	789	4951.65466	10	23	15	43						

It will be noticed that nearly a year would have been saved had the original discovery of the variability of this star been sent here for confirmation from the photographs, or had it been announced publicly. There is so little chance for error in a photograph that such cases are always examined here. Confirmation is not always obtained. A striking instance of this kind is furnished by a photograph, X 7524, taken at Arequipa with the 13-inch Boyden telescope on May 22, 1896, at $14^h\ 20^m$ G. M. T. Miss A. J. Cannon found that this plate shows the spectra of *A. G. C.* 17312, 17407, and 17453, magns. 7.0, 7.2, and 7.5, respectively, but fails to show the spectrum of the brighter star *A. G. C.* 17270, mag. 6.0. Apparently this is an Algol star observed at one minimum only. On 153 other plates the star appears of its normal brightness. On a photograph, C 7354, taken at Cambridge with the 11-inch Draper telescope on December 18, 1894, at $11^h\ 8^m$ G. M. T., the star, $+42°$ 4182, mag. 9.1 was found by Miss L. D. Wells to be

absent, although stars two and a half magnitudes fainter were shown. On plate C 7353, taken twelve minutes earlier, and on 259 other plates it appears of its normal brightness. An adjacent defect in the film of the first plate is perceptible, and perhaps explains the absence of this star.

EDWARD C. PICKERING.

June 10, 1899.

NOTE ON METEOR PHOTOGRAPHY.

IN his note on meteor-photograms, taken simultaneously at Cambridge and Blue Hill (*Harvard College Observatory Circular* No. 40), Professor Pickering describes a method for determining by photography radiants and velocities of bright sporadic meteors.

At both stations the cameras were pointed to the zenith and clamped; from the plates obtained thus the declination of the radiant is found. In order to find Right Ascension, Professor Pickering proposes to install at both stations equatorially mounted cameras, driven by clockwork. To find the angular velocity he intends to photograph the spectra of the meteors on a plate oscillating like a pendulum.

I wish to point out that Declination, Right Ascension, angular velocity, and time of apparition can be found easily by four cameras, pointed to the zenith and fixed, three of them at one station, the fourth at another station. In the first of the three cameras the plate is fixed, in the second it revolves once in the time of exposure, say in four, six, or eight hours. The third plate rotates quickly, say once in eight seconds.

The mechanism of the cameras permits them to be pointed exactly to the zenith, and the image of the zenith-point coincides with the center of rotation, which is indicated beforehand on each plate by a circle traced near the periphery. The positions of the platés in cameras 1 and 2 at the beginning of exposure is marked by equal dashes on the periphery of this circle.

The meteor's trails on both plates have the same form and position. If we now superpose the two plates and bring both trails and centers into coincidence, we can find the angular displacement of the two dashes, from this the time of apparition, and from that time the position of the stars around the zenith at both stations. We can now transfer both trails onto a chart of stars and deduce the Declination and Right Ascension of the radiant.

The quickly rotating plate in the third camera serves to determine the angular velocity of the meteor at every point of its trace. By the rotation of the plate and of the meteor's trace a characteristic curve is formed, showing at first sight the direction of the meteor's trace. The origin of the trails will be continued in the same direction as the rotation of the plate.

By superposing this curve and the trail upon the fixed plate, different intersections in different positions of the plate will be obtained. The distance between the origin of the trail on the fixed plate and between the intersection is equal to the arc described by the meteor during the time deduced from the angular displacement of the origin of trail on the quickly rotating plate and on the first fixed plate.

My method is equally applicable with all four cameras directed to a point different from the zenith.

JOSEF JAN FRIČ.

PRAGUE,
April 15, 1899.

REVIEWS.

Die Photographie der Gestirne, von DR. J. SCHEINER, Professor der Astrophysik an der Universität Berlin und Astronom am K. Astrophysikalischen Observatorium zu Potsdam (pp. iv + 382, 1 plate and 52 figures, with an atlas of 11 plates. Leipzig: Engelmann, 1897).

ASTRONOMICAL photography may be said to have begun with Daguerre himself, who upon Arago's suggestion took a photograph of the Moon in 1839. While this photograph showed none of the details of the Moon's surface yet it suggested the possibilities that lay in photography as an aid to astronomy. How great these possibilities were, the achievements of the last few years have amply testified; and no doubt this branch of astronomy is still in its infancy if we may judge by the rapid increase in the number and importance of the researches that are made with its aid.

Excepting the excellent little work by Konkoly (*Anleitung zur Himmelsphotographie*), which deals mainly with the manipulation of instrument and plates, no treatise on the subject has appeared until the present work by Professor Scheiner. This lack is no doubt due to the rapid strides which are being made in the subject and the large number of questions still unanswered. With the risk of being soon behind date, Professor Scheiner has wisely thought it time to put together in one convenient volume an account of whatever has been done on the subject. The work may be considered as making one of a uniform Potsdam series, the others being *Die Spectralanalyse der Gestirne*, 1890, by the same author, and *Die Photometrie der Gestirne*, 1897, by Professor Dr. G. Müller.

The author divides his subject into three parts :

I. The production of astronomical photographs and their utilization.

II. Photographic photometry and the nature of photographic images.

III. History of astronomical photography and the results it has yielded.

Part I opens with a discussion of the peculiarities of various emulsions, developers and processes of development, as a result of which the author is enabled to set down a number of valuable precepts, one or the other of which is to be followed according to the requirements of the case in hand. For example, if a photograph is taken with the intention of measuring it, the treatment will be different from that employed where mere detail is sought after in the object photographed. The author points out very clearly the requirements which distinguish an objective intended for photographic work from the ordinary refractor. Some points that are of little or no interest in the latter case become important for photography. Thus *diffraction rings* occasion little trouble in visual observations because of their faintness compared with the central image; but the sensitive film of the photographic plate has the power of accumulating whatever light energy falls upon it, and images may thus be obscured by even faint diffraction rings. For the same reason *spherical aberration* must be corrected with the greatest care, and *ghosts* due to reflection from the lens surfaces must be avoided. It should be observed, however, that these cause annoyance only in long exposures. Photographs intended for measurement are usually exposed for only a few minutes at the most, and in this case the requirements are hardly more exacting than for visual work.

The author distinguishes two kinds of distortions to which photographic images are liable: *geometric*, or those due to the fact that the plate is plane instead of spherical, and *optical*, due to peculiarities in the objective. The former are well understood and easily allowed for, but the latter can hardly be predicted, and in the most refined work ought to be investigated for each particular instrument. For any objective in which spherical aberration has been corrected the image of a star which lies on the optical axis will be small, well-defined and capable of exact measurement. The case is different with a star far from the axis, and unless especial care has been taken in the manufacture of the objective the image of such a star will present an unsymmetrical appearance, the densest part not occupying the center. The measures of such images are difficult and liable to systematic error. Still a third kind of distortion is sometimes spoken of, namely, that of the sensitive film after development. The author adduces some experimental evidence to show that such distortions are not greatly to be feared. The reviewer may be permitted to add that, by the inter-comparison of eight plates[1] each of which contained the images of

[1] *Annals of the N. Y. Academy of Sciences*, 10, 273.

thirty-three particular stars in Praesepe, he found the probable error of a single coördinate to be

$$\pm 0.070''$$

which includes not only distortions of the film, but optical distortions, uncertainty in division errors and many other possible sources of error. Moreover, the plates were the old wet plates, taken in 1870 and 1877, but not measured until 1897; it is not too much to assume that distortions on such plates will be larger than on modern plates measured soon after exposure.

After an admirable chapter devoted to the description of particular photographic instruments, follows the longest and perhaps the most important chapter in the book, on the methods employed for the measurement and reduction of photographs. Here, as elsewhere, the author has adhered to the historical mode of treatment, and this, in the reviewer's opinion, lessens somewhat the value of the chapter. To an astronomer seeking information as to the best procedure to attain a particular end it would have been better to omit some of the earlier methods which, though undoubtedly of historical interest, have been superseded by later methods. It would have added much to the utility of the chapter to have had numerical examples, or better, the same example worked out by various methods. These methods have been reproduced by the author in the original notations and with essentially no change in the manner of presentation. The reason assigned for pursuing this course is that reference to the original memoirs is thus made easier, but the author has thereby neglected an opportunity for introducing a much needed uniformity of notation which would have been highly appreciated by the class of readers for whom the work is intended.

The author characterizes Turner's method as not being *rigorous*. It will be remembered that Turner applies no correction for refraction to the measured coördinates, and then shows that the "standard coördinates" of any star may be obtained by equations of the form:

$$X = a\,x + b\,y + c$$
$$Y = d\,x + e\,y + f.$$

If corrections for refraction had been applied we should have had four instead of six coefficients in the second members, because of the relations

$$a - e = 0\,,\ b + d = 0.$$

In other words, Turner obviates the necessity of computing the refrac-

tion and substitutes two least-square solutions of three unknowns each for a single one with four unknowns. But the solution in the latter case is by no means as formidable as is usual, because of the way in which some of the coefficients are repeated in the equations of condition.[1] Moreover, we may employ abridged refraction formulæ,[2] which are simply applied and entail the use of no more than four unknowns. Thus Turner's method is very little if at all shorter than the method here very briefly outlined, and has the disadvantage of introducing two additional unknowns which can be determined with far greater precision from the well-understood laws of differential refraction than from the positions of a few images upon a photographic plate.

Two of the best methods for the reduction of single plates, those of Jacoby and Henry, do not appear in the work because, in one case at least, they were published too late to admit of insertion. The *réseau* is not mentioned in this chapter, and only a few words are devoted to it in a previous one. While originally intended to eliminate possible distortions of the film, the *réseau* permits much simplification in the measuring machine, and insures greater accuracy and rapidity in the measures. On the other hand, it has been found very difficult to make them sufficiently permanent, and no small labor is involved in the investigation of their errors. But the advantages arising from their use are so great that, no doubt, the efforts of astronomers and instrument makers will be directed toward making them more durable, and perhaps also easier of investigation.

The last chapter of Part I deals with the application of photography to the automatic registration of star-transits, etc. In spite of the great advantage arising from the annihilation of personal equation, the practical utility of such instruments must still be regarded as in the experimental stage.

Part II is the most complete exposition of photographic photometry which has as yet appeared. Many of the results are due to the author's own labors, and some are here announced for the first time. The most important practical problem in this subject is to determine the magnitudes of stars from a given plate. It thus becomes necessary to investigate the nature of photographic images and inquire how their diameters vary with the duration of exposure and the intensity of the star's light. Early investigators assumed the erroneous law,

[1] *M. N. of R. A. S.*, May 1896, and *Annals of the N. Y. Academy of Sciences*, **10**, 243.

[2] *Astronomical Journal*, No. 430.

intensity $\times$ time of exposure = a constant, that is, they took it for granted that if the time of exposure was doubled for a certain star, the resulting image would be identical with that of a star twice as bright. It is now well known that this law is not even a good approximation. Indeed, for long exposures, while the image steadily broadens, the center, instead of becoming more intense, may actually become fainter. The case is not dissimilar to that of some chemical solutions which form precipitates on the addition of certain reagents, but are again dissolved when the latter are added to excess. As to the law which governs the broadening of the image, no really satisfactory exposition has yet been given. So many and such complex causes seem to enter that perhaps the best that can be done is to determine the relation experimentally. The author gives preference to the form

$$r = A + B \times \log t,$$

where r is the radius of the image, t the duration of exposure, and A and B are constants. Although derived originally from experiment, some considerations are adduced to show that this relation has a physical basis, at least to the extent of being a first approximation.

The author recommends the following process for deducing magnitudes from a plate: determine a and b in the formula

$$\text{magnitude} = a + b \times \text{diameter of image}$$

in such a way as to secure the best agreement between the photographic and visual magnitudes of as many stars upon the plate for which the latter are known. The formula may then be safely applied to unknown stars so long as there is a range of not more than four or six magnitudes. It is interesting to compare this simple formula with the one used by Kapteyn in his work for the Cape Durchmusterung, viz.,

$$\text{magnitude} = \frac{A}{B + \text{diameter}},$$

where A and B are determined as for Scheiner's formula.

Part III is an historical review of astronomical photography, and forms a most interesting addition to the theoretical discussions of the former part of the work. The somewhat novel but successful plan is followed of dividing the subject in subtitles, such as Sun, Moon, planets, etc., and of treating the history of each one of these independently of the others.

The work is concluded with an exhaustive bibliography of the entire subject. An index to the work is not wanting, nor do any pains seem to have been spared to make the book as useful and attractive as possible; it may be recommended as well-nigh indispensable to those interested in any branch of the subject. F. S.

UKIAH, CALIFORNIA,
July 8, 1899.

THE
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THE EFFECT OF CERTAIN IMPURITIES ON THE SPECTRA OF SOME GASES.

By PERCIVAL LEWIS.

IT has frequently been observed that under certain conditions small traces of a foreign substance in a gas may affect the spectrum of the latter to an unexpected extent. Heretofore little systematic investigation of such phenomena has been made, and it was thought that it might be of interest to examine the interactions of some substances which are rarely or never absent from vacuum tubes, namely, mercury vapor, hydrogen, oxygen, and water vapor. The following cases have been studied:

1. *The spectrum of hydrogen.*—(*a*) Pure; (*b*) containing traces of mercury vapor; (*c*) containing traces of oxygen; (*d*) containing traces of water vapor.

2. *The spectrum of oxygen.*—(*a*) Pure; (*b*) containing hydrogen; (*c*) containing mercury vapor.

The method of investigation was first to observe the spectrum of the gas in as pure a condition as possible, particularly with respect to the substances whose influence was to be studied. It was impracticable to note all details, but the general appearance of the spectrum was observed, and photometric measure-

ments of the luminosity of selected parts or lines made, at various pressures of the gas. Small quantities of the foreign substances were then introduced and the observations repeated.

In order to guard against disturbances due to the absorption or giving off of gases by internal metallic electrodes, external electrodes, such as those described by Salet,[1] were used. For this reason no measurements of current strength were practicable.

THE APPARATUS.

The general arrangement of the apparatus is shown in Fig. 1. Hydrogen is generated in the voltameter *V* from distilled water containing a small quantity of phosphoric acid. The traces of oxygen always present in hydrogen generated in this manner were removed by passing through a strong solution of pyrogallic acid in the vessel *A*. The gas is stored in the drying tubes *C*, containing respectively calcium chloride, solid potassium hydroxide, and phosphorus pentoxide. *G* is a sulphuric acid valve to exclude mercury vapor coming from the pump, and *F* is a tube containing solid potassium hydroxide to absorb the vapors coming from the sulphuric acid. The vacuum tube *D* is of the H-shaped end-on type. The electrodes are made of four pieces of brass tubing separated from the glass by mica. Without this precaution the tubes were invariably sparked through, owing to the high potentials used. The capillary part of the tubes generally used was about 10 cm long and 4 mm internal diameter. *E* is a reservoir containing mercury, which, by opening a cock, may be admitted into the discharge tube at a pressure corresponding to the temperature of the reservoir, which was always lower than that of the vacuum tube. *B* is a glass bulb containing potassium permanganate from which oxygen may be generated by heat.

At first all parts of the apparatus were fused together. This gavé rise to inconvenience on account of frequent change or renewal of parts of the system, and finally several sealing-wax joints were used. No injurious effects were noticed unless

[1] SALET, *Ann. Chim. et Phys.*, **28**, 20, 1873.

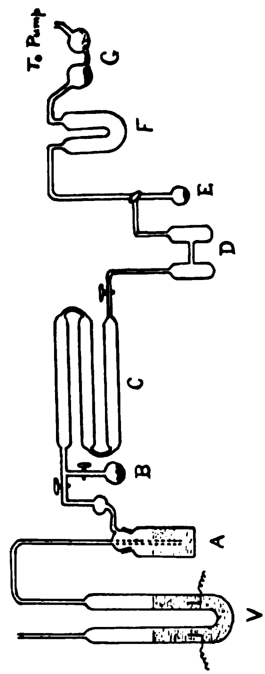


Fig. 1

the discharge reached the joints, when carbon monoxide bands always appeared. The same was found to be true of stopcocks, which were avoided at first on account of the grease, the gas under investigation being introduced through a barometer tube. In studying the effects of mercury vapor this method was of course impracticable, and stopcocks were necessary. At low pressures the carbon monoxide bands always appeared after the discharge had passed for several minutes. In these experiments, however, the discharge tube was always connected with the pump, and observations were made on fresh gas which had not had time to become contaminated by vapor from the cocks. The diffusion of the vapor was also hindered by capillary contraction in the tubing between the discharge tube and the cocks.

A small induction coil was used which, under the usual conditions, gave a spark of some 5 cm length. It was fed by a current from the city leads, at a potential of 110 volts, through a Wehnelt interrupter. The tube with external electrodes is practically a condenser, and it was found that the interrupter would not work so easily and uniformly as with a closed secondary circuit. Furthermore, it was impossible to always maintain constant conditions, owing to frequent renewal of the interrupter, change in the concentration of its acid solution, etc. During a given series of experiments, however, the conditions usually varied little, and different series have been as nearly as possible reduced to the same scale by assuming that in a pure gas at a given pressure in a given tube, the luminosity is proportional to the current strength.[1]

Photometric measurements were made with a Glan spectrophotometer, with an ordinary glowlamp, behind a screen of oiled paper, as a standard. As relative values only were required, the luminous intensity was taken as equal to the square of the tangent of the angle read multiplied by a convenient constant factor.

[1] FERRY, *Phys. Review*, 7, 9, 1898.

RESULTS.

I. *Hydrogen.*—The first experiments were made for the purpose of determining the relation between luminosity and pressure in the case of pure hydrogen. The luminosity is also a function of the current strength, which in these experiments was unknown; but the source of the current supply was kept as constant as possible, so that the results are comparable.

Measurements were usually made on fresh hydrogen, so that in all probability the results are very little affected by the presence of carbon compounds or gases given off by the glass. In some cases observations were repeated on the same portion of hydrogen, with its pressure reduced by pumping out. Usually such readings (indicated by the sign [–] in the table of results) were a little smaller than those made on fresh hydrogen.

For the sake of more accurate photometric measurements, a slit of about 5 mm width was used. For this reason, the compound spectrum of hydrogen, which was invariably present, appeared as a group of wide bands in the red and orange, and as a continuous spectrum in the green. Photometric measurements were made on $H\alpha$ (λ=6563), $H\beta$ (λ=4861), and the compound spectrum in the neighborhood of the green mercury line (λ=5460). Only a few observations were made on $H\beta$, as its luminosity seemed to follow the same law as $H\alpha$, and measurements in this part of the spectrum are difficult. The results are given on page 142.

These results for $H\alpha$ and $H\beta$ are plotted in the curves (I. $H\alpha$) and (I. $H\beta$), and for the compound spectrum in the upper sinuous curve (I. H^{II}) of Fig. 2. They show that, keeping the conditions of current supply constant, the luminosity of both the elementary and the compound spectrum of pure hydrogen reaches a maximum at about 3 mm pressure, and then diminishes very rapidly with the pressure. With increasing pressure the elementary spectrum diminishes in luminosity faster than the compound, and at pressures of more than 4 or 5 cm the latter alone remains visible, although too feeble to be measured. When these experiments were repeated with a tube having internal electrodes, the

I.

Pressure	Luminosity		Pressure	Luminosity	
	$H\alpha$	Compound		$H\alpha$	Compound
0.7	33	4.5	−11.8	24	6.2
0.7	35	4.9	12.5	18	6.2
−0.8	33	4.0	15	17	4.5
1	44	4.5	15	16	4.5
1.4	65	8.8	−16	13	4
1.3	70	7.3	−23.5	4	2.8
−1.5	55	8	−28	3	2.5
1.5	65	9.4	−27	3	2
1.7	81	12.6	−36	2.5	1
−1.8	65	8.8	42	1	2
−2.3	78	8.8			
2.6	96	12.8			
3	107	13		$H\beta$	
3.2	103	12			
3.9	93	9.4			
−4.2	90	12	0.9	52	
−4.5	81	8.8	1.5	100	
5.5	73	9	2.6	106	
5.8	70	8.8	3.5	86	
8	63	8	4.5	92	
−9	45	8.8	11	42	

luminosity increased down to the lowest pressures obtainable with the sulphuric acid valve (about 0.6 mm). Lagarde,[1] using interna electrodes, found that with a current of 115×10^{-6} amperes there was no change in the luminosity of $H\alpha$, $H\beta$, and $H\delta$ with the density between 1.8 and 0.27 mm pressure. With smaller currents the luminosity diminished, with greater currents it increased, as the pressure was reduced from 1.8 to 0.27 mm. Ferry[2] found a continuous increase in luminosity of $H\alpha$ and the compound spectrum as the pressure diminished down to 0.7mm, with internal electrodes and currents ranging from 1 to 6 mm. The point of maximum luminosity is therefore a function of the current strength as well as the density, and no conclusion can be drawn regarding the differences between tubes with internal and those with external electrodes without a knowledge of the current strength in the former. From the general conditions and appearance of the discharge,

[1] LAGARDE, *Ann. de Chim. et Phys.*, 4, 352, 1881.

[2] *Phys. Review*, 7, 6, 1898.

it would hardly seem possible that the current strength diminishes as rapidly as the luminosity below 30 mm pressure.

These results are of course functions of the current density rather than of current strength, and may vary greatly with the size of the tube. With a very wide capillary (1 cm in diameter) only the compound spectrum appeared, and with increasing current, or smaller capillary, the line spectrum increases in luminosity more rapidly than the compound. Although the latter was never absent, it was weaker when traces of mercury vapor, oxygen, or water vapor were present. This point will be discussed later.

THE EFFECT OF TRACES OF MERCURY VAPOR ON THE HYDROGEN SPECTRUM.

Mercury vapor from the pump was kept out by the sulphuric acid valve, but it was exceedingly difficult before beginning a series of observations to get rid of the last traces of mercury vapor carried into the system with the air from the room. The green line ($\lambda = 5460$) persisted after the tube had been repeatedly heated, pumped out, and filled with fresh hydrogen. Even when all mercury had apparently been removed, the green line would slowly reappear in the spectrum after the discharge had passed for some minutes or on heating the tube. It usually required a day or more of steady work to remove all traces of the mercury spectrum, and it was necessary to repeat this process every time after air had been admitted into the tube. It seems inconceivable that mercury vapor should have remained persistently in the tube under such conditions. It is probable that a film of mercuric oxide had been deposited on the glass, which was gradually decomposed by heat or by the discharge. Like results were noted in later experiments with oxygen, in which mercuric oxide was certainly present. Several times, also, the green line flashed out brightly when the discharge first passed, and then faded away, as though the mercury vapor had been torn from combination, and gradually dissipated by diffusion.

These facts indicate a persistency and spectral sensitiveness

on the part of mercury vapor of which little mention has been made in the literature of the subject, and which seems worthy of closer investigation. Plücker and Hittorff[1] note the remarkable spectral sensitiveness of mercury vapor, but many other investigators have seemed to consider its effects negligible. Hertz,[2] after showing how small the saturated vapor pressure of mercury vapor at ordinary temperatures is, says: "The smallness of this pressure, and not any characteristic property of mercury, may be considered the cause of the minute effect which the ever present mercury vapor has on the discharge phenomena in Geissler tubes."

In many cases tubes filled with sulphur to absorb the mercury vapor, and with copper turnings to absorb the sulphur vapor, are interposed between the pump and the discharge tube, but little has been written concerning the effectiveness of this arrangement. Ames,[3] who used such tubes, found some mercury lines on his photographic plates. Warburg[4] found that this method did not completely exclude mercury vapor. In the foregoing series of experiments it was found that this arrangement eliminated some mercury lines, and noticeably weakened the green line, but it was never entirely absent from the hydrogen spectrum without the use of the sulphuric acid valve.

E. Wiedemann[5] studied the spectra of mixtures of air and of hydrogen with saturated mercury vapor between ordinary temperatures and about 240°. No photometric measurements were made by him, but he showed that in each case the mercury spectrum rapidly grew brighter, while the spectrum of the hydrogen or nitrogen grew weaker and finally disappeared at very high temperature. On cooling the tube the reverse phenomena were observed.

[1] PLÜCKER and HITTORFF, *Phil. Trans.*, **155**, 25, 1865.

[2] HERTZ, *Wied. Ann.*, **17**, 200, 1882.

[3] AMES, *Phil. Mag.*, **30**, 49, 1890.

[4] WARBURG, *Wied. Ann.*, **31**, 576, 1887.

[5] E. WIEDEMANN, *Wied. Ann.*, **5**, 517, 1878.

Koch[1] found that the mercury lines completely disappeared from the spectra of hydrogen, oxygen, and nitrogen when the vacuum tube was cooled down to $-80°$.

It was decided to study the effect of mercury vapor upon the hydrogen spectrum at ordinary temperatures, and the relation between luminosity and vapor density of the mercury vapor, and to determine the minimum quantity of the latter necessary, under the conditions, to produce luminosity. The yellow and blue lines of mercury became visible only when the reservoir E (Fig. 1) was nearly at room temperature, the highest reached in these experiments, so measurements were taken on the green line alone.

After making observations on pure hydrogen, the reservoir E (Fig. 1), containing mercury and kept at a constant low temperature by a freezing mixture, was placed in communication with the discharge tube, and the relative intensities of $H\alpha$ and the green mercury line measured, after allowing time for the diffusion of the mercury vapor. The luminosity of the mercury line reached a maximum within a few minutes, which is rather surprising considering that the vapor had to traverse about half a meter of tubing averaging about 6 mm in diameter. Measurements were repeated with different pressures of hydrogen, and with the reservoir E at different temperatures. The first measurements made when E was at temperatures below $0°$ were irregular and unsatisfactory. This was partly due to the great difficulty in measuring the very feeble intensity of the green line under these conditions; partly, perhaps, to traces of mercuric oxide still remaining in the tube. The principal difficulty, however, came from the method of estimating the intensity of the background formed by the hydrogen spectrum. With a wide slit and fresh hydrogen this was practically uniform and continuous in this region, and the true intensity of the mercury line was assumed to be the difference between its apparent intensity and that of adjacent parts of the compound hydrogen spectrum. It was observed, however, that at low pressures, after the discharge had passed for a few minutes, several faint and diffuse

[1] KOCH, *Wied. Ann.*, **38**, 216, 1889.

green lines appeared in the neighborhood. Careful investigation with mercury vapor excluded led to the discovery of another line resembling these in all respects and almost exactly coincident with the mercury line. All these lines were found to vary exactly alike in intensity, so in future observations the intensity of the mercury line was taken as the difference between its apparent intensity and that of the nearest of these foreign lines. These lines apparently did not belong to the hydrogen spectrum, as they, with the mercury line, disappeared when a slow stream of pure hydrogen flowed through the tube. They also appeared in a tube with internal electrodes and in one filled with nitrogen. They disappeared with pressure exceeding 4 or 5 mm and when much mercury vapor was present.

Estimates of the luminosity of the mercury line made in this manner were uniform and consistent. With the reservoir E below -5°, however, it was too feeble for measurement. At -20° it was barely perceptible to the eye, and at -40° had vanished completely. These limits would, of course, be changed by using a capillary discharge tube of a different diameter or a different current.

At high pressures of hydrogen, the mercury line changes very slowly in intensity, and when the reservoir E is at a temperature of 10° or above it remains visible up to pressures of 10 cm or more, when only the exceedingly faint compound spectrum of hydrogen can be seen.

Measurements were made of the intensities of *Ha*, the green mercury line, and the neighboring foreign line (or the adjacent compound hydrogen spectrum when the latter did not appear) while the mercury reservoir was at temperatures of -5°, $+3^\circ$, $+7^\circ$, $+11^\circ$, and $+21^\circ$, and in communication with the vacuum tube. The vapor pressure of the mercury in the latter was equal to the saturation pressure of the mercury in the reservoir. These pressures were calculated from Hertz's formula[1]

$$\log p = 10.59271 - 0.847 \log T - \frac{3342}{T}.$$

[1] HERTZ, *Wied. Ann.*, **17**, 199, 1882.

Usually the readings were taken after the introduction of fresh hydrogen (allowing sufficient time for diffusion of the mercury vapor), but sometimes observations were repeated on the same gas, pumped out to lower pressures. Such results are indicated by the sign – in the tables given below. Many similar observations were made at various times, with the same qualitative results, but those given below were made immediately after those on pure hydrogen previously given. At the end observations were repeated on pure hydrogen, which showed that the conditions had remained unchanged, and that the results are comparable.

The lower sinuous curve (VI.H″) represents the luminosity of the compound spectrum, with the mercury reservoir at 21°.

An independent set of readings was made on $H\beta$, at another time, and is given in Table VII (compare with Table I).

With the mercury at $-5°$ and 3° the luminosity of $H\alpha$ is almost precisely the same as with no mercury vapor present. With the mercury at 7°, there is a slight diminution in the luminosity of $H\alpha$, which becomes more marked with the mercury at 11° and 21°. $H\beta$ also has its luminosity greatly diminished by the presence of mercury vapor (compare with Table I). The luminosity of $H\alpha$ still has a maximum at a pressure of about 3 mm (possibly at a slightly greater pressure when considerable mercury vapor is present). The luminosity of the mercury vapor begins to increase very rapidly with diminishing pressure at about the point where $H\alpha$ begins to grow weaker. The compound spectrum fluctuates somewhat irregularly (as seen by comparing Table IV with the others), but in the main follows the same laws as $H\alpha$.

The effect of mercury vapor on the luminosity of hydrogen is shown very strikingly in Table VIII. After the admission of fresh hydrogen the luminosity of $H\alpha$ and of the compound spectrum faded gradually as mercury vapor diffused over from the reservoir at 20°, and grew brighter when the reservoir was cooled.

	Pressure	Intensity			
		$H\alpha$	Foreign line + compound spectrum	Hg. app.	Hg. corr.
II. Temperature of mercury, —5°. Vapor pressure 0.000116 mm.	0.7	39	8.3	11.8	3.5
	1.8	65	8.8	10	1.2
	2.7	78	7.8	8.3	0.5
III. Temperature of mercury +3°. Vapor pressure, 0.000220 mm.	— 0.7	36	5.3	12.6	7.3
	— 1	41	7.3	13.3	6
	1.8	69	9.4	11.2	1.8
	— 4	61	7.3	10	2.7
	9	36	3.6	4.9	1.3
IV. Temperature of mercury 7°. Vapor pressure, 0.000340 mm.	0.9	33	2.5	10.6	8.3
	1.2	34	2.8	8.2	5.4
	3.6	90	5	8.8	3.8
	— 4.4	68	5.3	8.8	3.5
	5	70	5	10	5
	10	33	5.3	7.8	2.5
	13	16	3.6	6.7	3.1
	22	4	2.5	4.5	2
	31	1.5	1.5	3	1.5
	44	0	1.5	2.5	1
			Compound Spectrum	Foreign lines no longer visible	
V. Temperature of mercury 11°. Vapor pressure, 0.000530 mm.	— 0.8	19	3.3	14.8	11.5
	1.9	53	10.6	15.5	4.9
	2.5	57	10	13.3	3.3
	— 2.7	4.9	6.2	10.6	4.4
	3.2	61	8.3	11.9	3.6
	— 4	51	8.2	11.2	3
	— 4.5	57	10	13.3	3.3
	9	26	5.3	7.8	2.5
	26	3.3	1.5	3.6	2.1
	34	1	0.8	2.8	2
VI. Temperature of mercury 21°. Vapor pressure, 0.001350 mm.	— 0.6	16	2.5	30.7	28.2
	1.1	38	4.2	32	27.8
	— 1.5	34	6.2	31	25
	— 2.6	39	7	23.8	16.8
	— 2.7	36	5.7	19	13.3
	4	33	7	23.8	16.8
	— 5.8	37	7.3	17	9.7
	6.5	33	7.3	14.8	7.5
	—14	13	4.9	10.6	5.7
	19	7	3.9	9.3	5.4
	21	6	3.6	8.2	4.6
	36	1	1.1	3.3	2.2

These results are shown graphically in Fig. 2, and are numbered to correspond with the above ables.

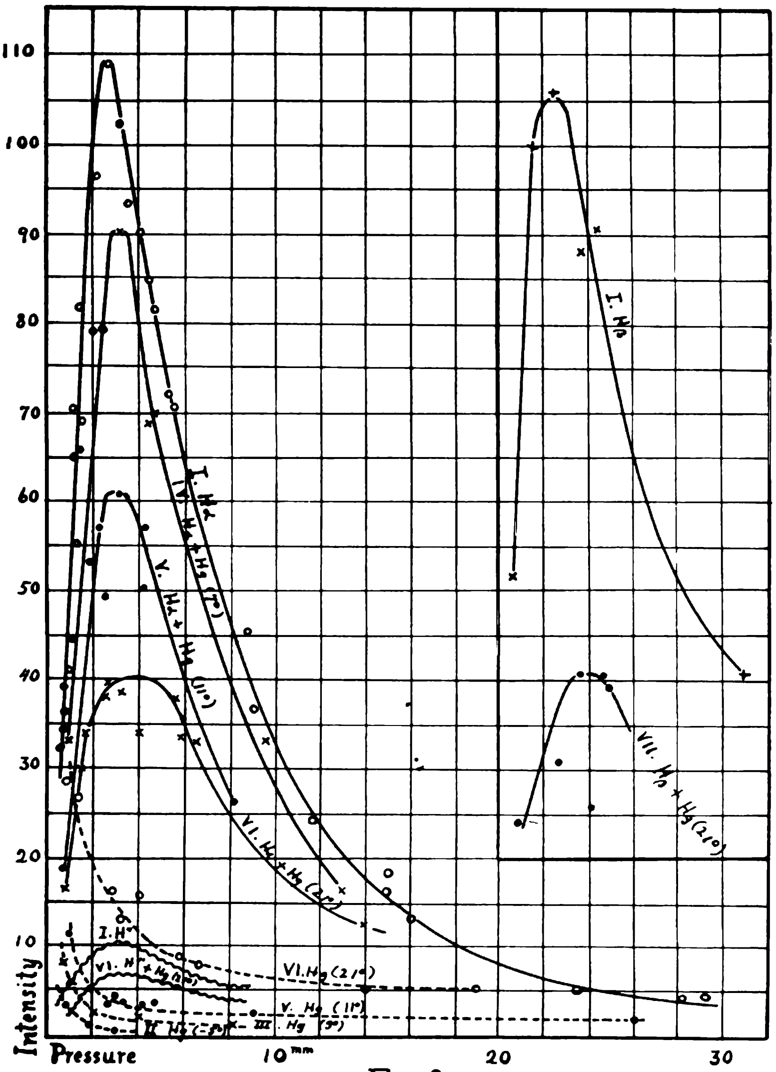

110
100
90
80
70
60
50
40
30
20
10
Intensity
Pressure
10mm
20
30
I. Hα
V. Hα + Hg (11°)
VI. Hα + Hg (21°)
I. Hβ
VII. Hβ + Hg (21°)
I. H
VI. Hg (21°)
V. Hg (11°)
III. Hg (9°)

Fig. 2

VII.

MERCURY AT 21°. VAPOR PRESSURE 0.001350 MM.

Pressure	$H\beta$	Compound spectrum	*Hg* app.	*Hg* corr.
1	24	8	20	11.3
2.5	31	5	17	8.7
3.5	42	10	20	10.1
4.5	39	11	16	4.6

VIII.

Pressure	$H\alpha$	Compound	*Hg* app.	*Hg* corr.	Remarks
0.9	36	4.5	0	0	Fresh hydrogen
	15	2.5	18	15.5	*Hg* at 20°
	28	6	11	5	*Hg* at 3°
1.1	59	13	0	0	Fresh hydrogen
	38	4.2	32	27.8	*Hg* at 21°
	63	12.5	19.8	7.3	*Hg* at 5°
4	100	12	0	0	Fresh hydrogen
	33	7	23	16.8	*Hg* at 21°
6.5	59	11	0	0	Fresh hydrogen
	33	7.3	14.8	7.5	*Hg* at 21°

The first and second sets of readings were obtained on different days and under different conditions, and are not comparable in absolute values.

Several times when the tube contained mercury vapor it was heated strongly with a Bunsen flame. No change of luminosity of either the hydrogen or the mercury spectrum was observed. Little change in density of either gas could be caused by the rise in temperature, as the tube was closed by cocks near each end. It would appear that, within this range of temperatures, the relation between the intensities of the two spectra is a function of the relative quantities of the two gases present, not of the temperature.

Although the luminosity of mercury reaches no maximum, the relative densities and relative luminosities of mercury and $H\alpha$ are nearly proportional for all pressures of hydrogen less

than 6 mm, as shown by the following table. *K* is a constant of proportionality. The mercury reservoir was at 21°.

IX.

Pressure	$A=K\,\delta_H\,\delta_{Hg}$	$B=I_H\,I_{Hg}$	$B\,A$
1	1	0.75	0.75
2	2	1.7	.85
3	3	2.3	.76
4	4	3.2	.80
6	6	4.2	.7
8	8	3.5	.4

The relation between the intensity of the mercury line and the vapor pressure of mercury, as deduced from the curves in Fig. 2, is shown below:

X.

Temp.	$P\times10^6$	Pressure=1 mm		Pressure=2 mm		Pressure=5 mm	
		I	$P\,I$	I	$P\,I$	I	P/I
−5°	116	3	39	1.3	90	...	...
+3°	220	5.5	40	3	73	1.6	140
+11°	530	11	48	6	88	3.5	150
+21°	1350	30	45	18	75	10	135

These results vary irregularly but within the limits of experimental error, and indicate proportionality between luminosity and vapor density, assuming that in each case the current at a given hydrogen pressure is the same. It seems hardly likely that such small quantities of mercury vapor can materially affect the current strength. Warburg[1] has shown that mercury vapor does not appreciably alter the cathode fall in a hydrogen tube, although present in sufficient quantities to give a bright spectrum.

The curves in Fig. 2 show that, while the luminosity of *Hα* at a pressure of 3 mm is practically unaffected by mercury vapor from the reservoir at temperatures below 7°, it is reduced by

[1] WARBURG, *Wied. Ann.*, 31, 574, 1887.

more than half by mercury vapor from the reservoir at 21°. The same is true of $H\beta$, and to a slightly less extent of the green region of the compound spectrum. No measurements were made in other parts of the compound spectrum, but they were observed to diminish in luminosity in about the same proportion, so that the visible spectrum as a whole loses at least half its luminosity. The relative densities of mercury vapor at 3° and at 21°, and of hydrogen at 3 mm pressure are

$$\frac{0.00022}{3 \times 2} \times .200 = 0.0073 \text{ and } \frac{0.00135}{3 \times 2} \times 200 = 0.045.$$

The addition, therefore, to hydrogen at 3 mm pressure of less than 4 per cent. of its own mass of mercury vapor, or of one molecule of mercury to every 2500 molecules of hydrogen, will deprive the latter of more than one half its visible radiant energy.

The cause of this phenomenon is not easily seen. E. Wiedemann[1] suggested that in general metallic vapors may conduct electricity better than non-metallic vapors, as indicated by his experiments previously referred to. Similar phenomena have been observed by others. It is well known that the presence of small quantities of sodium or potassium in the electric arc will almost extinguish the carbon spectrum. Trowbridge[2] states that the presence of 30 per cent. of iron in the arc will completely obliterate the carbon spectrum. On the other hand, J. J. Thomson[3] and others have found that while some metallic vapors conduct about as well as dissociable non-metallic gases, hot mercury vapor is almost an absolute nonconductor. The facts shown in these experiments, that very small quantities of mercury vapor do not affect the hydrogen spectrum, that the luminosity of the mercury vapor is proportional to its density, and that the relative luminosities of hydrogen and mercury are proportional to their relative densities, do not seem to indicate that any disproportionate share of the total energy is imparted to the mercury vapor by the current. It seems probable that some other explanation

[1] WIEDEMANN, *Wied. Ann.*, **5**, 517, 1878.

[2] TROWBRIDGE, *Phil. Mag.*, **41**, 450, 1896.

[3] J. J. THOMSON, *Phil. Mag.*, **29**, 364, 441, 1890.

must be sought. Similar effects are observed in flame spectra, in which there is no question of division of current. When salts are placed in a flame, only the spectra of the metallic components (or of the compound if undissociated) can be seen, and in the solar spectrum hydrogen, helium, carbon, and silicon are the only non-metals which have contributed recognized lines.

Professor Warburg has suggested to me that the luminosity may be an indirect effect of the current, the direct effect being the generation of invisible rays of some kind (comparable to cathode rays), by the absorption of which the luminosity is produced. If, now, these rays are strongly absorbed by the mercury vapor, the spectral sensitiveness of this substance, as well as its diminishing effect on the luminosity of the hydrogen, may be explained.

THE EFFECT OF OXYGEN ON THE HYDROGEN SPECTRUM.

Some observations taken on hydrogen which had not been entirely freed from oxygen showed marked irregularities, and the luminosity of $H\alpha$ did not reach a maximum except at much lower pressures than in the case of pure hydrogen, or the maximum was less sharply defined. The same phenomenon was also observed when traces of air (and, therefore, oxygen) remained in the tube. Under these conditions mercury vapor appeared to have a less marked effect in reducing the luminosity of the hydrogen spectrum.

By gently heating the bulb B (Fig. 1), pure oxygen was evolved from potassium permanganate and introduced into the vacuum tube. The addition of some 3 per cent. of oxygen to pure hydrogen (as determined by change of pressure) reduced the luminosity of the discharge so much that no spectrum was visible. By diluting with more hydrogen a faint, apparently continuous, spectrum appeared; on further dilution $H\alpha$, $H\beta$, and the green mercury line (from traces of mercury left in the tube) were also seen. It was only after several dilutions in the drying tubes, when the proportion of oxygen remaining was very small, that the intensity of the hydrogen spectrum became measurable. The results are given below:

Number of dilutions	Pressure	*Ha*	Compound spectrum in green
III	0.8	10.6	2.5
	1.8	8.2	2
	2.6	3.6	0.8
	3.2	2	0.8
	5	1.5	0.3
IV	0.6	18	3.1
	1.2	16.3	3.1
	2.2	8.2	3.1
	3.1	5.3	2.5
	5	3.6	0.8
V	0.7	18	3.6
	0.9	18	3.6
	1	18	3.1
	1.2	20	4.5
	1.3	20	3.9
	2.3	20	4.5
	2.9	20	4.5
	3.2	20	4.5
	4	18	4.5
	4.5	13	3.6
	5.7	6	3.1
	7.5		
VI	0.7	36	..
	1	42	..
	1.1	42	..
	1.7	42	..
	2.1	39	..
	3.6	22	..
	6	13.3	..
VII	0.6	47	..
	0.8	45	..
	1.1	49	..
	1.2	47	..
	1.7	49	..
	1.8	53	..
	1.9	49	..
	2.4	45	..
	4	28	..
	6	26	..
VIII	0.6	42	..
	0.9	70	..
	1.1	73	..
	1.9	81	..
	2	81	..
	2.2	76	..
	2.9	70	..
	3.6	65	..
	4	39	..

Number of dilutions	Pressure	*Ha*	Compound spectrum in green
Almost pure hydrogen	0.7	33	..
	1.2	57	..
	1.4	57	..
	2.6	76	..
	2.8	70	..
	3.6	70	..
	4.2	53	..
	5	36	..

The results are shown in Fig. 3. The lower broken curves represent the luminosity of the compound spectrum in the green in series II and V, above given.

These curves show:

1. A displacement of the maximum luminosity toward lower pressures, increasing with the proportion of oxygen.

2. A diminution of the luminosity of *Ha* at pressures above 1 mm, an increase at pressures below 1 mm. Paalzow and Vogel[1] observed a strengthening of the oxygen spectrum by the addition of traces of hydrogen.

A portion of the mixed gases, through which the discharge had passed, was left overnight in the tube. The next morning *Ha* again showed a sharply defined maximum at 3 mm. Apparently the gases had been combined by the current and the water vapor had been absorbed by the drying tubes.

When mercury vapor from the reservoir at 20° was admitted, it did not reduce the luminosity of hydrogen to such an extent as when no oxygen was present. In fact, *Ha* showed a greater luminosity at pressures above 2 mm, but smaller luminosity at smaller pressures. This is shown in Fig. 4. The explanation is probably the following: At pressures above 2 mm the mercury removes a part of the oxygen by combination, thus increasing the purity and intensity of the hydrogen spectrum. Below 2 mm the absolute quantity of oxygen present is so small that it is completely removed by the mercury, which then reacts on the hydrogen in its characteristic way.

[1] PAALZOW and VOGEL, *Wied. Ann.*, 13, 337, 1881.

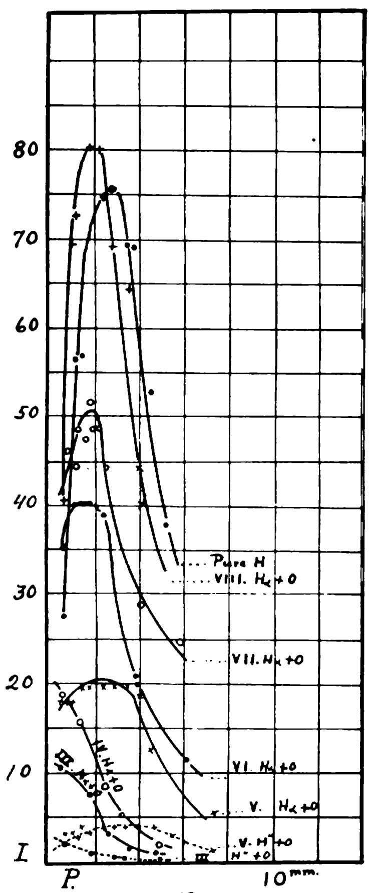


Fig. 3

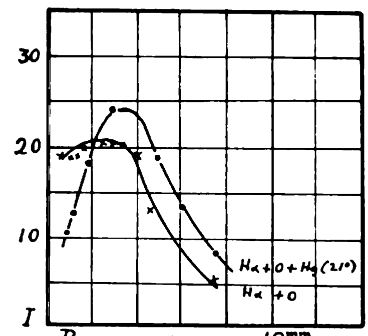


Fig. 4

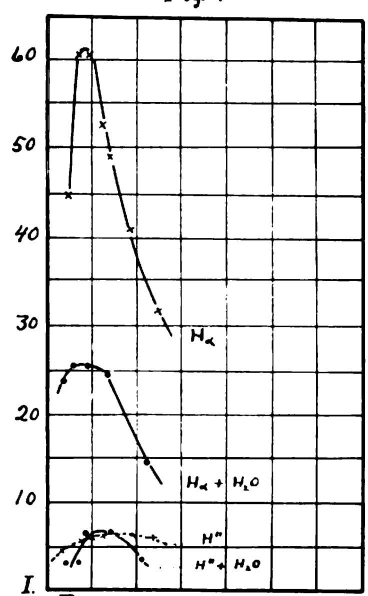


Fig. 5

With still smaller quantities of oxygen present mercury vapor was several times introduced. The initial effect was always smaller than with pure hydrogen, but after a little time the intensity of *Ha* diminished to about the value obtained with no oxygen present. This is consistent with the above explanation.

The presence of oxygen changed the color of the discharge in the capillary from pale pink to a dull feeble red.

THE EFFECT OF WATER VAPOR ON THE HYDROGEN SPECTRUM.

The mercury reservoir *E* was replaced by a bulb containing an almost concentrated solution of sulphuric acid in water, which was cooled to 3°. The vacuum tube was several times pumped out, heated, and filled with pure dry hydrogen, and measurements made of the luminosity of *Ha*. The position of the maximum indicates that small traces of oxygen still remained.

Water vapor was then admitted from the bulb. The vapor pressure must have been exceedingly small, but a very noticeable effect was almost instantly observed.

The results are given below and shown in the curves of Fig. 5. The lower broken curves represent the luminosity of the compound spectrum.

XII.

	Pressure	*Ha*	Compound (Green)
Dry hydrogen	0.9	45	5
	1.4	61	6
	1.7	61	7
	2.3	53	7
	2.9	49	7
	3.8	42	7
	4.9	33	7
Water vapor present	0.8	24	3.6
	1.1	26	3.6
	1.6	26	7
	2.7	20	7
	4.2	15	4.5

The effect of water vapor is quite similar to that of oxygen, as might be expected. These observations were repeated many times with similar results. Usually the effect on the compound spectrum was greater than that shown above.

After shutting off the water vapor and opening communication with the drying tubes the hydrogen gradually increased in luminosity.

These effects may be related to those observed by Warburg,[1] who found that very small quantities of water vapor noticeably influenced the cathode fall in hydrogen. The great reduction in luminosity, caused by water vapor, however, need not necessarily imply that the current strength is greatly reduced by its presence. The total radiation, invisible as well as visible, as well as the nature of radiation in vacuum tubes, must be known before we can determine the exact relation between luminosity and current strength.

THE COMPOUND HYDROGEN SPECTRUM.

In none of these experiments was the compound spectrum of hydrogen absent. The presence of traces of mercury vapor, oxygen, or water vapor materially reduced the luminosity of both the elementary and the compound spectrum, and with a very narrow slit the latter often seemed very feeble. It is probable that, working with a very narrow slit and ordinary, not "end-on," tubes or with very great dispersion, the compound spectrum might have been entirely invisible in the presence of mercury vapor, water vapor or oxygen. It seems doubtful whether an absolutely isolated elementary spectrum has ever been observed in pure hydrogen at low pressures. Lagarde[2] obtained what appeared to be a pure line spectrum, with no trace of the compound, only when water vapor was present, and then the intensity of the line spectrum itself was greatly reduced by the presence of the water vapor. He did not use "end-on"

[1] WARBURG, *Wied. Ann.*, 31, 575, 1887.

[2] LAGARDE, *Ann. Chim. et Phys.*, 4, 265, 1885.

tubes. Salet[1] and Cornu[2] obtained a "pure" elementary spectrum only after the tube had been repeatedly washed out with oxygen. It seems possible that traces of oxygen remained, and made the compound spectrum very weak. Ames,[3] Schumann,[4] and Hutton[5] never succeeded in eliminating the compound spectrum. Schumann found it strongest in the tubes which had been most carefully cleaned, and weakest in those which had not been cleaned at all, and which, therefore, probably contained compounds capable of giving off oxygen or water vapor. Hutton found that it was much weaker after washing the tube with oxygen. Trowbridge,[6] using a very large condenser, produced an elementary line spectrum of hydrogen in which no compound spectrum was seen. The three last-named observers, as well as myself, have observed that the discharge in hydrogen giving the purest elementary spectrum is red; when the compound spectrum is strong, it is very pale pink or nearly white at low pressures, bluish-white at higher pressure.

In these series of experiments it has been observed that in pure hydrogen the intensities of the elementary and the compound spectrum rise and fall together, with changes in current and density, although not in the same proportion. Changes in both spectra, due to mercury, oxygen, and water vapor, are qualitatively, if not quantitatively, the same. In spite of the many doubts which have been raised, it seems impossible to escape the conclusion that this compound spectrum is really due to hydrogen, not to impurities. Furthermore, since the two spectra seem almost invariably coexistent throughout a very wide range of physical conditions, it seems possible that they may not be essentially different, but parts of one and the same spectrum. The differences in their appearances under different physical

[1] SALET, *Ann. Chim. et Phys.*, **28**, 22, 1871.

[2] CORNU, *Jour. de Phys.* (2), **5**, 100, 1886.

[3] AMES, *Phil. Mag.*, **30**, 50, 1890.

[4] SCHUMANN, *Jahrbuch. f. Phot.*, **8**, 1894; *Wied. Beiblätter*, **18**, 752, 1894.

[5] HUTTON, *Phil. Mag.*, **46**, 338, 1898.

[6] TROWBRIDGE, *Phil. Mag.*, **43**, 137, 1897.

conditions are hardly more striking than those observed between lines generally accepted as belonging to the same spectrum — the green and yellow lines of mercury, for example, as the vapor density changes.

II. OXYGEN.

In some previous experiments, in which a tube with internal electrodes was used with no valve between the pump and the tube, it was observed that in an atmosphere of pure oxygen the spectrum due to mercury vapor from the pump was entirely absent. Eisig[1] also found that the mercury lines did not appear in oxygen tubes.

The addition of a very small trace of hydrogen caused the green line instantly to appear. Further experiments with internal electrodes were made to confirm this observation.

The tube was filled with pure oxygen generated from potassium permanganate. Even at low pressures the luminosity of the discharge was very small, and only an exceedingly faint, apparently continuous, spectrum could be seen. After the current had passed for several minutes, the discharge changed from dull pink to white, and the red hydrogen line and carbon monoxide bands appeared. After washing out the tube several times with oxygen the latter were permanently weakened.

Fresh oxygen being in the tube, communication was opened with the mercury reservoir at 20°. Although ample time was allowed for diffusion, the green line did not appear until $H\alpha$ appeared.

A trace of fresh hydrogen was added. The only effect was to slightly increase the intensity of the hydrogen and mercury lines. At no time did any oxygen lines appear, nor the green mercury line unless hydrogen was also present.

With the mercury reservoir open, the green line flashed out brilliantly at the instant the discharge began to pass, then faded to a small intensity. If the current was immediately turned on again, the flash did not reappear, but it always did after the few

[1] EISIG, *Wied. Ann.*, **51**, 750, 1894.

seconds necessary for fresh mercury vapor to diffuse into the capillary tube. This flash was evidently due either to the passage of the discharge through the uncombined mercury vapor or to the act of chemical combination itself. That combination did occur was evident from the reduction of pressure in the tube. In one case it fell from 1.9 to 0.8 mm within a few minutes, while the current was passing.

The mercury reservoir was cooled down to —13°. This produced no alteration whatever in the intensity of the green line, as in the experiments with hydrogen, in which the luminosity of the mercury responded rapidly and unfailingly to the changes of temperature of the reservoir. Heating the tube also caused a great increase in the intensity of this line (in one case from 13 to 87), and also brought out the yellow lines. A steady stream of fresh hydrogen did not weaken the intensity of the mercury spectrum. These facts indicate that the mercury was not present in the form of free vapor, but as mercuric oxide, and offer an explanation of the great difficulty previously experienced in completely eliminating the green line from the hydrogen spectrum.

As the oxygen disappeared, the intensity of both hydrogen spectra increased, that of the compound faster than the elementary. In one case the intensity of $H\alpha$ increased from 20 to 49, that of the compound spectrum in the green from 1 to 20. These changes are doubtless in part due to the change in pressure, partly to the varying purity of the hydrogen.

CONCLUSIONS.

1. Very small traces of an impurity in a gas may cause considerable changes in its spectrum, whether this impurity be chemically active or not.

2. The addition to pure hydrogen of very small traces of mercury vapor will cause the green mercury line to appear in the spectrum. Under the conditions prevailing during these experiments, the green line did not disappear until the supply reservoir was cooled to below —20°. At this temperature the saturated

vapor pressure of mercury is only 0.000016. At ordinary temperatures the green line remained plainly visible at hydrogen pressures above 10cm, when only an exceedingly faint, apparently continuous, hydrogen spectrum was visible. The luminosity of mercury vapor, mixed with hydrogen at a given pressure, was found to be nearly proportional to the density of the mercury vapor. The yellow and blue lines appeared only at temperatures above 10°.

3. The addition of small quantities of mercury vapor to pure hydrogen materially reduced the luminosity of both the elementary and the compound spectrum of the latter. The relative luminosities of the hydrogen and the mercury vapor seemed to be proportional to their relative densities below 6mm hydrogen pressure. At higher pressures of hydrogen the mercury possesses relatively greater luminosity. The addition to hydrogen, free or nearly free from mercury vapor, of 4 per cent. of its own mass (or one molecule of mercury to every 2500 molecules of hydrogen) reduced the luminosity of the entire hydrogen spectrum by more than one half.

4. In tubes with external electrodes pure hydrogen showed a maximum luminosity at a pressure of about 3mm, the conditions of current supply remaining constant. Tubes with internal electrodes under the same conditions gave no maximum down to pressures of about 0.6mm. The position of this maximum is probably a function of the current strength and the size of the tube.

5. The addition of small quantities of oxygen to hydrogen caused considerable changes in the luminosity of the latter. The luminosity was increased at pressures under 1 mm, and diminished at higher pressures. The maximum luminosity of the hydrogen was shifted to lower pressures with increasing quantities of oxygen present.

6. Water vapor produces effects quite similar to those of oxygen. Water vapor is probably formed when the discharge passes through the mixture of hydrogen and oxygen.

7. The so-called "compound" spectrum of hydrogen is

really due to hydrogen, not to impurities, as has been often claimed.

8. A very small quantity of hydrogen added to oxygen will excite luminosity in mercury vapor present. The cause of this is not clear.

My thanks are due to Professor Warburg for his continuous advice and help during this investigation.

PHYSICAL INSTITUTE OF THE UNIVERSITY OF BERLIN,
July 1899.

ON PROFESSOR KEELER'S PHOTOMETRIC RESEARCHES, BY PHOTOGRAPHIC METHODS, ON THE NEBULA IN ORION.

By J. SCHEINER.

IN the March number of the ASTROPHYSICAL JOURNAL, Professor Keeler has given an account of some investigations, in which a photographic method is used to show that the quality of the light emitted by the Orion Nebula is different for different parts of the nebula.

The method is briefly as follows: A negative of the nebula was made on an ordinary plate, with short exposure, by means of a reflecting telescope; and another negative was made, with a longer exposure, on an orthochromatic plate, which was protected from the action of the more refrangible rays by a yellowish-green plate of glass. The exposure times were so chosen that the bright part of the Huyghenian region appeared equally strong on both plates. The result was, that on the color-sensitive plate the fainter details in the outlying parts of the nebula were much less conspicuous than on the ordinary plate. From this fact the following conclusion was drawn: "We infer, therefore, that in the remote parts of the nebula the two lowest nebular lines are weak, or the hydrogen lines strong, as compared with the Huyghenian region. Thus the results of spectroscopic researches are confirmed, and are extended to parts of the nebula which are too faint for visual observation."

This conclusion involves the theorem, which is also directly stated in another place, that equally dense parts of the negative correspond to equally bright parts of the object. This theorem is, however, incorrect, and can only be justified under two limitations: equal times of exposure and similar plates. These two limitations are necessary, because, in the first place, the product of time and intensity is not a constant, and because in the second

place, the complicated law which is to be substituted for it involves constants which have very different numerical values for different kinds of plates. In the research under consideration both these limits are very greatly exceeded — first, in the variation of the time of exposure from six minutes to two hours and twenty minutes, and second, in that an ordinary plate was brought into comparison with one which had been sensitized with erythrosin. The fact that, even with equal times of exposure, the different degrees of darkening by different grades of brightness may vary widely for different kinds of plates is so generally known, that professional photographers give special orders to factories for "hard" or "soft" plates, each to be used for its appropriate purpose. A further consideration of this point might therefore be deemed superfluous; but I wish very briefly to sketch an example of such investigations as should have preceded the conclusion which I have cited.

An ordinary Schleussner plate *a* was exposed in a sensitometer for thirty seconds, and an orthochromatic plate *b*, protected by a yellow color-screen, for five minutes. The darkest square 1 of plate *b* corresponded with the squares 3–4 of *a*. The faintest perceptible impressions were, for *b* the squares 15–16; for *a*, 14–15. The interval on *b* was therefore 15.5 squares and on *a* 11 squares; the corresponding intensity intervals are 33 and 11.3. If these plates had been used on the Orion Nebula, the result would, on the assumption of uniform quality of light, have been the reverse of that found by Professor Keeler. On the orthochromatic plate the outlying parts of the nebula could have been traced farther than on the ordinary plate. But we should have to guard ourselves against drawing the conclusion that the hydrogen lines are relatively weaker in the outlying parts than they are in the Huyghenian region; we should rather, on the basis of the experiments with the sensitometer, just mentioned, explain the observed appearance as being in harmony with the fact that in this case the orthochromatic plate was the "softer." With equal intensity of the brighter regions, it had rendered visible portions which were three times fainter than

those which appeared on the ordinary plate; and it is known that even greater differences may occur than those given in this example.

A particular interest, on the other hand, attaches to Professor Keeler's statement that on the orthochromatic plate the *Proboscis Major* is materially stronger than the streamer which is parallel to it, and which on an ordinary plate nearly equals it in intensity. If these two objects were in fact exactly equal in brightness this experiment would prove the correctness of the views held by Professor Keeler; but if there exists even a small difference of brightness, the conclusion is again inadmissible, since, according to the example of differences of plates, which I have given, small contrasts are vastly exaggerated photographically.

With respect to the method used by Professor Keeler, of weakening the brighter parts of the nebula, I have merely to say that the application of such methods, in the comparison of brightnesses which are already near the limit of visibility, seems to me a doubtful proceeding.

ROYAL OBSERVATORY, POTSDAM.
June 1899.

NOTE ON THE FOREGOING ARTICLE BY PROFESSOR SCHEINER.

By James E. Keeler.

The peculiarities of photographic action and of photographic plates referred to by Professor Scheiner are very well known to me, through both reading and daily experience, and I must observe that I have not really made the statement with which I am credited. My words are: "About all that it is safe to assume is, that (setting aside certain limiting conditions not likely to be met with in nebular photography) equally dense parts of the negative correspond to equally bright parts of the object." Here the reference is to one and the same negative, and, the exposure-times, the plate, and (by hypothesis) the quality of the light, being the same, the truth of the statement is self-evident.[1]

This, however, is not an important matter. Professor Scheiner's main objection has a valid theoretical basis; and if the plates I used had been more unlike in their properties, if the differences I have described were less strongly marked, and if no other evidence were available than that criticised by Professor Scheiner, I might have felt some doubt as to the justness of my conclusions. The difference between the plates is, however, not sufficient to explain the difference between the negatives. There is also the other evidence afforded by the photograph to be considered, to say nothing of the spectroscopic observations. If the question were merely one of the plates, all parts of the image which are equally dense on the ordinary plate should also, if the quality of the light is everywhere the same, be equally dense on the isochromatic plate, which is not the case. And, as I have pointed out, all the stars, even the

[1] Possibly the source of the misunderstanding lies in the fact that a noun in its widest or most general sense is used in German with the definite article, but in English with the indefinite article.

faintest, in the outlying regions, are at least as strong on the isochromatic as on the ordinary plate, though the nebulosity which surrounds them is missing.

In the special case of the Messerian branch which I have brought forward, Professor Scheiner objects that the brightness is still not exactly equal to that of the parallel streamer with which I have compared it, but somewhat greater. This is true; but again I would say that the effect is disproportionate to the cause which Professor Scheiner assigns to it. The brightness is in fact so nearly the same that to obtain a comparison free from theoretical reproach it is only necessary to choose a part of the Messerian branch lying toward its eastern side, for the brightness falls off gradually in that direction. The difference between the negatives remains, however, and the conclusion is unaffected.

With regard to the experiments in which the spectrum of the Huyghenian region was dimmed, for comparison with the spectrum of the region surrounding the star Bond 734, I would say that the observation is really not a difficult or delicate one. There is plenty of light for the purpose. I am confident that Professor Scheiner's own doubts would be resolved at once if he could repeat this observation with our thirty-six-inch refractor.

LICK OBSERVATORY,
University of California,
August 1899.

THE POSITION OF THE STARS OF TYPE IV AND OF THE VARIABLE STARS OF TYPE III IN REFERENCE TO THE MILKY WAY.

By T. E. Espin.

Dr. Dunér has shown (*Sur les étoiles à spectres de la troisième classe*, p. 26) that the stars of Type IV have a distinct tendency to collect in the Milky Way. As in the last few years the discoveries of the stars of Type IV have made it almost certain that we now know all of them down to 8.5 mag. in both hemispheres (excluding those that are unsteady in light), it seemed of interest to examine Dr. Dunér's statement afresh. For this purpose two charts were drawn representing the heavens, and the places of 10°, 20°, 30°, etc., of Galactic longitude and latitude were calculated and laid down. The stars of Type IV were inserted, and the number of stars in each zone of 10° of Galactic latitude counted. As regards the Galactic longitude it was found that the IV Type stars were fairly evenly distributed, no special grouping being noticeable with the exception of that at G. long. 43° in the constellation of Cygnus.

The results in latitude are shown in the following table:

TABLE I.

Gal. lat.	N. lat.	S. lat.	Total
0°—10°	60	63	123
10 —20	24	19	43
20 —30	15	12	27
30 —40	4	9	13
40 —50	7	1	8
50 —60	1	4	5
60 —70	1	1	2
70 —80	2	0	2
80 —90	0	1	1
Total	114	110	224

This result entirely confirms that of Dr. Dunér. Of the stars of Type IV, 74 per cent. lie within 20° of Galactic latitude.

The variable stars of Type III were next treated in a similar manner. Stars where the variation is small and where the hydrogen lines are not bright were rejected. Some of the later discoveries have not been examined with the spectroscope, and in a few cases may turn out to be of Type IV, but there are too few to affect the general results. The following table gives the results:

TABLE II.

Gal. lat.	N. lat.	S. lat.	Total
0°—10°	25	23	48
10 —20	29	25	54
20 —30	32	33	65
30 —40	17	29	46
40 —50	10	14	24
50 —60	17	15	32
60 —70	10	3	13
70 —80	1	4	5
80 —90	1	5	6
Total	142	151	293

This table seems to show that the variable stars have a tendency to collect on the borders of the Milky Way. There is also a distinct tendency to grouping, such groups occurring at:

Gal. long.	Gal. lat.	Constellation
30.8°	17° S	Delphinus
53.3	19 N	Cygnus
173.9	12 N	Canis Minor
321.0	23 N	Libra
325.9	18 S	Sagittarius
346.9	14 S	Sagittarius

The results obtained in Table II seemed worthy of further investigation. For this purpose the latitudes of all the variable stars were approximately obtained from the two charts between 10° and 30° of Galactic latitude. The mean of the latitudes was

then taken for N. and S. in each map. The following are the results:

TABLE III.

	No. of stars	Mean Galact. lat.	No. of stars+25°	No. of stars−15°
Map I, S	23	19.0°	3	5
N	22	20.8	5	3
Map II, S	36	20.7	7	4
N	40	19.9	5	6
	Total 121	Mean 20.7°	Total 20	Total 18

The fourth and fifth columns give the number of stars 25° to 30° and 10°–15°. They show that the decrease on each side of 20° is nearly equal. Further, added together they give 38 for the ten degrees, while the zone 15° to 25° has 83, thus showing the aggregation to be real. Rejecting then the stars more than 5° from the mean found in Table III, we get

	No. of stars	Mean Gal. long.
Map I, S	15	19.7°
N	14	20.5
Map II, S	25	20.2
N	29	20.3
Total	83	Mean 20.3°

An inspection of Table II shows that there is a second apparent maximum at from 50°–60°. This treated in a similar manner gave as a result mean Galactic latitude 57.3°, but with great uncertainty.

Summarizing the results of this paper the following propositions seem warranted:

(1) The stars of Type IV, and the variable stars of Type III are distinctly related to the Galaxy.

(2) Of the IV Type stars 55 per cent. are found within Gal. lat. 10°.

(3) The IV Type stars are fairly evenly distributed in Gal. long.

(4) The largest number of the variable stars of Type III are found on the borders of the Galaxy.

(5) This zone is from 15° to 25° of Galactic latitude.

(6) These variable stars show a tendency to form into groups.

I have in addition investigated the variation and period of these stars and can find no connection with their position in reference to the Galaxy. Lastly to trace any connection between the variable stars and the stars of Type III, I plotted down all the known variables of III !! and III !!!. These show a distinct tendency to grouping, but not with any reference to the Galactic latitude of 20° as in the case of the variable stars. The third type stars increase in number as the Milky Way is approached, and in two of the cases, where there are groups of variables in Cygnus and Delphinus, the third type stars group as well; on the other hand, there is a strong group at Gal. long. 60°, lat. S. 28°.

TOW LAW, DARLINGTON, ENGLAND.
July 6, 1899.

OBSERVATIONS OF COMET SPECTRA.

By W. H. WRIGHT.

COMET 1898 I (*Perrine*).

THE spectrum of this comet was observed visually on May 9, 1898. It was of the usual type: the three characteristic bands superposed on continuous spectrum, which was relatively strong. Photographic observations were not made.

COMET 1898 VII (*Coddington*).

Observations on June 11 by Professor Campbell and the writer showed the spectrum to be of the usual type, except that the banded spectrum was very faint as compared with the continuous. The yellow and blue bands were seen only with difficulty. To the writer the banded spectrum seemed to be stronger in the outlying parts than in the nucleus. The spectrum was again observed on June 16 with practically the same results, except that the bands in the nucleus seemed to be relatively stronger than on the previous occasion. The spectrum was too faint to be photographed.

COMET 1898 X (*Brooks*).

Observations were made on November 3, 1898, by Professor Campbell and the writer. The three chief bands were easily visible, the one in the green being apparently much brighter than usual. It was roughly estimated as being from four to six times as bright as the others. The continuous spectrum was very weak, being visible only in the nucleus with a wide slit Nothing was visible above or below the three bands.

COMET 1899 *a* (*Swift*).

The visible spectrum of this comet resembled that of 1898 x, inasmuch as it consisted almost entirely of bright lines. The

three carbon bands were bright, but the dispersion used was too low to show further details.

The spectrum was photographed with a spectrograph attached to the twelve-inch equatorial on May 4, 8, and 11. The photograph of May 8 was the best, showing the blue band resolved, and the cyanogen lines at λλ 3871 and 3883, with considerable detail between.

Further photographs were obtained on June 5 and 6 with a spectrograph attached to the thirty-six-inch telescope. By the use of a correcting lens, placed one meter in front of the slit, a color curve satisfactory for this class of work is obtained over the entire region covered by the spectrograms. The following are the results of measurements:

June 5	June 6		
3870	3869	Very bright, slightly hazy to violet	3871.5 Cyanogen
3881	3879	V. V. bright, very sharp	3883.5 Cyanogen
399	3987	Faint and somewhat hazy	
4019	4014 } 4019 }	Bright, resolvable with difficulty	
4041	4042	Bright and rather broad	
4053	4052	V. bright, sharp	
4074	4074	Fairly bright; looks like head of band extending toward violet	
4101	4100	Faint	
	413	V. faint	4128.1 Cyanogen
421	421±	V. faint	4216.1 Cyanogen
	4313		
	435 } 440 }	Suspect faint band with faint line at 4369	436 } 438 } (?) 5th carbon group
	472	Center of gravity of blue band, which is out of focus	4th carbon group
	4883	Suspected bright line	

The slight correction (—¼ A. U.) for motion of the comet in the line of sight has been taken account of, but the results are uncorrected for the effect of slit-width as discussed by Professor H. Kayser (*A. and A. P.*, 13, 367). The general shift toward the violet indicated by the spectrogram of June 6, is not, however, entirely to be accounted for by the Kayser effect, as will be seen by the following consideration: The hydrogen tube used on

this date for comparison contains an impurity giving the bands at λλ 3871 and 3883. With reference to these the corresponding comet bands are perceptibly shifted toward the violet. The spectrograph used in these determinations is one constructed for the purpose, and which has not otherwise been tested. Although it is in many respects most efficient, I am inclined to consider that the shift referred to is instrumental, the result probably of manipulation of the slit while photographing the comparison spectrum.

Most of the lines here observed have counterparts in the spectra of carbon and cyanogen, and the majority of these are doubtless to be accounted for by the presence of the two substances. In addition to those indicated above there is a line of wave-length 4099.2, which approximates closely to that of the comet line 4101. Professor Kayser, however, thinks the two are not identical, the cyanogen line being in his opinion much too faint. The identity of the comet line being open to doubt, it is of interest to note that it is very close to $H\delta$ (λ 4101.9). The proximity must not, however, be held to prove identity.

In appearance, the head of the comet was very diffuse, with a nucleus some 4″ in diameter. In the spectrum of the nucleus λ 4052 is fully as bright as λ 3870, but the latter line extends out into the fainter parts of the comet's head more than four times as far as the former. In fact the lines λλ 3870 and 3880 experience only a gradual increase in brightness in the region of the nucleus, and extend the entire length of the slit. In the cases of all the other lines the change is quite abrupt. This must be taken to indicate a marked difference between the spectrum of the nucleus and the spectra of the outer parts of the head.

The spectrograms have been compared with some of comet 1893 *b* (Rordame) and 1894 *b* (Gale), secured by Professor Campbell with a spectrograph attached to the thirty-six-inch telescope. The correspondence is not exact, as Professor Campbell had not the advantage of the photographic corrector referred to above. This accounts sufficiently for such differences as exist in the estimates of relative intensity of the lines. Taking into

consideration the difference in color curves there is no evidence of any variation in the type of spectrum.

The constants of the spectrograph used in these determinations are:

Length of collimator - - - -	81. cm
Length of camera - - - - -	30.5 cm
Slit width - - - - -	0.1 mm
Effective aperture of lenses - - -	4.27cm

Dispersion between D and $H\gamma$, $57'\ 10''$.

THE SPECTROSCOPIC BINARY CAPELLA.

By W. W. Campbell.

An examination of six spectrum plates of α Aurigae, obtained with the Mills spectrograph in 1896–7, leaves no doubt that this star is a spectroscopic binary. The spectrum is composite. The component whose spectrum is of the solar type furnished the following velocities with reference to the solar system:

1896	Aug. 31	+ 34 km
	Sept. 16	+ 54
	Oct. 3	+ 49
	Oct. 5	+ 44
	Nov. 12	+ 4
1897	Feb. 24	+ 3

On the first photograph the spectrum is of essentially normal solar type; on the others it is unmistakably different. There appears to be a second component whose spectrum contains the $H\gamma$ line and the rather prominent iron lines. On the plates of September 16, October 3, and October 5, these lines are shifted toward the violet with reference to the solar type spectrum; and in the spectra of November 12 and February 24 they are shifted toward the red.

Lick Observatory,
August 10, 1899.

THE VARIABLE VELOCITIES IN THE LINE OF SIGHT OF ϵ LIBRAE, h DRACONIS, λ ANDROMEDAE, ϵ URSAE MINORIS AND ω DRACONIS.

By W. W. CAMPBELL.

ϵ LIBRAE ($\alpha = 15^h\ 18.8^m$, $\delta = -9°\ 57'$).

THE variable velocity of this star, detected several months ago, is indicated by the following results:

1899	April 13[1]	− 8 km
	May 10	+12.2
	15	+ 7.5
	June 12	− 7.5
	14	− 7.0
	26	−11.2
	July 13	−10.8

The period is undetermined, but it seems to exceed three months.

h DRACONIS ($\alpha = 16^h\ 55.4^m$, $\delta = +65°\ 17'$).

The velocities obtained for this star are:

1899	June 26	−26 km
	July 11	−36
	16	−32
	24	−16

The period remains undetermined.

λ ANDROMEDAE ($\alpha = 23^h\ 32.6^m$, $\delta = +45°\ 56'$).

The reality of the variations indicated by the first three plates, noticed in June, is amply confirmed by five later plates. The velocities obtained up to date are as follows:

[1] An underexposed plate, not suitable for accurate measurement.

1897	Nov.	16	+16km
1898	Oct.	18	− 2
		26	+13
1899	July	5	+15
		11	+ 3
		12	+ 2
		16	+ 1
		24	+14

The observations are apparently satisfied by a period of about 19.2 days.

ϵ URSAE MINORIS ($\alpha = 16^h\ 56^m$, $\delta = +82°\ 12'$).

The velocities observed for this star are as below:

1897	May	5	+ 3km
		27	−35
	July	21	−10
	Aug.	4	+ 9
1899	July	31	−40

The period remains undetermined.

ω DRACONIS ($\alpha = 17^h\ 37.5^m$, $\delta = +68°\ 48'$).

The velocity of this star in the line of sight varies rapidly. Four spectrograms give the following results:

1899	July	25	+18 km
	Aug.	8	−45
		9	−12
		29	−53

Compared with the whole number of stars for which plates have been secured with the Mills spectrograph, the nine or ten spectroscopic binaries recently discovered here seem to indicate that these systems are at least as plentiful as visual binaries. The observations for determining the orbits of these bodies are well up to date, and for several of them are practically completed.

Acknowledgments are due to Mr. Wright for his skillful assistance in the observations.

THE VARIABLE VELOCITY OF α URSAE MINORIS IN THE LINE OF SIGHT.[1]

By W. W. CAMPBELL.

POLARIS furnishes an interesting case of variable velocity in the line of sight. Six spectrograms were obtained in 1896, as follows:[2]

Gr. M. T. 1896, Sept.	8^d	22.8^h	—20.1 km
"	15	22.8	—19.1
"	23	21.4	—18.9
Oct.	5	21.0	—19.0
Nov.	11	19.3	—20.1
Dec.	8	16.7	—20.3
		Mean	—19.6

The agreement of these results was satisfactory, and gave no evidence of variable velocity.

Gr. M. T. 1899	Velocity	Measured by
August 9^d 0.8^h	—13.1	Campbell
9 20.1	—11.4	Campbell
14 22.8	— 9.0	Campbell
16 0.1	—14.1	Campbell
23 0.3	—10.9	Campbell
24 0.8	—15.2	Campbell
26 0.9	— 9.4	Campbell
*	— 8.6	Wright
27 0.3	—10.6	Campbell
27 16.2	—14.0	Campbell
28 0.8	—14.7	Campbell
*	—14.3	Wright
28 16.3	—13.7	Wright
29 0.4	—12.1	Wright
29 18.8	— 9.6	Wright
30 0.0	— 8.9	Wright
30 16.2	— 9.3	Wright

* Measures of the same plate by Mr. Wright.

[1] Read at the Third Conference of Astronomers and Astrophysicists, Sept. 8, 1899.

[2] Published in this JOURNAL, October 1898, p. 149.

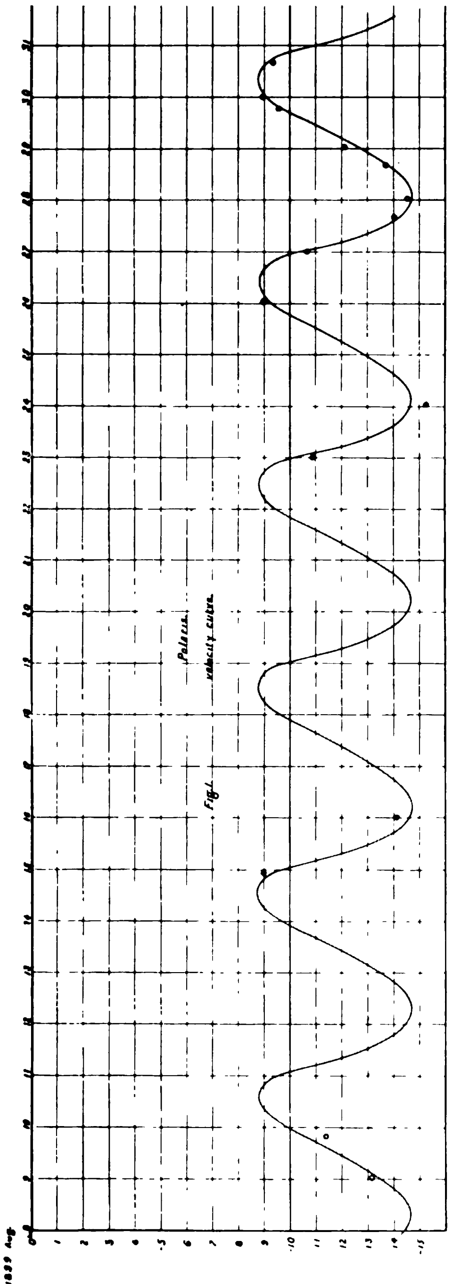


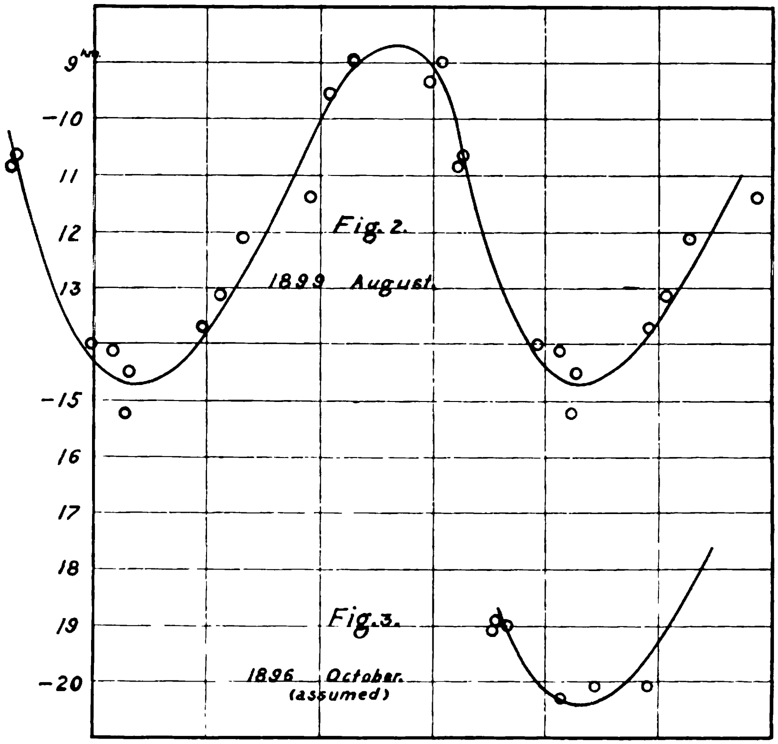
FIG. 1.—VELOCITY CURVE OF POLARIS.

In order to test the current results of our work with the Mills spectrograph, another photograph of the spectrum of Polaris was obtained on Aug. 8, 1899. This yielded a velocity of —13.1 km, and led to the suspicion that we were dealing with a variable. Two additional plates were secured on August 9 and 14, which yielded velocities of —11.4 and —9.0 km, respectively. Inasmuch as a range of 4 km is not permissible in the case of such an excellent spectrum, the star was suspected to be a short period variable, and further observations were obtained, as on page 180.

On plotting these observations, Fig. 1, it became evident that Polaris is a spectroscopic binary, having a period a little less than four days. The 1899 observations have been collected and plotted, in Fig. 2, on the assumption that the period is

$3^d\ 23^h$. The velocity, at present, seems to be included between —8.6 and —14.6 km, having an extreme range of only 6 km. The velocity of the binary system seems to be about 12 km.

The determinations of velocity made in 1896 lie entirely outside of the present range of values, and leave no doubt that the



velocity of the binary system is changing under the influence of an additional disturbing force. I think it is certain, therefore, that Polaris is at least a triple system.

The 1896 observations were made at intervals differing but little from multiples of the period of the binary system, and therefore fell near the same point in the velocity curve. Assuming a period of $3^d\ 23^h \pm$, there is no difficulty in selecting the epoch of minimum so that these six observations will fall on the

curve satisfying the 1899 observations. The residuals will be negligible if we assume the observations to fall near the lower part of the curve, as in Fig. 3; and I have no doubt that future determinations of the orbit will definitely place them there. It will be seen, on comparing Figs. 2 and 3, that the velocities of the binary system in 1896 and in 1899 differ about 6 km.

LICK OBSERVATORY,
Sept. 1, 1899.

THE VARIABLE VELOCITY OF POLARIS.

By EDWIN B. FROST.

POLARIS has not been on the regular working list of stars whose velocities are to be determined at the Yerkes Observatory; but in view of the interest in Professor Campbell's important discovery, it seems desirable to give at once the results of the measurements I have just made on three plates recently secured, which confirm the short period variation in the velocity.

The velocities are:

1899, August	10,	20.8^h G. M. T.		-12.0 km
Sept.	20,	19.2		-17.7
	27,	16.0		-10.6

The first plate was taken under unfavorable circumstances, with a short camera of 271 mm focus, and would be called a poor plate. The velocity is based upon the displacements of six star lines, titanium being used for comparison. The result, however, may be expected to be reliable within three kilometers.

The second plate is an excellent one, taken with a camera of 456 mm focus. The probable error of the above mean of the determinations from 12 lines is ± 0.59 km. The third plate is equally good. The probable error of the mean of the 16 lines measured is ± 0.48 km. The three plates were photographed by Mr. Ellerman.

On locating the second observation on Professor Campbell's curve by reckoning forward with a period of $3^d\ 23^h$ from his minimum of -15.4 km on August 24, it appears that the observation (Sept. 20) falls within a little over an hour of a minimum. It will be seen that the third plate also falls close to its expected position on the curve.

A fourth plate, secured, as was the third, while this was passing through the press, and not yet measured, appears on inspection to give a result accordant with the curve.

The range of variation, 7 km, is within a kilometer of that found by Professor Campbell.

Of course my observations cannot give testimony on the question of a long period motion in the system of Polaris.

YERKES OBSERVATORY,
Sept. 26, 1899.

THE WAVE-LENGTH OF THE GREEN CORONAL LINE, AND OTHER DATA RESULTING FROM AN ATTEMPT TO DETERMINE THE LAW OF ROTATION OF THE SOLAR CORONA.[1]

By W. W. CAMPBELL.

A DETERMINATION of the law of rotation of the solar corona would no doubt be valuable on many accounts. Interest in this problem was aroused by Deslandres' attempt to solve it at the 1893 eclipse, in Senegal. The reality of his result has been questioned on the ground that the H and K calcium lines, used by him, do not have their origin in the corona, but in the prominences and chromosphere. It seemed proper that another determination should be attempted, basing it upon light radiations which are of unquestioned coronal origin. Accordingly, as one of many problems, it was undertaken by the Crocker expedition[2] sent out from the Lick Observatory to observe the eclipse of January 22, 1898, in India.

It was evident that this investigation, involving the Doppler-Fizeau principle, and requiring high dispersion, could apply, with any hope of success, only to the bright-line portion of the corona. Existing data seemed to show that the green line and those near λλ 423 and 399 were the only reasonably strong and unquestioned coronal lines in the available portion of the spectrum; and further, that the photographic action of the green radiation was vastly stronger than in the case of the other two lines, even though the green line lay in a region of weakness on isochromatic plates. It was therefore decided to base the observations on the green line. The justification of this decision

[1] Read at the Third Conference of Astronomers and Astrophysicists, September 7, 1899.

[2] The expenses of the expedition were defrayed by the late Hon. C. F. Crocker, a regent of the University of California. He was also the patron of the expeditions to Cayenne (1889) and Japan (1896).

seems clear, in view of Newall's experience.[1] His ingenious and powerful apparatus was constructed for recording the blue line at λ 423; but his plate, on development, showed no trace of any impress of the coronal line.

Since prismatic dispersion in the green is relatively weak, the question of using a large-sized grating was considered. Two gratings with ruled surfaces about 2.5 × 3.9 inches were kindly loaned by Mr. Brashear; but some simple experiments indicated clearly that a large number of suitable prisms would be more efficient for this purpose than a grating, besides permitting greater compactness and stability of mounting.

The optical train of the instrument employed was as follows:

Image lens; clear aperture, $2\frac{1}{16}$ inches; focal length, $19\frac{7}{16}$ inches; forming an image of the Sun on the slit, with diameter 0.184 inch.

Collimator lens; clear aperture, $2\frac{5}{32}$ inches; focal length, $20\frac{3}{4}$ inches.

Four compound prisms, altitude of each, $2\frac{5}{8}$ inches; face, $2\frac{1}{8}$ inches; combined minimum deviation for λ 5317, 152° 37′.

Two single 60° prisms; altitude, $2\frac{1}{2}$ inches; face, $3\frac{1}{8}$ inches; combined minimum deviation for λ 5317, 113° 04′.

Camera lens; clear aperture, $2\frac{1}{2}$ inches; focal length, 20 inches.

The combined deviation of the six prisms being 265° 41′, the beam of light in the camera crossed the beam in the collimator nearly at right angles.

These optical parts were mounted in wood (Spanish cedar) from my drawings, by the Observatory carpenter, and finished in shellac. The instrument was easily adjusted, and worked well.

Immediately in front of the plate-holder were three rectangular sliding diaphragms for controlling the exposures. These could be withdrawn, singly, in a direction parallel to the Fraunhofer lines of the spectrum. One of these (A) covered the red end of the spectrum up to within $\frac{1}{4}$ inch of λ 5317. Another (B) covered the violet end up to within $\frac{1}{4}$ inch of λ 5317. A

[1] *Proceedings Royal Society*, **64**, 56, 1898.

third (C), about $\frac{5}{8}$ inch wide, covered the central half inch of the plate, overlapping (A) and (B) very slightly.

A Cramer isochromatic plate, backed with Carbutt's liquid backing, was employed.

The slit of the spectrograph was set at 0.04 mm and directed upon the solar equator. About 15 minutes before totality the red end of the plate was uncovered by withdrawing diaphragm A. The solar crescent was caused to drift rapidly along the slit, thereby recording the solar spectrum in the region A, for reference. Diaphragm A was inserted, and the polar axis was set in motion by the clockwork. Two seconds after totality, diaphragm C was withdrawn, allowing the region of the coronal spectrum between $\lambda\lambda$ 5260 and 5430 to record itself on the plate. The diaphragm was inserted at $1^m\ 52^s$ after the beginning of totality. After third contact, diaphragm B was withdrawn, and the solar crescent caused to drift along the slit, recording the solar spectrum on the violet end of the plate, for reference.

On developing the plate, in camp, the solar spectrum at the red end of the plate was found to be suitably exposed, whereas that at the violet end was underexposed—but measurable—probably on account of my underestimating the reduced brightness of the thin crescent. The continuous spectrum of the inner corona was strongly recorded, as was also one strong bright line. However, I was struck with the fact that the bright line fell much further to the violet side of the region uncovered by diaphragm C than I had expected. Being busy with further photographic developing, in the intense heat, the matter escaped my attention. While engaged in measuring this plate, in January 1899, I learned that Lockyer had assigned a new wave-length to the green coronal line. Reducing my measures, I at once obtained a result in substantial agreement with his.

The details of my determination of the wave-length are given below. The first column contains Rowland's wave-lengths of the solar lines used for reference. The second contains these wave-lengths corrected for the fact that the lines under diaphragm A had their origin at the E. (approaching) limb of the

Sun, and those under *B* had their origin at the W. (receding) limb. The micrometer readings on the solar lines, and on the bright lines, are contained in the third and fourth columns, the value of a revolution of the screw being 0.25 mm. The readings on the bright coronal line were made at points 2′ from the E. limb and 1′ from the W. limb.

Rowland's W.-l. in ☉	Corrected W.-l. in ☉	Micrometer measures E. of ☉	Micrometer measures W. of ☉	Computed W.-l. E. of ☉	Computed W.-l. W. of ☉
5183.79	5183.83	12.162	12.062	5183.83	5183.83
5227.2[1]	5227.2[1]	27.883	27.790	5227.26	5227.21
Bright	Line	53.176	53.185	5303.21	5303.32
5463.33	5463.29	98.908	98.957	5463.30	5463.36
5476.9[1]	5476.9[1]	102.402	102.427	5476.96	5476.90
5497.74	5497.70	107.603	107.650	5497.70	5497.70
5528.64	5528.60	115.137	115.177	5528.67	5528.60
5603.10	5603.06	132.161	132.226	5603.02	5602.95
5688.44	5688.40	150.080	150.152	5688.53	5688.38
5763.22	5763.18	164.441	164.562	5763.18	5763.18

The wave-length of the bright line was computed by means of Dr. Hartmann's exceedingly valuable interpolation formula,[2]

$$\lambda - \lambda_0 - \frac{C}{R_0 - R} = 0\,.$$

Substituting 5183.83, 5497.70 and 5763.18 successively for λ, and their corresponding micrometer readings for R, and solving for λ_0, C and R_0, there resulted,

$$\text{For E. side, } \lambda = 3805.43 + \frac{[5.850829]}{526.744 - R}\,;$$

$$\text{For W. side, } \lambda = 3806.57 + \frac{[5.850844]}{527.088 - R}\,.$$

Substituting the various values of R in these equations, and solving for λ, we readily obtain the wave-lengths in the last two columns of the table. The three wave-lengths on which the formula were based naturally reproduce themselves. The accordance

[1] A double line.

[2] This JOURNAL, 8, 218, November 1898.

of the computed values of the wave-lengths with Rowland's values furnishes an indication of the accuracy of the results. No doubt a slight improvement of the residuals would have resulted from a least-square determination of the values of λ_0, C and R_0; but on account of the character of the bright line, to be described later, this would have been superfluous.

The wave-lengths obtained for the coronal line are

For E. side,	5303.21
For W. side,	5303.32
Mean,	5303.26

The difference of the determinations for the E. and W. sides is 0.11 t.m., corresponding to a relative velocity in the line of sight, for the two sides, of 6.2 km, or a rotational velocity of 3.1 km per second. However, I regard this last result as subject to a possible error of at least ± 2 km per second: partly on account of unavoidable errors of observation, but principally on account of the character of the bright line. The inner ends of the bright line are overexposed, while its outer ends are underexposed, and it does not seem to be monochromatic. The suitably exposed portions of the line are not only ill-defined, but unsymmetrical, and accurate settings on them could not be made. It is possible that the original radiations were reasonably monochromatic. A good photograph of the green coronal ring was secured with another of our instruments, an objective grating spectrograph, showing this ring to be extremely irregular in form; and I believe the observations by Lockyer, Newall and others led to the same conclusion. Such being the case, we should expect rapid movements to occur within this atmosphere. The recorded appearance of the line $\lambda 5303$ finds, possibly, an easy explanation in the distortions due to relative velocities, in the line of sight, of the different portions projected on the slit.

In planning the apparatus, it was taken for granted that this line would fall at $\lambda 5317$. Inasmuch as some of the earlier observers mention having seen one or two additional bright lines in this vicinity, the diaphragms were arranged so that one half inch of the plate was reserved for coronal exposure, hoping to

record any of these additional lines. This is responsible for the great distance between the bright line and the solar reference lines; and, besides, the whole purpose of the main problem was to measure *difference* of wave-lengths. Had it been intended to determine the wave-length of the green line, the comparison spectrum would have been arranged very differently. However, the above value of the wave-length should not be in error by more than ± 0.15 t.m.

As confirmatory of the above value, the green ring on the objective grating spectrograph, referred to above and to be described later, is at $\lambda 5303$.

It is desirable that the position of the line should be determined as accurately as possible at the next eclipse.

The radial length of recorded bright line at the E. limb is 4′, and at the W. limb 2′. It does not seem wise to attempt to measure the coronal rotation at the short eclipse of 1900, but it is possible that useful results might be obtained at the East Indian eclipse of 1901, lasting 6½ minutes.

The continuous spectrum of the inner corona was recorded out to a distance of 2.5′ on the E. side, and of 1.5′ on the W. side. The greater strength of coronal radiation on the E. side is very apparent for both bright-line and continuous spectra. While the dark lines in the recorded comparison spectra are sharp and strong, there is not the slightest trace of dark lines in the recorded continuous spectrum of the corona. This radiation seems to be of coronal origin, and not due to reflected photospheric radiations.

I do not think it is difficult to explain the origin of the error which has prevailed for many years in the accepted value ($\lambda 5316.87$) of the wave-length of the bright coronal line; and the following explanation is respectfully suggested.

The strongest chromosphere line in this region is the one at $\lambda 5317$. On one of my photographs, giving a continuous record of the spectrum of the Sun's edge, both when the thin crescent was gradually disappearing at contact II, and reappearing at contact III, the line $\lambda 5317$ is the brightest in this region, and

"persisted" slightly longest.[1] Likewise, in visual observations at contact II, it would no doubt be the brightest line visible, and "persist" longest. Inasmuch as my moving plate recorded many of the faintest chromosphere lines in Professor Young's list, but made no record of a bright line at $\lambda 5303$, it is pretty certain that the true coronal line would be difficult to observe so long as the chromosphere spectrum was visible. The observers made it their first duty to fix the position of the green line. The persisting chromosphere line, very conspicuous just before and at the instant of totality, was naturally assumed to be identical with the true coronal line, and its position was fixed at 1474K. Later, when this line had disappeared, rather suddenly, and the background had become dark enough to allow the line at $\lambda 5303$ to be seen, the observers were interested in determining the extent, and other allied properties, of this line; and no further micrometer settings were made for determining its wave-length. This illustrates one of the advantages of photographic methods, now happily available.

The photograph above described is not suitable for mechanical reproduction; but, with the Director's assent, I should be willing to supply copies on glass to those investigators who are planning for observations, based on the green coronal line.

Professor Young contributed largely to this determination of the position of the coronal line, by arranging for the most generous loan of the four large compound prisms, described above, and belonging to Princeton University; and also, by the fact that the instrument was manipulated during totality, in a perfectly satisfactory manner, by one of his former students in Dartmouth College, the Rev. J. E. Abbott, long a resident of Bombay, who extended many favors to the expedition.

LICK OBSERVATORY,
August 23, 1899.

[1] This persist[illegible] greater b[illegible]ss of the line, rather than to a thicker stratu[illegible]

THE RING NEBULA IN LYRA.[1]

By JAMES E. KEELER.

ON taking charge of the Crossley 3-foot reflector of the Lick Observatory, about a year ago, it was my intention to devote the instrument to spectroscopic work, for which a spectroscope, designed by Professor Campbell, had been partially completed by the Observatory instrument maker. It was first necessary to make numerous small changes in the mounting and in the guiding apparatus. The pier was also cut down two feet, and a new and powerful driving-clock was made at the Observatory from plans by Professor Hussey.

To test the capabilities of the instrument, a number of photographs were then made of well-known celestial objects, and these were of such excellence that I determined to pursue this photographic work for the present, and to put off the spectroscopic investigations until some future time.

Among the objects photographed was the ring nebula, which I have chosen for the subject of the present paper; not because it is specially well suited to display the capabilities of the telescope (for this is not the case), but because it is a very well-known object, to which observers, with photographic telescopes in particular, have paid a great deal of attention, and which possesses in itself many features of interest.

It is very doubtful whether the ring nebula in Lyra has ever been photographed with an entirely suitable instrument. In this connection it may be well to recall the fact that the focal length of a camera must be from thirty to sixty times its aperture in order that the photographic and optical resolving powers may be equal. A photograph taken with such an instrument, under perfect atmospheric conditions, should show all that the eye can see. Practically, however, mechanical difficulties of construction, and the faintness of the light emitted by some

[1] Read at the Third Conference of Astronomers and Astrophysicists, Sept. 7, 1899.

objects, make it necessary to modify the theoretical ratio of focal length to aperture. For photographing faint, diffuse nebulae, in particular, the focal length must be short.

In all modern reflectors the focal length is quite small for the aperture. But the ring nebula is photographically a bright object. It is also a small object (80″ × 60″), and hence could be most advantageously photographed by a reflector of unusually great focal length.

In photographic refractors the ratio of focal length to aperture is usually much greater than in the reflector; but aside from the fact that the absolute focal length of such instruments is, in general, small, the absorption of the chemically active rays by the glass lenses is so great that the nebula can no longer be regarded as a bright object. Thus, I find that exposures of nine, and even up to twenty hours have been given to the ring nebula with refracting telescopes. With the Crossley reflector such exposures would yield nothing but a large black blotch on the negative.

The Crossley photographs.—As an example of the exposure-times required for the Crossley reflector, I give the following list of negatives made here under the finest conditions:

1899, July 13. Exposure two hours. All parts of the nebula greatly over-exposed, though the plate was treated for over-exposure.

July 12. Exposure one hour. Over-exposed.

July 14. Exposure thirty minutes. Good photograph; treated for over-exposure.

July 14. Exposure ten minutes. Best general picture of the nebula.

July 14. Exposure two minutes. Distinct image.

July 14. Exposure one minute. Faint image.

July 14. Exposure thirty seconds. Barely visible image.

The focal length of the Crossley telescope is seventeen and one half feet, and the longer diameter of the ring nebula on the plate is about 2 mm. If the focal length of the telescope were increased four times, the diameter of the image would be about 8 mm, and the length of exposure required would be about three hours, which is not excessive. With such an instrument a far better photograph could doubtless be obtained than any that has yet been made.

But it is impracticable to change the focal length of a telescope, while the aperture is easily varied. Considering the Crossley telescope, therefore, as an instrument of fixed focal length, the only advantages to be expected from reducing the aperture are (1) diminution of aberration, (2) diminution of atmospheric disturbance.

1. The aberration of a parabolic mirror has been discussed (among others) by Professor Schaeberle, who found it excessive at a short distance from the axis; but the case is not really so bad as he made it out to be, as may be shown by applying the more rigid methods of physical optics to the same problem. The excellent star images on Sir Isaac Roberts' photographs are a practical support of the above statement. With the Crossley reflector the star images are quite good one or one and one half inches from the axis, and at half an inch from the axis they are practically perfect. For such an object as the ring nebula the aberration is therefore insensible.

2. The photographs mentioned in the preceding list were taken on nearly perfect nights, when no improvement in the definition would have resulted from cutting down the aperture.

I have tried the Crossley telescope on objects still smaller than the ring nebula. A photograph of the planetary nebula *G. C.* 4628 ($26'' \times 16''$), taken on the night of July 30 with two minutes' exposure, shows the nearly circular outline, the distorted inner bright ellipse, and the central nucleus, almost exactly as drawn by Professors Holden and Schaeberle.[1] The exposure is about right. Another plate, to which was given an exposure of ten minutes, shows the projections or "ansae," strong at the outer ends, and faintly connected with the main nebula, which is greatly over-exposed. For such objects, however, visual observation with the 36-inch refractor is more satisfactory than photography.

The photographs of the ring nebula made with the Crossley reflector show features which have been described by observers with powerful visual and photographic telescopes, and others of which I can find no description and which appear to be new. In

[1] *Monthly Notices*, **48**, Plate 4.

this connection I have consulted a large number of papers on the nebula, and such drawings and photographs as have been published and are in our library. I have also re-observed the nebula with the 36-inch refractor of this Observatory.

The most satisfactory drawing is one made by Professor Holden at Washington in 1875. The original is at the Lick Observatory. Trouvelot's drawing in *H. C. O. Annals*, Vol. VIII, is also excellent. Neither drawing shows the central star, and both are somewhat too regular and symmetrical.

Form of the nebula.—The outline of the ring nebula, as shown by the Crossley photographs, is oval rather than elliptical, the more pointed end being toward the northeast. M. Stratonoff's diagram in *A. N.*, 3388 shows the form well, though the inner dark space is, owing to the long exposure and the spreading of the photographic image, considerably too small. From both sides of the oval project faint masses or fringes of nebulosity, much as drawn by Lord Rosse,[1] but less uniform in shape and brightness, and having no structure. The most important of these projecting nebulosities are tabulated below, the position angles being measured from the central star as center. Not having made trails on any of the plates, I take the position angle of the bright star following the nebula to be 87.8°, as determined by Professor Burnham.[2]

Projection.	Limiting pos. angles	Projection beyond ellipse.
a	7°– 27°	2″
b	31 – 52	10
c	60 – 82	5
d	88 –102	3
e	138 –178	4
f	195 –222	11
g	222 –240	5 0
h	240 –260	7
i	270 –276	2
j	281 –292	2
k	345 – 0	4

[1] *Phil. Trans.*, 1844, Plate 19, Fig. 29.

[2] *Monthly Notices*, 52, 42.

The projection *e* is so much brighter than the others that it is perhaps to be regarded as a part of the nebula proper, rather than as attached nebulosity. Without it the outline of the nebula would be much nearer to a true ellipse.

Structure of the ring. — The ring, as shown in these photographs, has a quite complicated structure. It seems to be made up of several narrower bright rings, interlacing somewhat irregularly, the spaces between them being filled wiih fainter nebulosity. One of these rings forms the outer boundary of the preceding end of the main ring. Sweeping around to the north end of the minor axis it becomes very bright, perhaps by superposition on the broader main ring of the nebula at this place. It crosses this ring obliquely, forming the brightest part of the whole nebula, and then forms the inner boundary of the main ellipse toward its following end. The remaining part of the ring is not so easily traced, as several other rings interlace on the south side of the ellipse. One of these forms the projecting arc *e* already referred to, and then, crossing the main ring obliquely, it forms the inner boundary of the ellipse on the north side.

The main ring contains many bright patches and condensations, but I find no evidence in my photographs of general resolution into detached knots of nebulosity, such as Denza thought he had observed.

Dimensions of the nebula.— When an object is, like this, not sharply bounded, but fades outward somewhat gradually into the sky, its dimensions determined from a photograph must depend upon the length of exposure given to the negative; further, and for the same reason, its dimensions determined by visual micrometer measures must depend to some extent upon the aperture of the telescope employed. It is also possible that the exterior fringes of nebulosity may, like the outlying streamers of the nebula of Orion, emit chiefly the hydrogen radiations, in which case they would be visually weak and photographically strong. The photographic image would then be larger than the visual. In the case of a photograph taken with long exposure, the

image may also exceed its proper dimensions by the spreading of the photographic action.

In *A. N.*, 3354, Professor Barnard has given his measures of the ring nebula with the Lick telescope, and in *A. N.*, 3388, M. Stratonoff has compared these measures with his own, made on photographs taken with a refractor of 0.33 m aperture. I give these results below, together with my own. It will be seen that the dimensions determined by photography are, in general, larger than those obtained visually.

	Barnard (36 in. visual.	Stratonoff (phot. 10 h.)	Keeler (phot. 10 m.)
Major axis outside ellipse - -	80.89″	90.19″	87.3″
Major axis inside ellipse - -	36.52	29.13	40.8
Minor axis outside ellipse (extreme)		63.11	63.9
Minor axis outside (mean) ellipse	58.81		58.6
Minor axis inside ellipse - -	29.36	24.52	32.0

I have assumed that Barnard placed his micrometer wire tangent to the mean ellipse, in measuring the length of the minor axis outside. Stratonoff, with his long exposure, certainly measured the extreme diameter, *i. e.*, to the outside of the projection *e* of my table. His plate with 20^h exposure shows a still greater divergence from the visual measures. The irregularities of the outlines, and the somewhat gradual fading of the light, make the measures of my plates somewhat uncertain, though the definition is excellent.[1]

Structure of the interior space.—Lord Rosse's drawing in the *Phil. Trans.*, 1844, shows the interior space of the nebula crossed by a series of dark and bright bands in the direction of the major axis, and this drawing has, I think, generally been regarded as fanciful. Nevertheless, the structure it represents is confirmed by the Crossley photographs, and confirmed, so far as I am aware, for the first time.[2] There are, however, only three dark and two bright bands within the ellipse, unless the

[1] On the plate with 10 m exposure the disks of stars 3″ apart are clearly separated.

[2] PROFESSOR HOLDEN'S partial confirmation with the Washington telescope was subsequently regarded by himself as probably a mistake. *Mon. Not.*, **48**, 384, 1888.

nebulous border around the inner edge of the ring is regarded as constituting two more of the latter. One of the dark bands is centrally placed. The central star is not precisely on the middle line of this band, but a little nearer the north edge. The direction of the bands is not exactly that of the major axis of the nebula, but in a position angle about 5° greater. At the following end of the interior ellipse the nebulosity is brighter, as described by Holden and illustrated in his diagram (*Monthly Notices*, **48**, 386), forming a bright patch without definite outlines. The bright bands become fainter toward their intersection with the minor axis. It is quite probable that on photographs of still better definition, this band-like appearance would be resolved into a more complex structure, to which it is perhaps only incidental.

I have tried to verify this band structure of the photographs by visual observation with the 36-inch refractor, and have fancied that at times I could catch glimpses of it; but the observation is a most difficult one. The image as seen in the telescope is sufficiently large, bright, and well defined, but the contrast of the light and dark bands, which is exaggerated by the photograph, is almost too slight to affect the eye.

The central star.—The actinic power of the central star[1] of the ring nebula has been remarked by many observers. My own photographs also demonstrate this peculiarity of the star, for it is perfectly distinct on the plate taken with an exposure of one minute, and is faintly visible on the plate exposed for thirty seconds. But a more careful study of my negatives shows that the actinic power is less remarkable than I had at first supposed it to be. Other stars, which have never been regarded as possessing any unusual properties, are also strongly impressed on plates with short exposures.

On plates to which long exposures were given, the central star is just about equal to Lassell's star 3, outside the ring. With shorter exposures the superiority of the central star becomes apparent, and on the plate exposed two minutes the

[1] Estimated by Burnham as 15.4 visual mag. *Mon. Not.*, **52**, 42.

comparison star is no longer visible, while the central star remains. The difference is however, not very striking.

I may note here that the strength of the central star relatively to that of the nebula depends upon the nature of the photographic instrument employed ; the brightness of the star follows one law, that of the nebula another. This relative brightness, therefore, considered by itself, has little physical meaning.

From observations made in 1891 with the 36-inch refractor, I concluded that, as in an ordinary star, the maximum intensity of the light in the spectrum of the central star of this nebula falls in or near the yellow. The spectrum itself is too faint for observation, and the above conclusion rests on the fact that the nebula is best seen with the eyepiece drawn out a little from the position which gives the most distinct vision of the central star.[1] The nuclei of planetary nebulae, have, according to Scheiner,[2] the same remarkable actinic power as the central star of the ring nebula ; nevertheless, in the spectra of these stars, many of which could be observed with the Lick spectroscope, the maximum brightness was found to be in the yellow region. I have suggested in my memoir on the spectra of the nebulae, in Vol. III of the *Publications of the Lick Observatory*, that the photographic strength of these central stars may be due to bright lines in the upper spectrum (very probably the ultra-violet hydrogen series). With suitable apparatus, which is in preparation, I anticipate no difficulty in photographing the spectrum of the central star of the ring nebula. The results, if successful, can hardly fail to throw light on the particular question involved, and indeed on the whole question of stellar evolution.

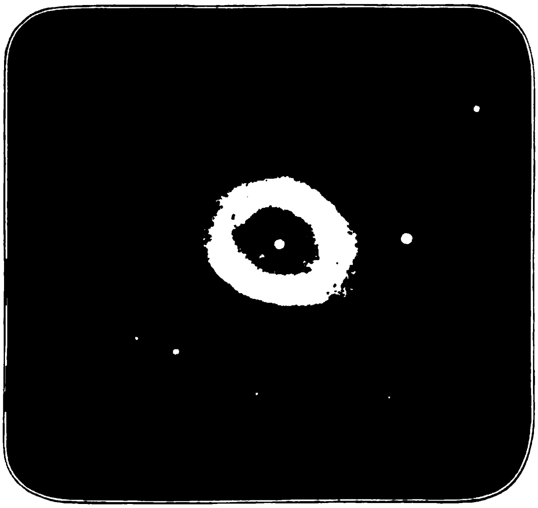
On all of my photographs the central star is as clearly defined as are other stars outside the nebula ; there is no evidence of blending into the nebulous background. This is also the appearance of the star as seen with the 36-inch refractor.

Other stars in the nebula.—Besides the central star, only one other star is shown on the photographs within the darker

[1] *Publications of the Lick Observatory*, 3, 210.

[2] *A. N.*, 3086. See also my own photographic observations of *G. C.* 4628 above.

PLATE VII.



Ring Nebula in Lyra. Exposure 10^m.

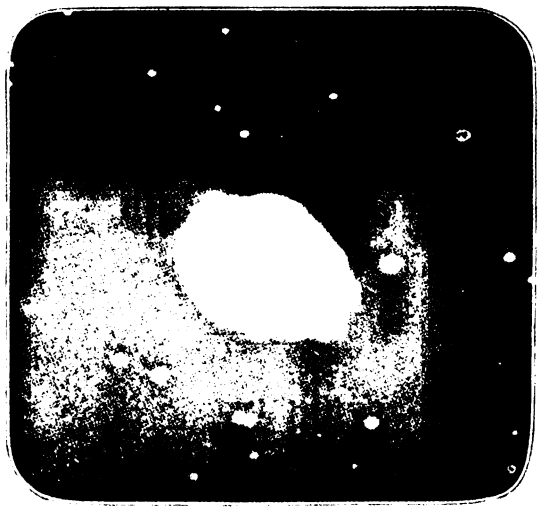
Photographed with the Crossley Reflector of the Lick Observatory.

PLATE VIII.

RING NEBULA IN LYRA. EXPOSURE 30^{m}.

Photographed with the Crossley Reflector of the Lick Observatory.

PLATE IX.



RING NEBULA IN LYRA. EXPOSURE 2^h.
Photographed with the Crossley Reflector of the Lick Observatory.

space enclosed by the ring. Its position with reference to the central star is

$$p = 297^\circ \qquad d = 10.7''.$$

It is evidently the star *d* of Holden's diagram.[1] This star is very distinct on the negative made with 10 m exposure, and is just visible on that exposed for 2 m. As it is at the very limit of vision with the 36-inch refractor, it must, like the central star, possess unusual photographic energy.

Holden's star *f*, at the preceding extremity of the major axis, also appears on the photographs. On some of the plates it is larger than other stars of the same magnitude, and slightly irregular, so that it is perhaps not a true star, but a very small bright patch of nebulosity. The other stars of the diagram seem to be bright patches due to the interlacing of the narrow rings, but in such cases as this the evidence of the visual telescope is perhaps to be preferred.

Barnard's small nebula.—The negative obtained with two hours' exposure shows that the small nebula discovered by Professor Barnard[2] with the 36-inch refractor in 1893 is a left-handed, two-branched spiral. The extreme diameter on the photograph is about 30.

The positive enlargements (11.1 diameters, 1 mm = 3.55′), sent with this, in the form of lantern slides, reproduced in Plates VII, VIII, and IX, show most of the details described in the foregoing article.

In all work with the Crossley reflector I have had the efficient assistance of Mr. H. K. Palmer, Fellow at the Lick Observatory.

LICK OBSERVATORY,
August 15, 1899.

[1] *Mon. Not. R. A. S.*, **48**, 386.

[2] *A. N.*, **134**, 130, 1893.

PRESSURE IN THE ELECTRIC SPARK.

By John Fred Mohler.

In an article published in the Astrophysical Journal for June 1899, Eduard Haschek and Heinrich Mache give a method by which they obtain the amount of pressure produced when a spark is passed through a gas. They give results for various media, for different electrodes, and for variation of capacity, spark length, and pressure of the surrounding media.

The work touches some done by Dr. Humphreys and myself[1] in that we have found that when the arc is under pressure the period of vibration of the light emitted is a little greater than when the pressure is removed, and consequently the wave-length is increased a little when pressure is added. We have shown in the article referred to above that this change in wave-length or shift of the spectral lines is proportional to the pressure, and in some cases to wave-length, and that it varies with the element producing the line.

The measured shift of the lines of a few of the elements under twelve atmospheres pressure, together with the shift reduced to wave-length 400, is given in the following table:

	Wave-length	Measured shift	Reduced shift
Cadmium - -	4678.389	0.092	0.080
" - -	4800.097	.096	.080
" - -	5086.001	.106	.080
Iron, average of lines		.025	.025
Zinc, average of lines		.057	.057

Using these data as a starting point it would seem possible to measure the pressure at the source of light if we knew the displacement of the lines. This assumes that a spectral line will be displaced by pressure whatever may be the source of light producing the line. The question of the amount of pressure becomes very interesting in the light of the recent experiments

[1] This Journal, 3, 114, 1896; also 4, 175, 1896.

of Professor J. Wilsing,[1] who obtained a spectrum very similar to that of the new stars by passing a spark between electrodes immersed in water.

My experiments described below were made to test the results obtained by Haschek and Mache. If their results are near the truth the shift of the lines due to pressure should be very considerable; indeed, as in many experiments they found a pressure of more than fifty atmospheres, the shift of the lines should be four or five times the largest shift we found in previous experiments with the arc, and the displacement due to one atmosphere is a measurable quantity with some elements.

I used in this investigation a four-inch concave Rowland grating, mounted in the usual way on piers of solid masonry, in a room of nearly constant temperature. Photographs of the spectrum under consideration were taken along with the spectrum of the arc, at atmospheric pressure, for comparison. The camera was very solidly mounted and the plate was exposed to the arc spectrum both before and after it was exposed to the spark. For varying the pressure around the spark and for investigating the effect of different gases the electrodes were inclosed in a heavy brass vessel similar to that used in the previous work.[2] This vessel was mounted with the arc lamp on a swinging frame, so arranged that either spark or arc could be focused on the slit of the spectroscope without touching the mounting of the grating. For capacity I used a series of jars, some large and some small. Their capacity was not very accurately determined, but the error cannot be more than 4 or 5 per cent. The induction coil used was capable of giving an eight-inch spark when used without capacity. The spark gap was usually 3 mm or less. A filar micrometer in connection with a very low power microscope was used to measure the displacement of the lines. As the previous work had shown that cadmium gives a relatively large displacement under pressure I used that metal in most of my experiments.

[1] *Sitz. d. K. Akad. d. Wis. zu Berlin*, No. 24, May 4, 1899; this JOURNAL, **10**, 113, 1899.

[2] This JOURNAL, **3**, 116, 1896.

CAPACITY.

Messrs. Haschek and Mache found that under given conditions the pressure produced when the spark is passed through air varied with the capacity in a peculiar way. As the capacity increased the pressure increased to a maximum of fifty-one atmospheres and then decreased. Below is part of their table giving the relation of capacity to pressure:

Capacity in meters	Pressure in atmospheres
5.16	22
11.1	40
22.9	45
53.1	51
100.2	46
156.	36

With cadmium electrodes and at atmospheric pressure I found the displacement of the green and blue lines with varying capacity to be as given in the following table. The measured shift is given in thousandths of an Ångström unit, the capacity is given in meters. Considering 0.008 as the measured shift of the cadmium lines per one atmosphere pressure, I give in the same table the pressure in atmospheres deduced by this method.

Wave-length	Capacity	Shift	Pressure
5086.001 4800.097 4678.339	11.1	0.026	3.25
4800.097 4678.339	22.2	0.036	4.5
4800.097 4678.339	22.2	0.052	6.5
4800.097 4678.339	48.	0.084	10.5
4800.097 4678.339	70.2	0.088	11.

The above table shows that the pressure calculated from the shift of the lines is very much smaller than that given by Haschek and Mache. Considerable allowance must be made for errors in measurement of the shift, as the spark lines, with condensers in the circuit, become broader as the capacity is increased.

The last result with capacity of 70.2 meters gives but a slightly greater shift than a capacity of 48 meters. This seems to indicate that the pressure does not increase directly with capacity, but as indicated by Haschek and Mache, the pressure approaches a maximum value.

The effect of capacity on the position of the iron lines of the spark spectrum was also investigated. With small pieces of steel as electrodes several photographs were taken with a capacity of 22.2 meters. About twenty lines on these plates were measured. The shift was small, and the average measured displacement was 0.011 Ångström unit. The previous work indicated that the shift per atmosphere was about 0.002 Ångström unit for the iron lines. This would indicate a pressure in the spark of 5.5 atmospheres which is the average found for cadmium for the same capacity.

PRESSURE.

The effect of the pressure of the surrounding medium is shown by comparing the shift produced by the spark with a definite capacity when the pressure about the electrodes is one atmosphere with the shift produced when the pressure is four times as much, the capacity remaining the same. The capacity used for this experiment was 22.2 meters. The pressure was four atmospheres and the measured shift was 0.160 Ångström unit, corresponding to a pressure in the spark of 20 atmospheres. This, compared with the shift at atmospheric pressure, shows that the pressure in the spark varies very nearly with the pressure of the surrounding medium. This is altogether different from the results given by Haschek and Mache, who find that the pressure in the spark increases very much faster than the pressure in the surrounding medium.

The effect of the kind of gas surrounding the electrodes as given by Haschek and Mache indicates that carbon dioxide had three times the effect of atmospheric air, and strangely enough, illuminating gas with a density of 0.47 had produced a greater pressure than air.

The results of my experiments to test this point are given below. The measurements were made on the two blue cadmium lines of wave-length 4800.9 and 4678.3.

Capacity	Shift, CO_2	Pressure, CO_2	Pressure, air
22.2	0.067	8.4	5.5
48.	0.116	14.5	10.5

This gives an average ratio of the effect of carbon dioxide to that of air of 1.45 instead of 3.

The above results show, I think, that there is pressure produced when the spark passes through a medium, but that it is not nearly so great as supposed from the work of Haschek and Mache. These results also show that the amount of pressure varies with the density of the medium surrounding the electrodes, and that the kind of gas does not affect the result. With a medium such as water, 800 times as dense as air, with a small capacity (say one meter) a displacement of about 0.4 Ångström unit would be produced in the iron lines, which is only a little less than the average displacement of the lines obtained by Professor Wilsing.

DICKINSON COLLEGE,
Aug. 30, 1899.

MINOR CONTRIBUTIONS AND NOTES

ON TITANIUM FOR A COMPARISON SPECTRUM.

IN the preliminary observations for determining the radial velocities of stars with the spectrograph attached to the forty-inch refractor of the Yerkes Observatory, the spark between iron electrodes was employed as the source of the comparison spectrum. The spark apparatus was swung into place in the collimation axis of the refractor, and a brief exposure given to the spark either before and after or at the middle of the exposure to the star, according to the duration of exposure required for the star. The region of spectrum brought to the center of the plate was about $\lambda 4600$. The iron comparison spectrum proved, however, to be rather unsatisfactory on account of a scarcity of good lines in the spark in this region, and on account of the troublesome air lines which are here numerous and broad, obscuring some of the fainter iron lines. In looking about for some other element to substitute for iron, I decided that titanium ought to give satisfactory results. On obtaining metallic titanium, through the courtesy of Professors Smith and Lengfeld, of the chemical department of the University, I was gratified to find that the spark gave a large number of very sharp lines, very uniformly distributed throughout the spectrum from λ 4200 to λ 5000, and free from air lines. The spark passes much more steadily between titanium electrodes than in case of iron, and considerably shorter exposures are required — less than ten seconds as the apparatus is usually arranged. The consumption of the element is very slight, and it is only necessary to occasionally brighten up the points of the electrodes.

The wave-lengths of the lines in the titanium (arc) spectrum have been very accurately determined by Hasselberg,[1] and many of the lines are identified in Rowland's table of solar spectrum wave-lengths.

In stellar spectra of the solar type numerous coincidences can therefore be obtained, and the direct displacements of the star lines can be measured. When the wave-lengths of the comparison lines are

[1] This JOURNAL, 4, 116, 212, 1896.

accurately known, however, it is not important that direct displacements should be employed, and I have commonly selected the best star lines, and measured their position with respect to the nearest titanium lines, which are seldom more than five tenth-meters distant. The scale value, $\frac{d\lambda}{ds}$, at any point in the spectrum can be very accurately calculated from Hartmann's convenient interpolation formula.

In spectra of Type I*b* it happens that almost every important stellar line falls very close to a sharp titanium line. In spectra of Type I*a*, measures of the broad and difficult hydrogen lines are not commonly necessary, only the finer lines being employed.

In the procedure which I have adopted for the reduction of stellar plates for radial velocity, no auxiliary solar plates are employed, and every plate is reduced independently of all others. It is believed that in this way errors due to differences of dispersion and of camera focus, etc., on account of temperature, can be avoided, and the danger of systematic error somewhat diminished.

It is evident that the wave-lengths of unidentified stellar lines can be sharply determined throughout the whole region of spectrum on a plate, thanks to the uniform distribution of the comparison lines.

EDWIN B. FROST.

NOTE ON THE POSITION OF THE MAXIMUM IN THE SPECTRAL ENERGY-CURVE OF A BLACK BODY.

THE researches of Paschen on the spectrum of the ideal black body recently completed, which have been brought before the readers of the ASTROPHYSICAL JOURNAL in the articles by Paschen and by Paschen and Wanner in the June and May numbers, need no indorsement from me. They bear on their face the evidence of minute care, and are a brilliant confirmation of the remarkably simple law which now reads:

$$\lambda_{\text{max.}} \times T = 2891.$$

A few words in regard to the bearing of these results upon the objections urged by me against the proposed law (this JOURNAL, **2**, 316, November 1895; **4**, 38, June 1896) may be in order.

It will be evident that the law may be departed from rather widely

in the spectra of bodies which do not conform to the ideal of blackness. Moreover the definition of what constitutes blackness needs to be more exactly stated. For absorption, it is sufficient to say that the ideal black body must completely absorb rays of every wave-length, and this ideal appears to be very nearly fulfilled by a bolometer strip, coated thickly with soot and platinum-black in successive layers, and placed at the center of a reflecting hemisphere, pierced centrally by a minute window through which the rays pass, falling normally upon the blackened strip; but the perfect radiator is not so easy to obtain or define. Bodies whose spectral energy-curves have nearly the same form, and which will be called black if the relative emission of rays of different wave-lengths according to a special law is made the criterion of blackness, exhibit wide divergence in total emissive power, and this because the rate of emission depends not merely on the nature of the surface, but also on the conductivity of the underlying substance, and the readiness with which it transmits radiation from interior layers. If the blackest body is one which transfers the greatest amount of energy through the unit of surface in the unit of time by the radiative process, it is conceivable that this position may be taken by a body having a spectral energy-curve very different from that of lampblack, or other substances which have been arbitrarily assumed as standards. A further difficulty is the practical one that no single substance is available, or convenient, as a radiator through the entire range of temperature.

Assuming for the present purpose that blackness of a radiator means conformation to a spectral energy-curve which is very nearly that of lampblack or platinum-black, it appears probable that this standard is approximated by many other substances, at least within a middle range of temperature.

For the incandescent carbon of the electric arc-light, the formula gives:

$$\lambda_{\text{max.}} = \frac{2891}{3900} = 0.74^{\mu}$$

In an article by Captain Abney and Colonel Festing on "The Influence of Water in the Atmosphere on the Solar Spectrum and Solar Temperature" (*Proc. Roy. Soc.*, 35, 328, 1883), is a spectral energy-curve of the positive pole of an electric arc (Diagram II, p. 334) which I overlooked when writing my former articles. The maximum is at about

0.73μ, agreeing well with that deduced by Paschen's law. I presume that the spectrum of the arc-carbon which gave a maximum near $1.1\ \mu$, must have been too impure for establishing this position, but it is no longer necessary to discredit the measurements of the temperature of the arc-carbons in order to fulfill the requirements of the law of the maximum.

Comparing the values of the position of the normal maximum, given in my former paper, with those obtained from Paschen's law, the following differences are found:

T(Absol.) -	1088°	798°	603°	451°	392°	313°
$\lambda_{max.}$ ($V.$) -	3.96^{μ}	4.55^{μ}	5.24^{μ}	6.20^{μ}	6.80^{μ}	8.04^{μ}
$\lambda_{max.}$ ($P.$) -	2.66	3.62	4.80	6.41	7.38	9.24
Diff. ($V.$–$P.$) -	+1.30	+0.93	+0.44	−0.21	−0.58	−1.20

Two causes conspire to produce these differences. (1) The imperfect absorption of long waves by the simply blackened bolometer makes the wave-length of the apparent maximum too small, and this to a greater degree as the temperature is lower and the maximum in the spectrum nearer to these long waves whose absorption by the blackening substance becomes less and less as the wave-length increases. (2) The temperatures of thick radiating plates are estimated too high, on account of neglecting the sub-surface temperature-gradient which is larger and increases the assigned maximum more, the higher the temperature. The first of these causes of error is eliminated in Paschen's recent work by the use of what may be called a repeating bolometer, in which the strip has repeated opportunities for absorbing the reflected remnant of radiation; and the second has been obviated by the use of radiant cavities and extremely thin radiant strips, reducing the sub-surface gradient to a minimum.

The second cause of error does not seem to apply to the radiating surface of an electric arc carbon, at least provided the voltage of the current is high, and the section of the carbon small enough to secure the conditions conducing to constant temperature.

It is to be hoped that Professor Paschen will give us the ratio of radiations of every wave-length, as measured by the repeating and the ordinary bolometer, in order that observations made with the latter may be reduced to the standard of the ideal black body.

FRANK W. VERY.

THE THIRD CONFERENCE OF ASTRONOMERS AND ASTROPHYSICISTS.

At the Second Conference of Astronomers and Astrophysicists, held at the Harvard College Observatory in August 1898, a committee, consisting of Professors Newcomb, Comstock, Pickering, Morley, and Hale, was appointed to make arrangements for a third conference, and to draw up a constitution for a permanent organization. This committee was given power to add to its number, and accordingly Professors Boss, Michelson, Langley, and Ames were requested to serve with the members already appointed. A meeting of the committee was held in Washington in February 1899, and a constitution was drawn up. It was decided at that time to hold the next conference at the Yerkes Observatory. Details of the arrangements were left to Professor Comstock, secretary of the committee, and to the Director of the Yerkes Observatory.

The conference opened its session at the Yerkes Observatory on September 6, 1899, with Professor Harkness in the chair. The following papers were read:

A. S. Flint, The Repsold Transit Micrometer of the Washburn Observatory and Slat Screen Apparatus.

S. J. Brown, The Position of the Axis of Neptune from Perturbations of its Satellite.

E. E. Barnard, Notes on the Annular Nebula in Lyra, the Fifth Satellite of Jupiter, Triangulation of Star Clusters, and Measures of the Difference of Declination of the Stars Atlas and Pleione.

William Harkness, On the Semi-Diameters of the Sun and Moon.

F. R. Moulton, Problems in Modern Celestial Mechanics Treated by the Use of Power Series.

Ormond Stone, On the Motion of Hyperion.

E. C. Pickering, The Revised Harvard Photometry.

Kurt Laves, Determination of the Principal Term of Nutation from Observations of Eros.

W. W. Campbell, Wave-Length of the Green Coronal Line.

Asaph Hall, Jr., Aberration Constant from Meridian Zenith Distances of Polaris.

J. E. Keeler, The Ring Nebula in Lyra.

George E. Hale, Notes on Carbon in the Chromosphere, the Connection between Stellar Spectra of the Third and Fourth Types, and Some New Forms of Spectroheliographs.

E. B. Frost, Notes on the Reduction of Stellar Spectra, Titanium as a Comparison Spectrum, and Corrections of Absolute Wave-Lengths due to the Earth's Motion.

Frank Schlesinger, Suggestions for the Determination of Stellar Parallax by Means of Photography.

S. I. Bailey, Note on the Relations between the Visual and Photographic Light Curves of Variable Stars of Short Period.

S. I. Bailey, The Periods of the Variable Stars in the Cluster Messier 5.

H. S. Davis, A Statement of the Progress of the New Reduction of Piazzi's Meridian Circle Observations between 1792 and 1814.

W. W. Campbell, The Spectroscopic Binaries Capella and Polaris.

F. L. Chase, Refraction of Red Stars.

George C. Comstock, Some Researches in Stellar Color.

Kurt Laves, Inner Potential Forces Applied to Celestial Mechanics.

F. R. Moulton, Laplace's Ring Nebular Hypothesis.

G. W. Hough, On the Actinism or Photographic Power of the Moon in a Total Eclipse.

M. B. Snyder, The Phonochronograph and its Advantages in certain Astronomical Observations.

Abstracts of all these papers will be published in *Science*, and most of the astrophysical papers will be published in full in this JOURNAL.

Several committees appointed at the Harvard Conference presented reports. The committee on a permanent society offered the following constitution, which was adopted without material change:

CONSTITUTION.

Article I. *Name and Purpose.*

1. This association shall be called The Astronomical and Astrophysical Society of America.

2. The purpose of this society is the advancement of astronomy, astro physics, and related branches of physics.

Article II. *Membership.*

1. Those persons whose names were signed on or before September 15, 1899, to the annexed statement of desire to form such an association shall constitute the charter members of this society. Other persons may be elected to membership in the society by the council hereinafter provided.

2. The council shall prepare and publish, in the form of a by-law, uniform rules for the government of such elections.

Article III. *Officers.*

1. The officers of the society shall consist of a president, two vice presidents, a secretary, and a treasurer, who, in addition to the duties specifically assigned them by this constitution, shall discharge such other duties as are usually incident to their respective offices. These officers, together with four other members of the society, shall constitute a council, to which shall be entrusted the management of all affairs of the society not otherwise provided for. The president and secretary of the society shall serve respectively as chairman and secretary of the council, and every officer of the society shall be responsible to the council and shall administer his office in accordance with its instructions.

2. The council shall enact such by-laws as may be found needful and proper for administering the affairs of the society, and may, from time to time, modify or repeal such by-laws.

3. The president, the vice presidents, and the treasurer shall be elected annually, in a manner to be prescribed by the council, and shall serve until their successors are duly elected and qualified. Two members of the council shall be chosen at the first annual meeting of the society to serve for a period of one year, and two members shall be chosen annually to serve for a period of two years, or until their successors are duly elected and qualified. The term of office of the secretary shall be three years, or until his successor is duly elected and qualified.

Article IV. *Meetings.*

1. The council shall determine the time and place of each meeting of the society, and shall provide for an annual meeting, at which officers shall be elected.

2. The council shall have charge of the program for each meeting.

3. At meetings of the society, regularly called, twenty members shall constitute a quorum.

Article V. *Finance.*

1. The council shall levy an annual assessment upon the members of the society sufficient to provide the funds required by the society for the ensuing year; provided that this assessment shall not exceed the sum of five dollars per member in any year.

2. If at any time there shall be required, for the purpose of the society, a larger sum than can be obtained in accordance with section 1 of this article, the council shall present at an annual meeting of the society a statement of such need, and of the circumstances attending it, and the society shall thereupon determine by ballot a policy to be adopted in the matter.

3. No officer of the society shall receive any compensation for services rendered to it, but the council may by resolution direct the treasurer to

reimburse to any officer expenses necessarily incurred by him in the discharge of his official duty.

Article VI. *Amendments.*

1. This constitution may be amended by the affirmative votes of three fourths of the members present at any annual meeting of the society, but no amendment shall be voted upon unless a notice setting forth the nature of such proposed amendment shall have been forwarded to the several members of the society at least one month before the meeting at which it is proposed to be voted upon.

2. It shall be the duty of the secretary to forward such notices of a proposed amendment to this constitution when so requested in writing by ten members of the society.

A by-law subsequently adopted by the council provides that

Any person deemed capable of preparing an acceptable paper on some subject of astronomy, astrophysics or related branch of physics may be elected by the council to membership in the society upon nomination by two or more members of the society. At least once in each year the council shall consider all such nominations and may request the opinion of persons not members of the council with reference to the qualifications of the nominees. Blanks for such nominations to membership shall be furnished by the secretary.

It was decided that officers of the new society should be elected on the last day of the session, and the committee on organization was instructed to take nominating ballots on the afternoon of the previous day. The results were announced at the final session on Friday morning, when officers of the Astronomical and Astrophysical Society of America were elected as follows:

OFFICERS.

President	Simon Newcomb.
Vice Presidents	C. A. Young, George E. Hale.
Secretary	George C. Comstock.
Treasurer	C. L. Doolittle.
Councilors, for two years	E. C. Pickering, J. E. Keeler.
Councilors, for one year	E. W. Morley, Ormond Stone.

On account of the poor health of Professor Comstock, Professor E. B. Frost, of the Yerkes Observatory, has consented to serve as acting

secretary for the present. The list of the charter members of the new society includes one hundred and fourteen names.

At a meeting of the council held on September 8, it was decided that the next meeting of the society should be held in June 1900, at New York, in conjunction with the meeting of the American Association for the Advancement of Science.

The committee on the total solar eclipse of May 28, 1900, consisting of Professor Newcomb, chairman, Professor Hale, secretary, Professor Barnard and Professor Campbell, presented the following preliminary report:

THE TOTAL SOLAR ECLIPSE OF MAY 28, 1900.

The committee on the total solar eclipse of May 28, 1900, appointed at the Second Conference of Astronomers and Astrophysicists, presents herewith a preliminary report.

The aim of the committee has been:

1. To ascertain the opinions of astronomers regarding the best means of securing coöperation, the most important classes of observations and the best means of making them, and the plans of the various eclipse parties.

2. To collect other information likely to be useful to persons planning to observe the eclipse.

For the purpose of securing information on the various points referred to in paragraph (1) a circular letter was addressed to American astronomers. From an examination of these replies it appears:

1. That there is a general willingness to coöperate with the committee in securing thorough observations of the eclipse phenomena and effective distribution of stations along the line of totality.

2. That, in the opinion of those from whom replies were received, the most important observations include studies of the minute structure of the corona, both visually and by means of large scale photographs; photography of the flash spectrum and determination of the wave-length of the green coronal line; measurement of the heat radiation of the corona; photographic search for an intra-mercurial planet.

3. That several institutions, including the Princeton, Lick, Naval, Goodsell, Chabot, Flower and Yerkes Observatories, will probably be represented by well-equipped parties; while a considerable number of astronomers with good instrumental equipment will take part as individuals.

4. That no general appeal to the public for funds is required, as each institution will endeavor to secure the amount necessary for its work.

5. That the work already planned includes observations of contacts, photography of the corona with large and small cameras; visual and photographic observations of the spectrum of the Sun's limb and of the corona;

visual examination of the details of the coronal structure; measurement of the brightness of the sky at different distances from the Sun; search for an intra-mercurial planet; observations of the shadow bands.

Extracts from the letters of various astronomers are appended to this report.

A preliminary report on the weather conditions along the line of totality has been prepared by the Weather Bureau, at the request of the committee (p. 221). From this it appears that interior stations are probably to be preferred to those on the seacoast, in spite of the shorter duration of the total phase. The full report of the Weather Bureau, which will soon be published, will contain much valuable matter, including maps of the eclipse track, showing location of towns and railways; information regarding hotel accommodations, desirable sites; etc.

It is understood that the Naval Observatory will issue instructions to observers, and that a map of the eclipse track will be published by the Nautical Almanac Office. The Treasury Department has made arrangements by which the instruments of foreign parties will be admitted free of duty.

The committee, if authorized by the conference to continue its work, will be glad to receive and publish further information from eclipse parties regarding their plan of observations and location of stations.

In response to the circular letter referred to above the committee has received the following statements regarding the classes of observations which are considered most important.

Ordinary photographs of the corona with cameras of different kinds; photographs with the apparatus designed by Mr. Burckhalter. (I am convinced from an examination of the photographs secured by this method in India, that such photographs show the forms of the coronal streamers far more satisfactorily than a comparison of ordinary photographs taken with different exposures.) Photographs for determining the exact wave-length of the "coronal line." It appears that this line has been erroneously identified with a line in the solar spectrum. Possibly, photographs for showing the displacement of this line on opposite side of the Sun, like those made by Deslandres, though very effective apparatus will be required for this purpose. Photographs of the spectrum of the "reversing layer." In my opinion the best form of instrument for this purpose is an object-glass spectroscope, arranged as follows: the refracting edge of the prism is placed parallel to the disappearing limb of the Sun. At the focus is a fixed slit, placed radial or lengthwise of the spectrum, and the photographic plate below it is moved slowly at right angles to the slit, securing a continuous record of the spectrum. The same observations are repeated at the end of totality. The details of the apparatus require, of course, careful consideration.

JAMES E. KEELER.

I think a photographic search for an intramercurial planet, and a study of the brightness of various portions of the sky by means of photography, such as I made in Grenada in 1886 (*H. C. O. Annals*, 18) would be of value. I do not think that mere pictures of the corona, unless upon a large scale with good definition, will have anything more than a personal interest. . . .

I shall make a study of the brightness of the sky as above described. I should like to make a search for an intramercurial planet. I have already completed a series of photographic observations which show that during an eclipse it is possible to photograph stars as faint as the seventh magnitude. We are at the present time reasonably sure from visual observations that no such planet brighter than the 3.0 or 3.5 magnitude exists, but have no evidence with regard to smaller bodies. My observations show that a body of the size of an average asteroid, say, 20 miles in diameter, located within 15 million miles of the Sun, would certainly show upon my plates. The nearer to the Sun it lies the more conspicuous it would become.

W. H. PICKERING.

In. thinking about what the observatories having small incomes might do in connection with the total eclipse of the Sun of May 28, 1900, it has occurred to me that it might be useful to take photographs during totality at various points along the path of the shadow in America and also in Spain and from these possibly obtain definite evidence of change in the corona during the interval. Careful orientation would be necessary. To obtain a zero of position angle a *réseau* or a single line on the plate might be used (possibly a vertical line) and the crescent Sun photographed just before and after totality. A combination of three photographs taken at any given place with the same instrument, one before, one during, and one after totality, ought to fix the position angle of any given coronal ray pretty accurately.

ORMOND STONE.

(*a*) Little has been learned of the details of structure in the corona. A closer examination of special portions under good telescopic power is suggested in addition to photographs. (*b*) Corresponding observations made by observers in this country and in Portgual and Spain can be arranged to advantage in this eclipse on any agreed-upon subject. (*c*) A cloudy weather program, made up, *e. g.*, of meteorological notes, degree of darkness, change of color, change in light by jumps instead of gradually, it is well to arrange for, in case the sky is cloudy.

WINSLOW UPTON.

It is proposed to measure the rate at which the radiating power of the solar atmosphere diminishes with the altitude above the photosphere, and to obtain a rough quantitative comparison with photospheric radiation. The apparatus required is as follows: (1) a large siderostat with a good driving-

clock, and freely moving attachments permitting rapid but fairly accurate adjustment. The plane silvered mirror should be about a foot and a half in diameter. (2) A concave silvered mirror, about one foot in aperture and 30 to 40 feet in focal length, mounted on a tripod with adjusting screws. (3) A linear bolometer (exposed part of strip about 5 mm long by $\frac{1}{4}$ mm wide) in a cylindrical water jacket, the strip being viewed from behind by an eyepiece. The entire bolometer case must be capable of being moved radially in the solar image by a recording, slow-motion screw, the strip being set tangent to the limbs of the Sun and Moon at the point of internal contact by revolution of the case in its holder. The holder preferably slides on a vertical bar with massive tripod base, and adjustments resembling those of a cathetometer. (4) A delicate galvanometer, nearly dead-beat, with time of half-swing three to five seconds, and a wide-range shunt operated by the reader. The shunt may be a resistance box, with coils from $\frac{1}{100}$ to 1000 ohms, the resistance of the galvanometer being about 10 ohms. (5) A bolometer battery suited to the rest of the apparatus.

The reflection from the concave mirror need not return on the path from the siderostat, but may fall upon the bolometer placed in a shelter on one side, where also the galvanometer and battery are situated. The concave mirror and siderostat may have separate shelters.

The screens for exposing may be permanently withdrawn during totality, but the galvanometer's zero-reading must be approximately constant.

Final adjustments, with instruments almost entirely screened, must be made on the vanishing crescent a few minutes before totality.

One observer must be detailed to keep the siderostat accurately pointed to the Sun, *not*, as is commonly done, by the use of a finder attached to some part of the siderostat-movement, and sharing its lost motion, but by means of a telescope with adjustable cross-wires, receiving a portion of the beam from the mirror. A micrometer-reader at the bolometer, recording silently both the setting and the time, a galvanometer-reader, who also manipulates the shunt and calls the galvanometer deflections aloud, an observer at the eyepiece who exposes by withdrawing the screens in the path from the siderostat to the concave mirror, setting the bolometer thread in the coronal image, and calling out "read," a timekeeper and a recorder who has the faculty of doing several things at once, and who records galvanometer readings and times, with any remarks by the observer at the eyepiece, will be needed. The observer at the eyepiece controls the others by his movements, and must give directions and warnings.

Since we cannot tell beforehand what deflections to expect, there must be the ability to alter the sensitiveness of the galvanometer rapidly. Suppose that the first deflection, immediately after totality is announced, is 25 div. (the shunt being 1 ohm). The shunt resistance is at once increased until

a deflection of (let us say) 250 div. is obtained for tangency, or in the brightest part of the corona. The micrometer-reader notes the micrometer setting for the position of tangency at that instant. The bolometer is then moved radially outward by steps, the galvanometer-reader calling successively (*e. g.*, 200, 150, 100, 50, or whatever the readings may be) and the micrometer-reader noting the corresponding positions. Our reading must be made on the Moon during totality to determine atmospheric radiation.

If there is still time, shift 180° (most rapidly effected by the assistant at the siderostat) and repeat the measurements on the opposite side of the Sun, securing, if possible, tangential readings just before emergence.

Final measurements on the photosphere may be made conveniently by following the edge of the Moon, noting times. A variety of stops should be provided for the concave mirror, to be used in addition to the shunt in securing manageable deflections during this stage. I do not allude to the precautions required in bolometric measurement, as this would require a small treatise.

Since there is no present prospect of my securing such an outfit, I can only add that I should be very glad to undertake the work, if opportunity offers.

FRANK W. VERY.

As to the observations to be considered most important, I rather naturally think of spectroscopic, especially in the lower portion of the spectrum, which thus far has been only very imperfectly reached by photography. The questions as to changes in the "flash-spectrum" from second to second, and, in view of Mr. Lockyer's recent paper in *Nature*, and Mr. Campbell's observations of the eclipse of 1898, the verification or otherwise of their result for the wave-length of the coronal line. Photographic and visual observations should be combined, and both "analyzing" and "integrating" spectroscopes. I leave to others problems relating to photometry and polarization.

C. A. YOUNG.

By vote of the conference the committee was continued. Intending observers of the eclipse are requested to communicate with the committee regarding their plans of work.

The committee on the United States Naval Observatory, appointed at the Harvard Conference, reported that the opinions of astronomers regarding the organization of the Naval Observatory had been obtained and communicated to the Secretary of the Navy. In harmony with the suggestion of the committee, the Secretary of the Navy, with the advice and approval of the Superintendent of the Observatory, has appointed a Board of Visitors to visit, examine and report upon the United States

Naval Observatory. This board, which consists of Hon. William E. Chandler, chairman; Professor George C. Comstock, secretary; Hon. A. G. Dayton, Professor E. C. Pickering, and Professor George E. Hale, visited the Observatory on June 30, 1899, and will meet again in Washington on September 26 to complete its report.

As the third conference, like the two preceding ones, was well attended by representative astronomers and astrophysicists from all parts of the country, it seems safe to assume that these annual meetings are serving a useful purpose, and that the permanent organization now established will continue to advance the interests of astronomy and astrophysics. G. E. H.

PRELIMINARY REPORT FROM THE OBSERVATIONS OF 1899 TAKEN TO SURVEY THE CLOUD CONDITIONS OF THE ECLIPSE TRACK OF 1900.

By FRANK H. BIGELOW.

DURING the intervals of time, May 15 to June 15 inclusive, for each of the years 1897, 1898, 1899, respectively, series of observations were made on the state of the sky in general and near the Sun, at the morning hour 8 A. M. to 9 A. M., to discover the probable meteorological conditions likely to prevail at the different parts of the eclipse track of May 28, 1900. These observations were all made in exactly the same way, usually by the same observers in the several years, and recorded on a simple scale from which the percentage of cloudiness actually noted could be easily computed. As the final result of the three years' observations is alone of special interest to astronomers, who are engaged in locating eclipse parties for that occasion, this will be found in the accompanying table.

RESULT OF THE THREE YEARS' OBSERVATIONS FOR CLOUDINESS ALONG THE ECLIPSE TRACK, MAY 28, 1900.

States	1897	1898	1899	Means
Virginia	... ...	44.9 41.7	35.7 34.3	40.3 38.0
North Carolina	35.8 33.3	28.2 25.7	33.3 30.6	32.4 29.9
South Carolina	33.7 32.1	17.5 16.0	28.1 26.7	26.4 24.9
Georgia	18.4 16.0	12.2 10.8	18.5 17.4	16.4 14.7
Alabama	15.2 14.9	17.1 15.7	22.4 22.6	18.2 17.7
Mississipi	... ...	23.0 26.4	38.6 31.9	30.8 29.2
Louisiana	26.5 21.5	36.4 30.9	35.9 30.6	32.9 27.7

The first column of figures under each year gives the mean percentage of cloudiness for the entire month of observation for the sky in general, as seen by the observer; the second column gives the mean percentage in the immediate vicinity of the Sun. The last section gives the means of the three years. Two facts are very evident as the result of these observations: (1) The three years each give the same result and therefore this must be founded upon a definite meteorological phenomenon pertaining to that region and season of the year. (2) The general fact is that the eclipse track region in the states of Georgia and Alabama is decidedly less cloudy than in the other states which are nearer the ocean areas, and which lie at lower levels. The conclusion follows that the chances of fair weather are better for eclipse parties locating on the highland of the southern portions of the Appalachian Mountains than in the lower districts nearer the Atlantic Ocean and the Gulf of Mexico. Of course this mean result is no guarantee that such local cyclonic conditions will not prevail on the morning of May 28, 1900, as to entirely modify this calculation, but the indications are that it will be at least twice as safe to locate there, as near the coast line.

A more complete report is soon to be issued by the Weather Bureau, which will contain other useful information for eclipse observers, such as the approximate longitude, latitude, altitude, hotel accommodations, drift of smoke, and easily accessible heights, for the towns which are located quite near the central line of the eclipse track. It will also contain a map of the southern states, with the boundaries of the eclipse track marked upon it, the location of the towns, the topography, and the available railway transportation lines.

U. S. WEATHER BUREAU,
Washington, D. C.

THE NOVEMBER METEORS OF 1899.[1]

THE predicted time of maximum of the November meteors is November 15, 1899, at 18^h Greenwich Mean Time. As a similar shower may not occur again for thirty years, no pains should be spared to secure the best possible observations. The most useful observations that can be made by amateurs are those which will serve to determine the number of meteors visible per hour throughout the

[1] *Harvard College Observatory Circular* No. 45.

entire duration of the shower. *Circular* No. 31 was accordingly distributed last year, and numerous valuable observations were thus secured from observers in all parts of the world. The results, some of which are given below, are now being discussed by Professor W. H. Pickering, and will be published later in the *Annals*. Similar observations are desired this year, and it is hoped that they may be made on November 15, and also on the two preceding and following evenings. The most important time for observation is from midnight until dawn, as comparatively few meteors are expected earlier. Observations are particularly needed at hours when they cannot be made at the observatories of Europe and America. In general, the time required for ten or more meteors to appear in the region covered by the accompanying map, should be recorded. This observation should be repeated every hour or half hour. If the meteors are too numerous to count all those appearing upon the map, the observer should confine his attention exclusively to some small region such as that included between the stars μ Ursae Majoris, 40 Lyncis, δ and α Leonis. If the meteors occur but seldom, one every five minutes, for instance, the time and class of each meteor should be recorded. Also, note the time during which the sky was watched and no meteors seen, and the time during which that portion of the sky was obscured by clouds. Passing clouds or haze, during the time of observation, should also be recorded. The date should be the astronomical day, beginning at noon, that is, the date of early morning observations should be that of the preceding evening. Specify what time is used, as Greenwich, Standard, or Local Time. When a meteor bursts, make a second observation of its light and color, and when it leaves a trail, record the motion of the latter by charting the neighboring stars, and sketching its position among them at short intervals until it disappears, noting the time of each observation. If the path of a meteor is surely curved, record it carefully upon the map.

On November 14, 1898, thirty-four photographs were obtained of eleven different meteors. Their discussion has led to results of unexpected value. The greatest number of meteors photographed by one instrument was five. Only two meteors were photographed which passed outside of the region covered by the map, although the total region covered was three or four times as great. No meteors fainter than the second magnitude were photographed.

Photographs may be taken, first, by leaving the camera at rest,

when the images of the stars will trail over the plate and appear as lines, or secondly, attaching the camera to an equatorial telescope moved by clockwork, when a chart of the sky will be formed in which the stars will appear as points. A rapid-rectilinear lens is to be preferred in the first case, a wide-angle lens in the second. The full aperture should be used, and as large a plate as can be covered. The most rapid plates are best for this work; they should be changed once an hour, and the exact times of starting and stopping recorded. Care should be taken to stiffen the camera by braces, so that the focus will not be changed when the instrument is pointed to different portions of the sky, especially if the lens is heavy. If the first method is employed, the position of the camera should be changed after each plate, so as to include as much as possible of the region of the map on each photograph. If pointed a little southeast of ϵ Leonis, the radiant will reach the center of the field about the middle of the exposure. A watch of the region should also be kept, and the exact time of appearance and path of each meteor as bright as the Pole Star should be recorded. The plates should be numbered on the film side with a pencil, and should be sent to this Observatory with accompanying notes and other observations. After measurement here, they will be returned if desired. The value of the results will be much increased if similar photographs can be obtained by a second camera from ten to forty miles distant, and preferably north or south of the other.

EDWARD C. PICKERING.

September 1899.

A LONG PHOTOGRAPHIC TELESCOPE.

LAST spring a plan was proposed at the Harvard College Observatory for the construction of a telescope of unusual length for photographing the stars and planets. Anonymous donors have now furnished the means by which this experiment may be tried. The plan will, therefore, take definite shape, and it is expected that a telescope having an aperture of twelve inches and a length of a hundred feet or more will be ready for trial at Cambridge in a few weeks.

EDWARD C. PICKERING.

NOTICE.

The scope of the ASTROPHYSICAL JOURNAL includes all investigations of radiant energy, whether conducted in the observatory or in the laboratory. The subjects to which special attention will be given are photographic and visual observations of the heavenly bodies (other than those pertaining to "astronomy of position"); spectroscopic, photometric, bolometric and radiometric work of all kinds; descriptions of instruments and apparatus used in such investigations; and theoretical papers bearing on any of these subjects.

In the department of *Minor Contributions and Notes* subjects may be discussed which belong to other closely related fields of investigation.

Articles written in any language will be accepted for publication, but unless a wish to the contrary is expressed by the author, they will be translated into English. Tables of wave-lengths will be printed with the short wave-lengths at the top, and maps of spectra with the red end on the right, unless the author requests that the reverse procedure be followed. If a request is sent *with the manuscript* one hundred reprint copies of each paper, bound in covers, will be furnished free of charge to the author. Additional copies may be obtained at cost price. No reprints can be sent unless a request for them is received before the JOURNAL goes to press.

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THE

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ON THE RELATIVE BRIGHTNESS OF THE PLANETS MARS AND JUPITER, FROM MEASUREMENTS WITH A NEW PHOTOMETER.[1]

By J. HARTMANN.

ATTEMPTS to employ the photographic action of light for the measurement of luminous intensity were made very soon after Daguerre's process became known. In the year 1844 Fizeau and Foucault[2] compared in this way sunlight with terrestrial sources of light, proceeding on the principle that it would be permissible — if, perhaps, only within narrow limits — to place the intensity of a source of light inversely proportional to the exposure in which the particular source produced a definite impression upon the photographic plate. Bunsen and Roscoe,[3] employing chloride of silver paper, carried on further investigations, including the brightness of sunlight at different times of the day and year, as well as the distribution of light on the solar disk, after they had proven that the above mentioned

[1] *Sitzungsberichte der kgl. Akademie der Wiss. zu Berlin.* Session of the Physical-mathematical Section on July 20, 1899.

[2] *C. R.*, **18**, 746, 860.

[3] *Phil. Trans.*, 1863, 139; *Pogg. Ann.*, 117, 529.

relation between exposure and intensity actually holds good for chloride of silver paper within very wide limits.

G. P. Bond[1] was the first to subject any celestial body other than the Sun to photographic determinations of brightness. On March 22, 1851, he obtained with the 15-inch refractor of the Harvard College Observatory daguerreotypes of the Moon and the planet Jupiter, and he was at once struck by the fact that almost the same exposure was sufficient for a good plate of this planet as for one of the Moon, although the latter, according to its less distance from the Sun, is twenty-seven times as brightly illuminated as Jupiter. Interested by this peculiar condition he repeated these comparative photographs of the two objects, with the employment of the collodion process, particularly in the years 1857 and 1860, and he was able to fully confirm his earlier result. His experiments clearly indicated that the exposure for a good negative of Jupiter was only three times as long as for an equally clear plate of the full Moon ; in fact the brightest parts of the planet's surface appeared just as clearly as the central portions of the Moon's disk.

Similar investigations were later carried out by Lohse,[2] in Potsdam. On the occasion of the close approach of Jupiter and Mars, in October 1883, he obtained a number of plates of these two planets, which yielded as the ratio of brightness $\frac{\text{Jupiter}}{\text{Mars}} = 1.677$; or in other words, Jupiter was photographically 0.561 magnitudes brighter than Mars on October 24, 1883.[3] Bond and Lohse based their measurements upon the law of reciprocity between intensity and exposure.

There can be no doubt that the method of the measurement of surface brightness, especially with the use of photography, is capable of answering a great number of important photometric questions with a remarkable degree of certainty, and that in many

[1] *Memoirs of American Academy*, N. S., **8**, 221.

[2] *Potsdamer Publicationen*, **8**, 141.

[3] The number 2.176 given by Lohse refers only to the southern hemisphere of Jupiter, then the brighter ; but if we take the mean value for the whole disk we obtain the value given above.

cases this method is to be preferred to the comparison of intensity of objects in the form of points, hitherto chiefly employed by astronomers.

I will now only recall how much the study of the law of the reflection of sunlight from the Moon's surface is complicated by the fact that hitherto only the total light of the different lunar phases has been subjected to photometric measurements. If instead of this the change of the surface brightness of selected regions of the Moon were followed through all phases, accurate knowledge would be obtained of the law of reflection applying to the different parts of the Moon's surface, and from this the total brightness of the different phases would follow by the integration over the whole visible crescent. Similarly the final answer to the question as to the change in the brightness of comets, and of their tails, again brought into notice by Holetschek's recent investigations, can only be obtained when the surface brightness of particular portions is measured, instead of the total brightness, so difficult to define and still more difficult to observe, with bodies so variable in their form. Now while in many cases, as, for instance, in the observation of comets just mentioned, direct visual measurement of the surface brightness would be rendered difficult by the faintness of the object, the application of photography permits the photometry of even the faintest objects. In this respect the measurement of the distribution of brightness in the spectra of stars promises to be of special importance, since the distribution of energy in the spectrum of any incandescent body stands in the closest relation with the state of its incandescence.

The fact that hitherto the measurement of surface brightness has been so little employed in astronomical observations is probably to be chiefly explained by the fact that a photometer suitable for this purpose has been lacking. For a short time the Astrophysical Observatory has been in possession of such an instrument, made according to my designs by O. Toepfer, of Potsdam. The fundamental principle of this photometer for the determination of surface brightness consists in projecting in

the middle of the Lummer-Brodhun cube, an image of the object to be measured, by means of an objective and, by a second objective, an image of a photometer wedge. The special construction of this wedge then permits selected small portions to be cut out from the image of the object and their brightness to be measured with the greatest accuracy by means of the wedge. If the cube is placed in the focal plane of a refractor, the surface brightness of small portions of planetary and other surfaces can be directly measured with this apparatus. Hitherto, however, I have employed the apparatus only in combination with two microscope objectives, in order to determine the density of the silver precipitation on very small regions on photographic plates.

I have elsewhere[1] given an extensive description of the instrument and here I will communicate the first results of observations obtained with it. The following is to be remarked as to the procedure of measurement adopted. As suggested above, the oft-mentioned law of reciprocity is not to be employed as a basis for sharp measurements. If one should be compelled in certain cases to draw an inference as to the brightness of any object from the time of exposure employed, the relation between blackening, exposure, and intensity must be first determined every time, for the particular plates employed, by experiments in the laboratory. Since this relation may vary from plate to plate, the above procedure becomes, on the one hand, not very reliable, and, on the other, very troublesome. The more rational method is based on the following principle:

Two sources of light are photographically equally bright when they produce on the same plate equal blackening in equal times.

If only rays of the same wave-length from the light of two sources are compared, as in the case of the spectral photometer, this theorem is at once axiomatic. For light that is not monochromatic it represents the same approximation which provisionally justifies us in comparing the "brightness" of different

[1] *Zeitschrift für Instrumentenkunde*, 19, 97, 1899.

kinds of rays — provisionally, so long as any absolute measurement of the energy of the rays which the human eye recognizes as light is not possible. Since the sensitiveness of different compound emulsions exhibits just as great, if not greater, differences for the several spectral regions, as the sensitiveness of the eyes of different persons for the separate colors, every such comparison, by a photographic process of light that is not monochromatic will similarly possess only a subjective character. Nevertheless just as for the average normal eye, so also for the photographic plates of a definite preparation, can the relative intensity of different kinds of light be compared, at least within moderate limits of error. It should, however, be also taken into consideration that in cases of photographic determinations of brightness, two homogenous rays of different wave-length can produce equal blackening of the plate with equal exposure, so that they would be designated as photographically equally bright according to the above definition; and that with an alteration of the two rays in the same proportion, or with the alteration of the exposure, the two degrees of blackening would no longer be equal. In other words, we can assume that the relative brightness of two rays of different wave-length would be independent of the absolute intensity, and thus we should have a complete analogy to the Purkinje phenomenon arising in visual measurements. Such differences will probably only become appreciable in photographic measurements in cases of especially weak or strong impressions. Nevertheless it is desirable to secure, if possible, entire freedom from such influences, which would first have to be extensively studied, as in the case of the spectral photometer, and it is a special advantage of the photographic method I have employed that it makes it possible to carry out spectral-photometric measurements, hitherto quite tedious, not only with the greatest rapidity and accuracy, but also to extend them to fainter objects.

Every spectrograph whose slit-width can be changed by an accurately measurable amount can be directly employed for these observations. The observer makes a series of plates of

each of the sources to be compared, with different slit-widths and equal exposures, and then finds on measuring the spectra, by means of the above mentioned apparatus, the settings of the slit at which the two sources blacken the plate equally at a definite point of the spectrum. The intensity of the particular kind of rays is then inversely proportional to these slit-widths.

It is well known that in weakening the light by narrowing the slit, as in Vierordt's spectral photometer, precautions must be taken against too narrow slits, on account of the diffraction effects which then arise, since these may be very serious and affect the brightness of the image. If we make it a condition that in addition to the principal image of the slit its first two diffraction images show fully upon its collimator objective, we shall obtain for the slit-width s the condition

$$s \geqq \frac{2\lambda}{\sin \alpha},$$

where α is one half of the angular aperture of the collimator.

In the spectrograph A used in the following measurements, $\sin \alpha = \frac{1}{24}$, and for $\lambda = 434\,\mu\mu$ the condition holds: $s \geqq 0.021$ mm. It will, therefore, always be sufficient if the slit-width with this apparatus is not less than 0.02 mm.

The lower limit for the slit-width having been given, as above, the following consideration shows that in certain cases it also has an upper limit. Special attention must be directed, in making spectral plates for photometric measurement, to keeping a definite portion of the slit wholly filled by the light of the celestial body without interruption during the whole exposure. On the one hand, therefore, we cannot under any circumstances make the slit-width greater than the focal image, and, on the other hand, this image must be held with extreme care at the same place in the slit, by means of the slow motions of the refractor. Both of these conditions are rendered very much easier, we may even say are rendered possible, by the method of following introduced by H. C. Vogel. By employing the light reflected

from the surface of the first prism one can observe, during the exposure, the slit faintly illuminated artificially and can constantly check with certainty whether the image of the star always fills the whole slit-width at the same place.

It is clear that accurate following becomes more easy the greater the focal image of the body in question. It would seem to be entirely impossible to treat the stars in this way, but for them we can readily obtain with the objective-prism plates adapted for photometric investigations. On the plates of Mars, cited below, it was quite difficult to satisfy the last mentioned condition, since the diameter of the image of the planet's disk was 0.15 mm. The slit-width of 0.13 mm employed for plate No. 62 proved to be just permissible; it was possible to observe with perfect certainty that the planet's image always filled the entire slit. The great focal length of the new photographic refractor of the Astrophysical Observatory will present great advantages in this respect and I hope to be able, later, to carry out extensive investigations with this powerful auxiliary.

The series of measurements now to be communicated refer to the relative brightness of the Moon, Mars and Jupiter, and are intended to show how the new photometer can be used for spectral photometric measurements.

The first series of plates for comparison of the spectrum of the Moon and Mars was made May 23, 1899, with spectrograph *A* (described in Bd. VII of the Publications of the Astrophysical Observatory). With unchanged focus of slit and camera I obtained the following plates:

MARCH 23, 1899.

Plate No.	Greenwich Mean Time	Object	Slit-width
A 60	8^h 27^m	Moon	0.02 mm
61	40	Moon	.03
62	58	Mars	.13
63	9 22	Moon	.04
64	34	Moon	.05
65	56	Mars	.09

The six plates used for this exposure were cut from the same plate (13×18 cm) down to the size of 15×80 mm, and the plates were developed simultaneously in the same tray after an exposure of exactly ten minutes each; the Schröder refractor was used. The fine micrometer of 0.25 mm pitch serves for changing the slit-width of this spectrograph; the head being divided into 25 parts, one division corresponds to 0.01 mm.

On development it appeared that plates 60 and 65 were too faint. The measurement of the four remaining plates by means of the new photometer gave the following readings of the photometer wedge:

λ	First measurement				Second measurement			
	No. 61	No. 62	No. 63	No. 64	No. 61	No. 62	No. 63	No. 64
4485	48.2	45.6	40.7	37.0	50.0	45.9	40.5	37.8
4481	52.2	47.5	44.7	41.0	52.9	48.5	44.1	39.8
4478	48.7	45.3	40.2	36.7	49.0	45.7	40.2	37.0
4439	49.0	44.5	41.2	37.5	49.4	43.3	41.2	37.4
4398	51.6	44.2	43.0	39.7	50.1	44.2	41.7	38.8
4380	52.2	46.0	42.4	40.2	51.3	45.4	42.2	40.1
4335	51.0	46.0	43.4	40.4	51.8	46.4	43.2	40.6
4311	63.0	53.5	52.0	48.1	64.0	54.7	51.3	47.4
4279	52.6	47.3	44.3	41.3	51.8	47.2	44.1	41.4

The portions of the spectrum here selected contain only fine lines, which disappeared entirely with the slit apertures employed, so that the appearance of the spectrum at this place did not change appreciably upon varying the slit-width.

A glance at these figures shows at once that the blackening of the plate of Mars, No. 62, lies about midway between the two plates of the Moon, No. 61 and No. 63. If they had been equal to No. 61 it would follow, since the slit-width for these plates was in the ratio of 13 : 3, that the ratio of the brightness of Mars to the Moon is 3 : 13, or Mars would be 1.592 magnitudes fainter than the Moon. On the other hand, had No. 62 been equal to the plate of the Moon, No. 63, it would follow that

Mars was 1.280 magnitudes fainter than the Moon. Interpolation between these two values, taking into consideration the second differences given by plate No. 64, gives the following differences of brightness between the Moon and Mars expressed in magnitudes:

λ	I	II	Mean
4485	1.50	1.50	1.50
4481	1.42	1.47	1.44
4478	1.49	1.51	1.50
4439	1.44	1.39	1.42
4398	1.34	1.40	1.37
4380	1.44	1.43	1.44
4335	1.42	1.43	1.42
4311	1.35	1.41	1.38
4279	1.43	1.44	1.44

Since the two series of measurements were made on two different days with an entirely new adjustment of the apparatus, any systematic differences between the readings of the wedge should have disclosed themselves. For this reason the two series were separately reduced; and we see that the greatest differences occurring between the two measurements is 0.06 magnitudes. The probable error of the measurement is calculated to be $\pm$ 0.019 magnitudes.

One difficulty of the measurements was due to the fact that the objective of the refractor here employed was not intended for photographic purposes, but is visually achromatic. The consequence of this is that in focusing the slit for the blue and violet rays, the rays were sufficiently united for only a very small stretch of the spectrum. At the place where the spectrum was broadened on account of the size of the diffraction circles and consequently becomes too faint, it cannot be used for photometric measurements. Nevertheless for plates of planetary spectra, the stretch of the greatest blackening in the spectrum, which is very short for the stars, becomes longer the greater the apparent diameter of the disk, on account of the superposition of the

spectra produced by the different parts of the planet's disk. This difficulty is wholly removed in the case of the Moon, as we obtain the whole spectrum at a maximum of blackening, for any focusing of the slit.

A second series of spectra of these two objects was obtained on March 25. In order, however, to extend the measurements to the remaining parts of the spectrum in so far as the spectrograph employed permitted, a longer series of the spectra of Mars was obtained on this day with different focusing of the slit.

MARCH 25, 1899.

Plate number	Gr. M. T.		Object	Slit focus	Slit-width
	h.	m.			
A 66	7	5	Mars	15.0 mm	0.11 mm
67	7	18	"	18.0	0.11
68	7	30	"	21.0	0.11
69	7	43	"	24.0	0.11
70	8	8	"	27.0	0.11
71	8	21	"	30.0	0.11
74	9	26	Moon	21.0	0.02
75	9	37	"	21.0	0.03
76	9	48	"	21.0	0.04

The photographs of March 25 show very clearly how the most intense part of the spectrum is displaced with the change of the focus. If, for instance, the blackening at any selected wave-length is measured in the above spectra of Mars, it will have a maximum value at a definite focus. In the following summary of the measurements, these values (smallest readings of the wedge) are indicated by heavier type; these numbers only represent the true intensity of the spectrum. Here also the measurements were twice made independently:

FIRST MEASUREMENT.

λ	No. 66	No. 67	No. 68	No. 69	No. 70	No. 71	No. 74	No. 75	No. 76
4720			57.5	54.9	50.0	52.0		54.6	47.0
4667			56.0	52.7	49.1	55.0		53.9	47.0
4659			53.0	49.6	46.1	48.2		49.6	44.0
4628			50.0	46.0	44.0	48.2	62.7	47.1	42.2
4569			44.7	42.5	41.4	48.8	59.2	44.9	39.7
4565			47.2	44.6	43.2	50.3	60.3	47.0	40.7
4560			45.2	42.6	42.6	49.7	60.2	44.6	39.7
4535		59.3	46.6	44.5	45.5	52.5	64.0	47.2	41.8
4485		53.7	42.1	41.1	43.2	53.1	59.4	44.8	38.6
4481		54.2	44.9	45.0	46.7		62.8	47.7	41.7
4478		51.3	42.3	41.9	44.8		58.1	44.1	38.1
4439		51.0	40.3	42.9	41.3		56.0	44.0	38.9
4398	58.3	49.4	42.4	47.5	50.7		61.6	44.7	39.5
4380	54.7	49.5	42.6	47.3	51.7		62.0	46.6	40.9
4335	54.4	49.3	43.3	48.8	57.2		61.1	47.3	41.1
4311	63.0	57.0	51.6	58.2			70	56.5	50.2
4279	51.2	48.0	45.7	54.0			65.0	48.5	42.6
4225	55.4	53.4	55.3					55.6	46.8
4197	54.6	54.6	59.6					57.8	49.0
4194	51.1	51.3	57.9					53.6	46.6
4165	53.6	55.1	61.0					53.3	47.5
4146	52.8	55.1						57.0	47.1
4144	58.6	61.9						63.4	54.2
4120	54.0	57.4						54.3	48.4

SECOND MEASUREMENT.

λ	No. 66	No. 67	No. 68	No. 69	No. 70	No. 71	No. 74	No. 75	No. 76
4720				55.7	50.5	53.7		56.1	47.6
4667				53.5	49.5	56.0		55.7	47.4
4659				49.0	46.1	52.3		51.1	44.6
4628				46.5	44.2	49.3	64.5	47.8	41.6
4569			45.9	43.3	42.1		63.7	45.0	39.5
4565			48.0	46.1	44.2		65.0	47.4	42.0
4560			45.2	42.5	43.1		61.6	46.7	39.2
4535			47.0	45.2	45.8		73.0	48.9	41.8
4485			41.6	42.2	42.9		62.7	44.0	38.5
4481			45.6	45.5	48.2		68.2	49.9	42.6
4478			41.7	42.6	44.5		61.0	43.8	38.9
4439			41.8	44.3			58.0	44.8	38.5
4398			43.0	47.3			63.3	47.1	40.7
4380			43.5				70.0	46.3	42.0
4335		48.6	44.9				70.0	47.3	42.5
4311		58.1	53.2					60.6	49.6
4279	52.4	48.7	45.1					50.0	41.5
4225	59.1	55.4	56.9					57.6	48.2
4197	56.6	58.9	61.8					59.6	50.7
4194	52.6	53.2						53.7	46.1
4165	57.3	56.2						59.9	51.0
4146	54.1							56.6	48.4
4144	62.6							70.0	55.7
4120	54.4							58.0	49.6

Graphical interpolation of these measurements now gives the following differences of brightness between the Moon and Mars expressed in magnitudes :

λ	I	II	Mean	λ	I	II	Mean
4720	1.24	1.22	1.23	4398	1.29	1.23	1.26
4667	1.21	1.20	1.20	4380	1.21	1.23	1.22
4659	1.24	1.18	1.21	4335	1.23	1.28	1.26
4628	1.23	1.25	1.24	4311	1.18	1.22	1.20
4569	1.22	1.26	1.24	4279	1.28	1.25	1.26
4565	1.24	1.24	1.24	4225	1.34	1.35	1.34
4560	1.30	1.26	1.28	4197	1.32	1.32	1.32
4535	1.27	1.28	1.28	4194	1.32	1.37	1.34
4485	1.24	1.31	1.28	4165	1.43	1.34	1.38
4481	1.29	1.25	1.27	4146	1.30	1.32	1.31
4478	1.32	1.29	1.30	4144	1.27	1.28	1.28
4439	1.20	1.28	1.24	4120	1.40	1.29	1.34

From the differences of the measurements the probable error of a setting is computed as $\pm$ 0.034 magnitudes.

In order to free these differences of apparent brightness from the effect of the extinction we should have to have extinction tables for each of the different kinds of light of the different wave-lengths involved. Such tables being totally lacking, I have derived at least fairly reliable values for the extinction of these kinds of rays in the following manner: the factors f with which the figures of the Potsdam extinction table must be multiplied in order to yield the extinction for the particular wave-length were computed from the coefficient of extinction determined by Professor Müller [1] with the spectral photometer, for the wave-lengths from 680 $\mu\mu$ to 440 $\mu\mu$ and the corresponding value (0.385) from the Potsdam table of extinctions.[2] In this way the following table was obtained, the last, extrapolated figures of which, however, are uncertain.

If we now apply to the above differences of apparent brightness the extinction calculated by means of this table, the results of the two days of observation, which differed previously by

[1] *A. N.*, **103**, 241, 1882; and *Publicationen des Astrophys. Obs.* **8**, 7.

[2] MÜLLER, *Photometrie der Gestirne*, p. 515.

λ	f	Diff.
480 μμ	1.49	
		8
470	1.57	
		10
460	1.67	
		12
450	1.79	
		14
440	1.93	
		16
430	2.09	
		20
420	2.29	
		24
410	2.53	
		28
400	2.81	

about 0.2 magnitudes, come into absolute agreement. The following definitive values of the difference in surface brightness of the Moon and Mars are obtained.

DIFFERENCE OF SURFACE BRIGHTNESS, MARS – MOON.

λ	Mar. 25	λ	Mar. 23	Mar. 25	Mean.	λ	Mar. 25
4720	+1.40	4485	+1.54	+1.49	+1.52	4225	+1.61
4667	1.37	4481	1.48	1.48	1.48	4197	1.59
4659	1.38	4478	1.54	1.51	1.52	4194	1.62
4628	1.42	4439	1.46	1.46	1.46	4165	1.66
4569	1.43	4398	1.41	1.49	1.45	4146	1.60
4565	1.43	4380	1.48	1.45	1.46	4144	1.57
4560	1.47	4335	1.46	1.50	1.48	4120	1.64
4535	1.48	4311	1.43	1.44	1.44		
		4279	1.49	1.51	1.50		
Mean, 4613	+1.42	4398	+1.48	+1.48	+1.48	4170	+1.61

The probable error of a measure is found to be ± 0.027 magnitudes, calculated from the nine values measured on the two days; as this lies precisely midway between the probable errors o measurement derived from repeated measures of the same plate, we may properly draw from this the conclusion that the plates themselves are affected with only vanishingly small systematic errors — a conclusion which is confirmed, moreover, by the absolute agreement of the two daily means for the regions from λ 4485 to λ4279. The mean value for each wave-length in this region has therefore a probable error of ± 0.014 magnitudes. The extreme accuracy of the result is explained by the fact that it is possible on a photographic plate to reproduce small differences of brightness by very considerable differences of blackening. I must distinctly state that the above figures are far from expressing the limit of accuracy obtainable in this respect.

The above comparisons of the Moon's light refer to the brightest part of the southwest quadrant of the Moon's disk. Any selected point of this region was brought upon the slit, and since the instrument was driven by the clock on sidereal time, as a consequence of the lunar motion point after point was effective, so that the above lunar spectra represent the average brightness of the brightest part of the Moon's disk.

Having discussed the preceding observations of the brightness of Mars somewhat in detail, for the purpose of explaining the process of measurement, I think I may very briefly give the comparisons of Jupiter with the Moon, which were made in precisely the same manner. On May 24, 1899, between $8^h\ 45^m$ and $11^h\ 18^m$ Gr. M.T., I photographed seven spectra of the Moon and Jupiter with spectrograph *B* attached to the Schröder refractor. Instrument *B* is constructed similarly to *A*, but has one, instead of two, compound prisms. As the image of Jupiter had a diameter of 0.5 mm, a considerably greater slit-width could be used than in the case of Mars. It happened that the spectrum of the planet taken with the slit-width of 0.125 mm was blackened almost identically with one taken with the slit-width 0.065 mm, from which would follow a difference in brightness of 0.71 magnitudes. The accurate measurement of this and the other plates gave the following results:

DIFFERENCE OF SURFACE BRIGHTNESS, JUPITER – MOON.

λ	Apparent difference of magnitude	Extinction	True difference of magnitude
4763	0.71	0.55	1.26
4695	0.70	0.57	1.27
4646	0.62	0.59	1.21
4515	0.69	0.65	1.34
4412	0.70	0.70	1.40
4380	0.68	0.71	1.39
4335	0.69	0.74	1.43
4205	0.60	0.83	1.43
4159	0.58	0.87	1.45
4110	0.61	0.91	1.52
4036	0.66	0.98	1.64

It must be stated that these figures are exceedingly uncertain on account of the high amount of the extinction (the altitude of the Moon was 15°, of Jupiter 27°); it will be several years before a favorable opposition of Jupiter will occur again. There is a marked decrease of light with decreasing wave-length both for Mars and Jupiter; this condition remains under any assumptions as to the extinction having any degree of plausibility and may therefore be considered as conclusive, in spite of the uncertainty of the extinction. A simple explanation for this would be the assumption that the atmospheres of these planets, like the Earth's atmosphere, exert greater absorption for rays of shorter wave-length.

If we group together the differences of magnitudes found for the different wave-lengths, as in the case of Mars, we obtain the following comparison between the spectra of the two planets.

λ	Mars-Moon	Jupiter-Moon	Mars-Jupiter
476–451 $\mu\mu$	+1.42	+1.27	+0.15
448–428	1.48	1.41	0.07
423–411	1.61	1.47	0.14
			Mean +0.12

Jupiter is accordingly 0.12 magnitudes brighter than Mars in the blue and violet portions of the spectrum. If we reduce this to a mean distance from the Sun for both planets, Jupiter remains only 0.02 magnitudes brighter than Mars, so that the surface brightness of the two planets is very nearly equal. From this the ratio of the albedo of Mars to that of Jupiter may, therefore, be calculated as 1 : 11.9.

The comparison of the relative albedo here found with the results of other observers is of interest. Calling the albedo of Mars equal to unity we get:

1. From Professor Müller's photometric measures of the visual part of the spectrum:

Albedo of Jupiter = 2.8;

2. From my measures between λ 476 and λ 411, given in this article:

$$\text{Albedo of Jupiter} = 11.9\,;$$

3. From Professor Lohse's plates, mentioned above, in which, besides the blue and violet light measured by me, the ultra-violet was also effective:

$$\text{Albedo of Jupiter} = 18.8.$$

These figures show clearly how the albedo of Jupiter more and more exceeds that of Mars with decrease of wave-length.

KGL. ASTROPHYSIKALISCHES OBSERVATORIUM,
Potsdam.

THE VARIABLE VELOCITIES OF β CAPRICORNI AND υ SAGITTARII IN THE LINE OF SIGHT.

By W. W. CAMPBELL.

THE velocities of these stars were found to be variable, from the second plate of each. Later photographs confirm the variations.

β CAPRICORNI ($\alpha = 20^h\ 15^m$, $\delta = -15°\ 5'$).

1898	August 15	− 5 km
1899	May 15	−42.9
	May 30	−44.6
	May 30*	−44.4
	June 8	−42.4
	June 14	−42.4
	June 26	−41.1
	July 12	−40.3
	July 24	−39.1
	August 12	−37.8
	August 15**	−38.6
	September 4	−35.2
	September 10	−34.5
	September 25**	−33.0
	September 26	−33.8

υ SAGITTARII ($\alpha = 19^h\ 16^m$, $\delta = -16°\ 8'$).

1896	July 1	− 3 km
1899	August 23	−31
	August 28	−28
	September 5	−18
	September 12	+ 2
	September 19	+10

This is one of the "Bright $H\beta$" stars announced several years ago by Harvard College Observatory, and in which I observed a bright $H\alpha$. The $H\gamma$ line is dark. The lines in the spectrum are difficult to identify, and are not, as a rule, well defined, so that the velocities assigned above may be in error a few kilometers.

LICK OBSERVATORY,
Sept. 28, 1899.

* Measures of the same plate by Mr. Wright.

** Measures by Mr. Wright.

SUGGESTIONS FOR THE DETERMINATION OF STELLAR PARALLAX BY MEANS OF PHOTOGRAPHY[1]

By FRANK SCHLESINGER.

THERE are not more than twenty-five or thirty stars whose parallaxes, as now known, can be relied upon within 0.05″. It is no less than sixty years since the announcements by Bessel and Henderson of measurable parallaxes for 61 Cygni and for α Centauri; yet this branch of stellar astronomy has fallen far behind others of more recent origin. We know comparatively much of the proper motions of the stars and of their relative brilliancies; the work which has been done or is now in progress at Potsdam, Poulkowa, the Lick and the Yerkes Observatories will soon make us well informed as to the motions in the line of sight of most of the bright stars.

The neglect of the important subject of stellar parallax is all the more surprising in view of the ready means we now have for its pursuit, photography and the modern heliometer. Although other methods may be useful in isolated cases, there can be little doubt that these two methods are the best for an extensive attack upon stellar parallaxes. It would be out of place to discuss here the relative advantages of these two methods, but it may be said that while photography is not inferior in accuracy to the heliometer, it has much to recommend it for the work in view, especially on the score of economy of time. On the other hand, experience has shown that photographs are liable to peculiar errors, the means of obviating which I purpose to consider very briefly.

The star to be examined for parallax will usually exceed each of the comparision stars in brightness by 6 or 7 magnitudes; that is to say, it will be about 400 times as intense. Consequently on a plate which has been exposed long enough to bring

[1] Read at the Third Conference of Astronomers and Astrophysicists, September 8, 1899.

out the comparison stars well, the central star will be unduly broadened; not only is this fatal to exact bisection, but systematic errors are apt to creep in. Hence our first task is to cut out, in some way, most of the light of the central star; this may be done, in effect, by rendering the film less sensitive in the small area upon which the image of the star will fall. A central strip of the film may be washed with a colored fluid and then allowed to dry thoroughly before exposure. Experiment will be necessary to determine the best dye for the purpose, and how strong a solution will be necessary to reduce a star of given magnitude to say one of the ninth.

We are brought at once to a second source of error, distortion of the film after exposure. Objection will be made, and with reason, that the process just proposed might lead to considerable distortion. Consequently extra precautions will be necessary to guard against their effects. Dr. Wilsing, of Potsdam, has lately used the ingenious device of taking two pictures of the same region close together on a plate, but six months apart in time. Thus if there is any distortion of the film it will shift both pictures alike, and its effects will be eliminated from the parallax. We may modify this process slightly by taking two pictures on the same date very close together; then six months later two more may be taken exterior to the first two, and separated from the latter in such a way that the three spaces between the four images of the same star shall be equal. By this means any linear distortion, no matter how violent, will have no effect so far as parallax is concerned.

A third source of error is optical distortion, caused by peculiarities of the object-glass. Such errors are small and only in the most refined work need they be guarded against. With equatorials of the usual mounting, in following an object across the meridian, it is necessary to turn the telescope 180° upon both the polar and hour axes. Hence if there is any optical distortion, its full effect will be involved with parallax, because photographs made at an eastern elongation of a star will nearly always be taken with the objective reversed 180°

(with respect to a configuration in the sky) from the position for a western elongation. A simple means for obviating any ill effects from this cause, is to mount the object-glass in such a way that it may be revolved in its own plane, around the axis of the telescope tube. On the latter two stops may be placed so that the corresponding positions of the object-glass shall differ by 180°. Throughout the whole series of observations the observer must be careful to reverse the object-glass whenever he reverses the telescope. In this way he will be enabled to present the objective in the same relative position to a configuration, no matter whether in the east or in the west. Optical distortion will then shift all the images of the same star alike, and will not affect the parallax.

The last source of error which will be considered here has its origin in the construction of the measuring machine. It is hardly necessary to point out that all the measures should be differential; thus, the distance between a certain pair of stars on any plate should always be measured with the same parts of the scale and of the micrometer-screw. A simple arrangement of the plan of measuring is possible, by which the parallax is rendered practically independent of most of the instrumental errors.

Other precautions are necessary in parallax work, but as they are not peculiar to photographs they need not be discussed here.

To sum up, the whole process is as follows: put the unexposed plate into a properly constructed template and stain a narrow central strip to a degree previously determined by experiment for plates of the same manufacture. Permit the plate to dry for several weeks at least. Then insert it in the plate-holder of the telescope, in such a way that the stained strip shall be parallel with an hour-circle. Make two pictures of the same region, shifting the plate a millimeter in declination between the two exposures. Store the plate in a dark room, undeveloped. Six months later, with the object-glass reversed 180° if the telescope mounting demands it, take two more pictures on the plate. These should be 3 millimeters apart, and each 1 millimeter from one of the former pictures. Begin a new

plate by making two exposures 1 millimeter apart, and put this by for six months; and so on till a sufficiently long chain of plates is secured to give good values both for the parallax and for relative proper motions, with respect to the comparison stars.

I may remark in conclusion that if we confine ourselves to the measurement of distances only, much simplification is possible in the measuring machine, and some in the reductions. According to a conservative estimate, a single observer, working fifteen or eighteen hours per week at the telescope, and employing the rest of his time in measuring and reducing, could give us in three or four years the parallaxes of 200 stars with an accuracy hitherto attained for only a score.

UKIAH, CAL.
August 29, 1899.

THE DISTRIBUTION OF STARS IN THE CLUSTER MESSIER 13, IN HERCULES.

By H. K. PALMER.

IN all drawings of the great cluster in Hercules which were made prior to the introduction of photographie processes into astronomy, no attempt is made to plat accurately the places of individual stars, the observers having contented themselves with representing the general appearance of the cluster. Accurate charting, and a study of the distribution of the stars in such objects, first became possible with the aid of the photographic plate. Excellent photographs of the Hercules cluster have been made by the Henry brothers, Dr. Roberts, Mr. W. E. Wilson, and others, and with the 36-inch refractor of the Lick Observatory. Some of these have been reproduced in popular journals. Among the researches based on photographic methods is especially to be mentioned the elaborate investigation of Professor Scheiner,[1] in which the positions of 833 stars are determined with all possible precision and arranged in the form of a catalogue.

In the course of the photographic work with the Crossley reflector, which is being carried on by Professor Keeler with my assistance, the Hercules cluster was photographed four times, in June and July of the present year, with exposures ranging from ten minutes to two hours. The large scale of the negatives, the great light-gathering power and fine definition of the mirror, the purity of the sky and steadiness of the images at the time of observation, all combined to make these photographs of unusual excellence. The plate to which the very full exposure of two hours was given is the best. It was very lightly developed; with the result that the images of the brighter stars did not become too large and dense to obscure the multitude of minute stars which was brought out by the long exposure, while even

[1] *Abh. d. K. Akad. d. W. Berlin*, 1892.

the middle of the cluster is resolvable. The number of stars shown on this plate, fairly within the limits of the cluster, is over five thousand.

In view of the large number of stars shown on this photograph, an investigation of their distribution seemed to be desirable, and at Professor Keeler's suggestion I have undertaken this work in the present paper. An accurate determination of the places of all stars, such as was made by Scheiner, would also be very desirable, but it would require more time and labor than can at present be devoted to such a purpose.

To study the distribution of the stars in the cluster, a positive on glass, with an enlargement of 5.6 diameters, was made from the original negative. On the positive the cluster occupies the center of a rectangle which measures 192 mm east and west, and 241 mm north and south. The rings, which are referred to farther below, were traced in red ink on a clear plate of the same size as the enlargement, and the two plates were then bound together with their films in contact.

The focal length of the Crossley reflector is 17 feet 6.1 inches, so that on the original negative 1′ = 0.061104 inch, or 1.55 mm. On the enlargement 1′ = 8.79 mm. The average star disk is 3.5″ in diameter, the brightest being about 4″ and the faintest 3″.

The primary object of this examination was to compare the actual distribution of the stars with the distribution of stars in a uniform globular cluster; by which I mean a globular cluster in which the stars are uniformly distributed. To make this as simple as possible, the sphere was supposed to contain several concentric cylinders, whose volumes within its surface were in the ratio 1, 2, 3 . . . , *n*, and whose axis was parallel to the line of sight. Therefore, when projected on a plane perpendicular to the line of sight, the cluster would appear to be divided into several concentric rings, and if it were uniformly globular, each ring would contain an equal number of stars. The radii of these cylinders were obtained from the formula

$$n\frac{4}{3}\pi R^3 = \frac{4}{3}\pi R^3 - \frac{4}{3}\pi \sqrt{(R^2 - r^2)^3}$$

in which R is the radius of the sphere, r the radius of the cylinder, and n the ratio of the volume of the cylinder to that of the sphere. The limits of the cluster were very indefinite; so the radius of the sphere was assumed to be 80 mm on the enlargement, which corresponded to an angular distance of 9.2′. This seemed to include all stars which were so arranged as to come within the limits of a globular structure, and at the same time excluded those which had only a radial arrangement. This circle of 80 mm radius was then divided into eight rings, the radius of the different circles being as follows:

Circle	Radius of arc		Diameter
1	23.0 mm	2.7′	5.4′
2	33.2	3.8	7.6
3	41.4	4.8	9.6
4	48.6	5.6	11.2
5	55.4	6.4	12.8
6	62.2	7.2	14.4
7	69.2	8.0	16.0
8	80.0	9.2	18.4
Width of plate			22.2
Length of plate			27.8

Since the stars outside the last circle forming the rays seemed to belong to the cluster, and as these rays extended to the edge of the plate, the dimensions of the plate have been included. To facilitate the counting, and to see if there was any particular radial structure, the circles were divided into twelve sectors of 30° each.

In a general way it may be said that the stars in the Hercules cluster are of two distinct orders of brightness; for although intermediate magnitudes occur, they are less numerous than we should expect them to be in a purely fortuitous assemblage of stars of different sizes.

In counting, the number of faint stars was kept separate from that of the bright stars. The term "bright" was applied to all stars which, in the positive, showed clear glass, while the "faint" stars were those which contained visible silver grains. A comparison of several stars, which were just on the dividing line,

with Dr. Scheiner's chart,[1] showed the point of division to be about 13.5 magnitude.

The number of stars on the entire plate is 5482, of which 1016 are classed as bright and 4466 as faint. The number in each ring is given below. The first column gives the number of the ring, 1 referring to the inner and 8 to the outer ring, while the term "outside" refers to the region between the last circle and the edge of the plate. The second column gives the number of bright stars, the third the number of faint stars, the fourth the sum of the second and third, and the fifth the ratio of bright to faint stars.

Ring	Bright stars	Faint stars	All stars	Ratio $b:f$
1	551	580	1131	1: 1.05
2	148	671	819	1: 4.53
3	100	714	814	1: 7.14
4	43	623	666	1:14.5
5	29	504	533	1:17.4
6	25	398	423	1:15.9
7	22	279	301	1:12.7
8	27	306	333	1:11.3
Outside	71	391	462	1: 5.5
Total	1016	4466	5482	1: 4.40

Fig. 1 shows the second, third and fourth columns graphically. In this figure the ordinates represent the number of stars, and the abscissae the distances from the center of the cluster. The upper curve represents the whole number of stars, the middle one the faint stars, and the lowest one the bright stars. The number in the region outside the last ring has been divided by four because the area of that portion of the plate is four times that of the last ring and the stars are distributed quite evenly throughout.

From the manner in which the size of the rings was determined we should expect to find the same number of stars in each ring, if the cluster had a globular form in which the stars were distributed evenly throughout. In the center of the cluster, where the depth is much greater, we should expect to find, in a

[1] *Abh. d. K. Akad. d. W. Berlin*, Berlin, 1892, p. 57.

dense cluster like this, that many of the stars would be occulted by those in front, and only if we had star images of an infinitesimal size could we hope to see all of them. Since the size of

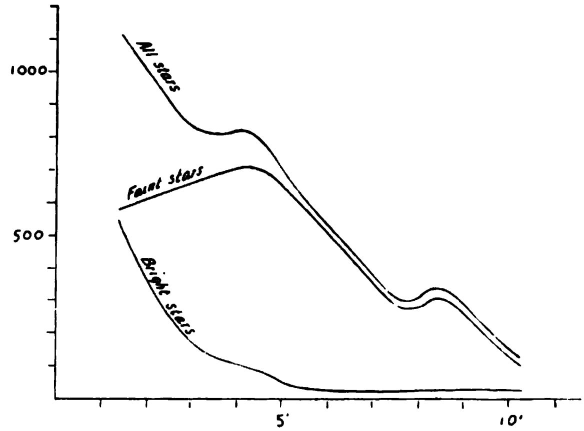


Fig. 1

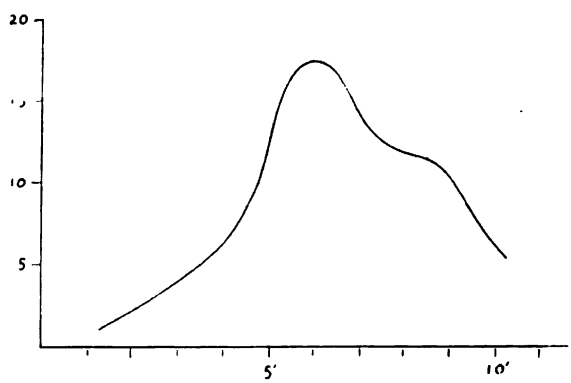


Fig. 2

the average star disk is 3.5", stars 2" apart would hardly be resolvable, though the elongation of the disk would be appreciable. From this consideration and on the hypothesis of uniform distribution, we should expect to find fewer stars in the first ring than in the outer rings, but the number of both bright and faint

stars found in the different rings shows that it is exactly the reverse, the number decreasing very rapidly as the distance from the center increases. This is especially true of the bright stars, there being only about one fourth as many in the second ring as in the first. Had the inner ring been divided into two, the same general distribution would have been exhibited. Within a distance of 1.5′ of the center, the stars are crowded so close together that they are just resolvable. Within this region no faint stars are visible, and it is only at some distance from the center that they reach their maximum. It cannot be doubted, however, that the faint stars, also, increase rapidly in number toward the center of the cluster.

This distribution of the bright stars easily accounts for the various sizes given to the cluster by various observers with instruments of different power. Messier[1] gives the diameter as 3′, — the diameter of the region within which the stars are so close together as to be hardly resolvable. Herschel[2] says of it, "The stars belonging to this cluster extend to 8′ or 9′ in diameter; the most compressed part about 2′ or 2½′; the latter is round, the former irregular." His outer limits would fall within the third ring, and as the faint stars were probably invisible to him, he placed the limit where the number of stars becomes almost constant. Schoenfeld[3] made the diameter 6′ or more, putting the limits within the second ring. Scheiner[4] and Trouvelot[5] both make the diameter about 13′, or about the limit of the fifth ring. At these limits Scheiner records only a few scattered stars, while Trouvelot shows only the rays of bright and faint stars. Other photographs, taken by the brothers Henry, and with the 36-inch refractor of this Observatory, show about the same amount. Roberts'[6] photograph shows the cluster

[1] *Mém. de l'Acad. des Sciences*, 1771, p. 441.

[2] *Phil. Trans.*, 1818, p. 439.

[3] *Beobachtungen von Nebelflecken und Sternhaufen*, Mannheim 1862 and 1875.

[4] *Abh. d. K. Akad. d. W. Berlin*, 1892, p. 57.

[5] *Annals H. C. O.*, 8.

[6] *Photographs of Stars, Star Clusters and Nebulae*, p. 93.

extending to about the distance of the outer ring, a diameter of 18′.

Outside the second ring the bright stars are grouped in rays, but it is only in the last two rings, at a distance of 8′ from the center, that we find the faint stars so arranged. Fig. 1 shows that the number of faint stars reaches its first minimum in the seventh ring, and it is about there that they are first found grouped more densely than elsewhere about the rays of bright stars. Outside the last ring the number of faint stars decreases rapidly, while the number of bright stars remains almost constant. Fig. 2 shows graphically the ratio of faint to the bright stars. It increases from almost unity in the first ring to 17.4 in the fifth ring and from there decreases to 5.5 outside the outer ring.

The rays of stars do not extend straight out from the center of the cluster, but curve, so that the distribution by sectors is not appreciably affected by them. The number of stars in the different sectors shows that they are distributed quite uniformly about the center.

In the center, where the stars are so closely packed together, we should naturally expect to find that the faint stars would be hidden by the bright ones, and that where there are several faint stars scattered among the bright ones so thickly that the distances between them hardly exceeds 1″, the bright stars would appear nebulous. This easily accounts for the nebulous appearance of the bright stars, and also for the apparent absence of faint stars in the center, where we have, instead, a continuous nebulous background. Just outside this dense region we find what appear to be wisps of nebulosity, resembling a row of faint stars too close to be easily separated. As we go further from the center we find that the stars begin to show resolution, and in the second ring there is nothing to suggest nebulosity. On carefully examining the negative, as well as the enlargement, neither Professor Keeler nor I could find any proof of true nebulosity in this cluster, and the same statement holds for other globular clusters that have been photographed with the Crossley telescope. The nebulous appearance seems to be entirely due to the

minute stars, which become more and more closely crowded together toward the center of the cluster, until their disks coalesce on the photograph to form a nearly continuous background.

As the globular clusters have always been regarded as nebulous, this seems to be a point of some importance. The star disks as seen with the 36-inch refractor are much smaller than those obtained photographically with any instrument, while with the great light gathering power of the 36-inch objective the smallest stars of the cluster can be steadily seen. Accordingly, a careful examination of the cluster, in which several of the astronomers of the Observatory took part, was made on the night of August 5. At first sight, the cluster had a decidedly nebulous aspect; but on fixing the attention on any small space among the brighter stars, the separate minute points of light came into view, on a background which seemed to be free from nebulosity. Thus the visual observations led to the same conclusion as the photographic. We do not, of course, assert that there is no nebulosity in this cluster, or in others of similar character, but we think that the existence of nebulosity has not yet been proved.

The wisps of nebulosity which have been referred to, are also apparently due to the grouping of the faint star disks. Only six instances were noted (on the enlargement) where the number of stars composing these apparent wisps was doubtful. These were not connected with any of the bright stars, nor with the central mass of the cluster, but seemed to be isolated, though all were within the first ring. None were so long as to have required for their formation more than six or seven close stars, and some could have been formed by three or at the most four. In so large a number of stars, gathered within so small a space, the law of probabilities would require that in a few instances enough stars would be situated sufficiently close together to form these "wisps." The "prominence forms" described by Ranyard[1] are obviously of the same character.

[1] *Knowledge*, **16**, 90, 109.

The three dark lanes discovered by Lord Rosse, as well as several others around the most compressed part of the cluster, are easily seen on this photograph. Professor Holden[1] speaks of these lanes as being regions which contain no stars, and bounded by one or two lines of bright stars. He counted thirteen points at which two or three of these lanes intersected, and suggested that they might be considered as centers of force. In all but two of these lanes — one, the largest of the three discovered by Lord Rosse, and the other, at quite a distance from the center of the cluster — from five to fifteen faint stars could be counted. A photograph which would not show a star of the fourteenth magnitude would probably show these lanes entirely empty of stars, but when the faint stars become visible the effect is that of gaps between the bright stars only. It is within these lanes that the faint stars can be seen nearest the center, showing that they probably exist in the center but are concealed by the bright stars.

The distribution of the bright stars is marked by the existence of these dark lanes and by the rays, while the arrangement of the faint stars is much more nearly globular.

LICK OBSERVATORY,
September 1899.

[1] *Pub. A. S. P.*, 3, 375.

THE PERIODS OF THE VARIABLE STARS IN THE CLUSTER MESSIER 5.[1]

By S. I. BAILEY.

THE cluster *Messier* 5 (*N. G. C.* 5904) contains about 900 stars on the photographs made with the 13-inch Boyden refractor. Of these about eighty-five, or one in eleven, are variable.

The period and light-curve of No. 18 were determined in 1896 by Professor E. C. Pickering. (See *A. N.*, **140**, 285.) The periods of Nos. 1, 42, and 84 were determined in 1898 by Professor E. E. Barnard, from visual observations with the great Yerkes refractor. These periods are confirmed by the Harvard photographs. (See *A. N.*, **147**, 243.)

Measures of sixty-three of these variables had been made on nearly one hundred plates, by Miss E. F. Leland and myself, when I left Cambridge in March of the present year. From a study of these measures the periods of about forty stars have been determined. The results are given in the following table, which gives in successive columns the star number, the period, the residual obtained by subtracting the mean period from each, the maximum and minimum magnitudes, the range, and the distance, to the nearest minute of arc, from the center of the cluster.

The star numbers were assigned in order of discovery and bear no relation to the positions of the variables. The periods are here given to the nearest thousandth of a day. It is believed that few, if any, of them are in error more than five ten thousandths of a day (seven tenths of a minute). A few of them have been determined to within one or two seconds. No. 42, which is the brightest star in the cluster, and which has a period of 25.75^{d}, has not been included in this discussion. In length of period it appears to belong to a different class, though the form of light-curve is similar to that of the others. No. 50 also

[1] Read at the Third Conference of Astronomers and Astrophysicists, Sept. 6, 1899.

appears from the photographs to have a period of a month or more, as was observed visually by Professor Barnard. No other variable, whose period has been found, is omitted from the table. Although the present study is provisional and includes only about half the variables, the general results should not be materially altered by later more elaborate investigations.

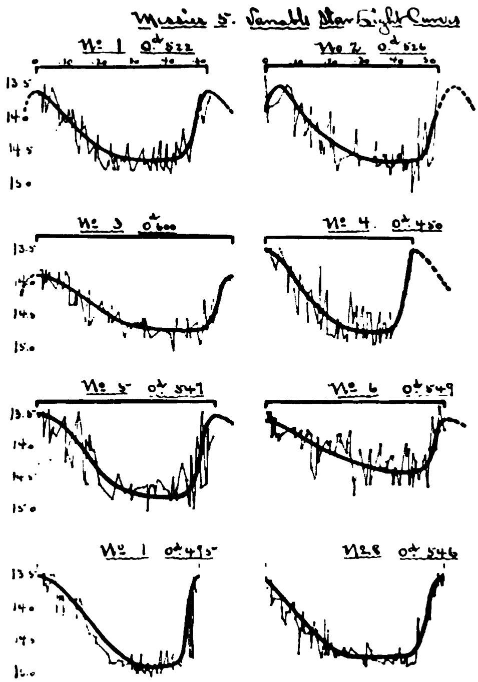
TABLE I.

Var. St. No.	Period	Resid.	Max.	Min.	Range	Dist.
1	0.522^d	− 4	13.6	14.6	1.0	3′
2	0.526	0	13.5	14.6	1.1	6
3	0.600	+ 74	13.9	14.7	0.8	3
4	0.450	− 76	13.5	14.7	1.2	1
5	0.547	+ 21	13.5	14.8	1.3	1
6	0.549	+ 23	13.6	14.4	0.8	1
7	0.495	− 31	13.5	14.9	1.4	3
8	0.546	+ 20	13.6	14.7	1.1	3
9	0.699	+173	13.8	14.8	1.0	3
10	0.531	+ 5	13.6	14.7	1.1	7
11	0.596	+ 70	13.5	14.5	1.0	3
12	0.468	− 58	13.5	14.8	1.3	3
13	0.513	− 13	13.4	14.6	1.2	1
14	0.487	− 39	13.7	14.7	1.0	3
16	0.565	+ 39	13.4	14.7	1.3	2
18	0.464	− 62	13.6	14.8	1.2	3
19	0.470	− 56	13.5	14.9	1.4	5
20	0.609	+ 83	13.6	14.5	0.9	4
21	0.605	+ 79	13.8	14.6	0.8	6
24	0.478	− 48	13.6	14.8	1.2	1
26	0.624	+ 98	13.5	14.8	1.3	2
27	0.471	− 55	13.5	14.9	1.4	1
28	0.544	+ 18	13.8	14.9	1.1	3
29	0.451	− 75	13.7	14.8	1.1	6
30	0.593	+ 67	13.9	14.8	0.9	4
32	0.458	− 68	13.5	14.8	1.3	4
33	0.501	− 25	13.7	14.8	1.1	2
34	0.568	+ 42	13.5	14.7	1.2	2
37	0.489	− 37	13.5	14.9	1.4	1
38	0.470	− 56	13.7	14.8	1.1	2
39	0.589	+ 63	13.5	14.7	1.2	4
41	0.488	− 38	13.6	14.9	1.3	4
47	0.540	+ 14	13.6	14.9	1.3	2
52	0.502	− 24	13.7	14.5	0.8	2
54	0.453	− 73	13.5	14.8	1.3	1
55	0.491	− 35	13.9	14.6	0.7	3
58	0.492	− 34	13.5	14.8	1.3	11
59	0.542	+ 16	13.5	14.7	1.2	3
61	0.569	+ 43	13.6	14.8	1.2	4
63	0.498	− 28	13.6	14.8	1.2	5

Mean period 0.526^d.

A brief examination of the above table will disclose a striking similarity among all these variables, not only in regard to length of period, but in magnitude and range of variation. The same is true of the form of the light-curve, as may be seen by reference to the figures, where are given drawings of the curves of the first eight variables in the group.

Of the forty variables given in the table, No. 9 has the longest period,



$0.699^d = 16^h\ 46.6^m$. This is an exceptional case however, for, aside from this star, the maximum period is that of No. 26, $0.624^d = 14^h\ 58.6^m$. The minimum period is that of No. 4, $0.450^d = 10^h\ 48.0^m$. Omitting No. 9, the extreme range is only $0.174^d = 4^h\ 10.6^m$. The mean period is $0.526^d = 12^h\ 37.4^m$, and the mean residual, $0.047^d = 1^h\ 7.7^m$. The greatest deviation from the mean (after No. 9, which is $4^h\ 9.2^m$) is No. 26, $0.098^d = 2^h\ 21.1^m$.

This remarkable tendency to equality in the length of the periods is matched by the similarity of the magnitudes. At maximum they range between 13.4 and 13.9, and at minimum between 14.5 and 14.9. The range of variation is between 0.7 and 1.4 magnitudes. Even these differences may be partially accounted for by the difficulties of measurement and the proximity of very close companions.

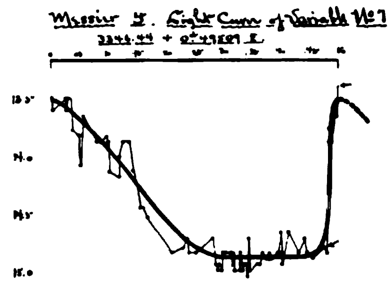
This uniformity of period, magnitude, and light-curve, among so many variables associated in the same cluster, points unmistakably to a common origin and cause of variability. Perhaps at one time the periods were precisely equal, but if so, the perturbations which have caused the present divergence have, apparently, no relation to the star's distance from the center of the cluster. If we group the stars according to the residuals given in the third column of the table, placing in the first group those stars whose residuals are between 0 and $\pm 0.025^d$, in the second, those from $\pm 0.026^d$ to $\pm 0.050^d$, in the third, from $\pm 0.051^d$ to $\pm 0.075^d$, and in the fourth all greater, the mean distances of the groups from the center are respectively 2.7′, 3.4′, 3.0′, and 3.6′. Grouped by positive and negative residuals the result is much the same.

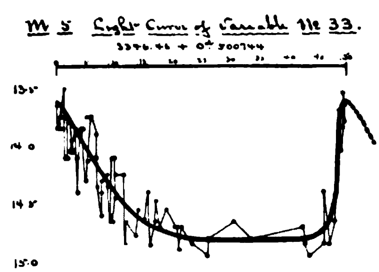
In the great cluster ω Centauri, in which I have determined the periods and light-curves of over 100 variables, no such uniformity was found. (See *H. C. O. Circular* No. 33.)

For the more exact determination of the form of light-curve a few of these variables in *Messier* 5 have been studied with special care. Two of these are represented on page 259. These periods are correct within 2^s. The black dots represent the individual measures, and a smooth curve has been drawn through these which probably well represents the photographic light-curve. A few of the residuals, from measures on poor plates, are rather large, but these could be much reduced by repeating the measures once or twice. These stars well represent what may be called the *Cluster Type* of variables. A large number of variables in ω Centauri are of this type and, so far as known at present, all the variables in *Messier* 5. The most striking feature

is the extremely rapid increase of light. As yet the photographs have not made clear just how rapid this increase is, since in the 13-inch refractor an exposure of nearly an hour is necessary to show these stars when at minimum. No. 7 furnishes, however, some interesting data.

Plate X 8544 was taken May 13, 1897. Exposure 50^m. The star was at a minimum. The mean time of exposure, which is taken in all cases as the time of observation, was *J. D.* 2414058.849. By the formula used the maximum of the star





occurred at *J. D.* 2414058.874, or 36^m later. But in order that the photographic magnitude should be at a minimum the star must have remained at minimum till near the end of the exposure, which closed 11^m before the computed maximum. On May 26, X 8586 was taken with exposure of 58^m. The star was at maximum. The mean time of exposure was *J. D.* 2414071.742, the computed maximum 7^m later. Hence the exposure began 36^m before the computed maximum. These two measures are marked on the chart by arrows. Interpreted literally, these results indicate that the star on the first date remained at

minimum till within 11^m of the computed maximum, and on the second date that it had reached maximum 36^m before the time of the computed maximum. A study of the star on numerous plates covering two years and fifteen hundred periods shows no change in the star's mean period. The most obvious conclusion would seem to be that the maximum phase is subject to small changes, either in time or rapidity, which do not affect the star's period as a whole. Another possible explanation would be that the last part of the first exposure and the first part of the second were more or less interrupted by clouds or haze. This, in connection with small errors in the measures would account for the facts if we grant that the greater part of the increase took place within a very few minutes.

The exact time of this increase could best be determined visually, with a telescope of the largest size, and may already have been accomplished by Professor Barnard, who has undertaken to investigate it. The form of the photographic light-curve is well shown by the two drawings in question. The decrease is rapid, but not nearly so rapid as the increase. It is doubtful if there is any duration of maximum, and at any rate it is exceedingly brief. At minimum the brightness appears to remain nearly or quite constant for a few hours. The whole period may be divided as follows:

Duration of maximum phase - - - - - -	0 per cent.
Duration of minimum phase - - - -	40 per cent.
Duration of decreasing phase - - - - -	50 per cent.
Duration of increasing phase - - - -	10 per cent.
	100

AREQUIPA, PERU,
May 20, 1899.

NOTE ON THE RELATION BETWEEN THE VISUAL AND PHOTOGRAPHIC LIGHT CURVES OF VARIABLE STARS OF SHORT PERIOD.

By S. I. BAILEY.

WITH a visual telescope of sufficient power a series of frequent observations of a variable star will give the true form of its light-curve, since each observation consumes so little time that it is not affected by the star's variability.

If we had a photographic telescope of sufficient power, so that the necessary time of exposure would bear an inappreciable ratio to any change of phase, the same would be true of the photographic observations; and the two curves should agree, except in so far as photographic magnitudes differ from visual. In the case of most long-period variables these conditions are approximately fulfilled. Certain short-period variables, however, notably those belonging to dense clusters, are so faint, and go through their changes, especially the increase of light, so rapidly, that the necessary exposure bears a very large ratio to the duration of any phase, and important modifications in the form of the light-curve follow. There are other modifications which depend on the essential difference between visual and photographic magnitudes, and on the nature of the plates employed, but in what follows, only those changes are considered which are related to the duration of the photographic exposure.

When the light of a star is changing at a uniform rate and in the same direction we may assume, as in general approximately true, that the measured magnitude on the photograph will represent the actual photographic magnitude at the mean time of exposure. The photograph thus becomes a single observation, whatever may have been the duration of the exposure. It is obvious, however, that sharp changes in the light of a variable will not be well registered on photographs of relatively long exposure. Let us assume a light-curve, where the increase and

decrease are rapid, and equally so, and the duration of maximum very brief, as illustrated in Diagram 1, where the complete change from minimum through maximum to minimum again, represented by the horizontal distance *A B*, takes place in two units of time, a single unit, represented by the heavy line below the curve, being the necessary time of exposure. It is obvious that no exposure, preceding maximum, which is finished later than *A*, or after maximum, which begins earlier than *B*, will record a complete minimum. So that, for example, the magnitude

Diagram 1.

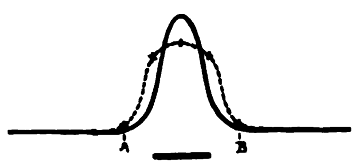
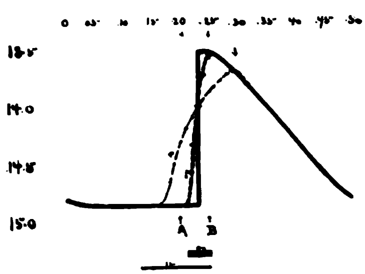


Diagram 2.



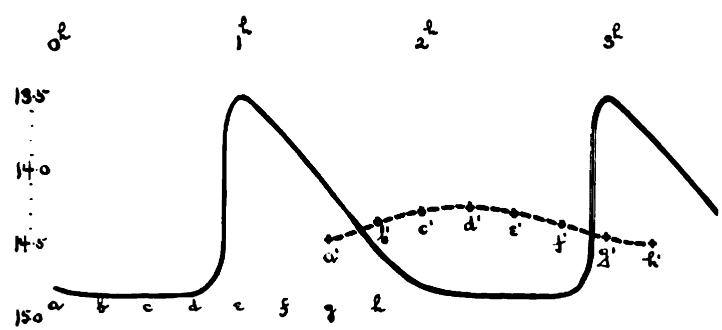
of the variable, measured on photographs whose mean times of exposure are at *A* and *B*, will not agree with the true curve but will fall above. It is also plain from an inspection of the curve that no photograph of the required exposure will directly give the true maximum, but that the photograph whose mean time of exposure is coincident with the true time of maximum will also give the photographic maximum which, however, cannot equal the actual, but will fall somewhere below. The light-curve resulting from photographs of such exposure will be approximately represented by the broken line. Inversely, the

same would be true of a brief minimum, so that the tendency of photographs is to smooth down the curve, reducing the star's apparent range of variation. The limit of this action is reached when the exposure equals the period of the star's variability, under which circumstances the curve becomes a straight line, which may be regarded as the light-curve of an invariable star.

In the above case, the photographed maximum and actual maximum are synchronous. In the case, actually true with most short-period variables where the increase and decrease are of different rapidity, the results are different. In Diagram 2 is represented the actual photographic light-curve of the variable No. 7, *Messier* 5, with the exception of that part between *A* and *B*, where the increase of the light has, for convenience, been represented as taking place instantaneously. Assuming this to be the true photographic light-curve, it is obvious that in order to represent the variable at a complete minimum the photographic exposure must close before the beginning of the maximum phase. Also, the photograph whose mean time of exposure is at the beginning of maximum will have the first half of its exposure at minimum and will record the variable as of less than maximum brightness, while that photograph will record the maximum, possible from such photographs, whose exposure begins at the beginning of maximum. The Harvard photographs of *Messier* 5 have had exposures of about 0.04^d (= about 1^h). With this exposure and the assumed light-curve, the photograph beginning an hour before maximum would record the variable at minimum, magnitude 14.85, and following photographs beginning at intervals each fifteen minutes later than the preceding would give respectively the magnitudes 14.33, 13.98, 13.71 and 13.55 approximately. These results are represented by the heavy broken line. If, with the same variable, we use exposures of 0.12^d (about three hours) the resulting curve is represented by the less heavy broken line. The first of these is interesting, since it agrees pretty closely with the light-curve of No. 7, as derived from the Harvard photographs. In such a case the retardation

of the photographed maximum from that of the actual photographic maximum (which in general agrees with the visual) equals one half the time of exposure. This retardation of the photographed maximum will be true, in a less degree, whenever the increase of light is more rapid than the decrease, even if not abrupt as represented in the diagram. If the increase is less rapid than the decrease the photographed maximum will precede the actual. In general the difference in time between the photographed maximum (or minimum) and the actual, is a variable which has zero for a limit as the length of the exposure

Diagram 3.



is reduced. Evidently a large telescope and very sensitive plates are desirable.

Probably the shortest period known at present is that of No. 91, in ω Centauri, which is $6^h\ 11^m$. It seems not improbable, however, that much shorter periods may be found, so that this relation of the exposure to the period becomes still more important. For example, to photograph the faintest stars of many faint clusters with the 13-inch Boyden refractor requires an exposure of two hours. Evidently variable stars with periods of two hours could never be studied on such photographs, since each exposure, whenever begun, would record the mean light of the star, a quantity interesting in itself, but which would give no key to the star's variations. Evidently the best remedy would be a larger telescope, or a more sensitive plate, or both. If

these are not obtainable, something may still be done by increasing the length of the exposure.

Let us assume the case of a variable star having a period of two hours, but so faint that two hours is the minimum exposure possible. Then, since nothing can be learned by photographs of two hours' exposure, let a series of photographs be taken at intervals of fifteen minutes, and having exposures of three hours. In Diagram 3 is represented the light-curve of variable No. 7, *Messier* 5. The period is here assumed to be two hours. Let exposures of three hours be begun at *a*, *b*, *c*, etc., and the resulting magnitudes, plotted at the mean times of exposure, be a', b', c', etc. The mean light of the star, derived from an exposure of two hours, is approximately 14.37. Uniting with this the additional light received during the third hour of exposure, we have for the different photographs the approximate magnitudes 14.47, 14.35, 14.28, 14.25, 14.29, 14.37, 14.45, 14.49. These results are represented by the broken line.

In such a case, unless the range of variation were large, the practical difficulties would be very great.

In the study of variable stars, as in many other branches of astronomy, photographic methods offer certain advantages over visual, but in all cases of photographic light-curves the duration of the exposures employed should be taken into consideration.

AREQUIPA, PERU,
June 10, 1899.

THE ANNULAR NEBULA *H IV.* 13 IN CYGNUS.

By JAMES E. KEELER.

OF the four annular nebulae known to the Herschels, only two are (easily) visible in the United States and Europe. One of these, the ring nebula in Lyra, has been described, drawn and photographed many times.[1] The other, *H IV.* 13, or *G. C.* 4565, in Cygnus, appears to have been very little observed, doubtless on account of its faintness and small size. Sir John Herschel's[2] drawing represents it as a faint, nearly circular ring. No stars are shown in or near the nebula. Lord Rosse's[3] drawing gives a little more detail — in particular the gradual fading away of the nebulosity, inside the ring, toward the center. In the description a conspicuous star (14.5 mag.) is mentioned as occurring on the inner edge of the ring, and another, more difficult, near this on the outer edge. No other drawings have been made, so far as I know, and I have found no record of any photographic observations whatever.

In the course of the systematic observations of nebulae now being made with the Crossley reflector, the Cygnus nebula was photographed on the nights of August 9 and August 10, 1899, with exposures of one and two hours respectively. On both occasions the sky was brilliantly clear, and the seeing good. From the negative of August 10, which is somewhat better than the other, a glass positive was made with an enlargement of 10.9 diameters, and as no magnifier was required for its examination, it was more convenient for some purposes than the original negative.

The nebula, as shown by the photograph, is an elliptical,

[1] For an account of photographs made with the Crossley reflector, see this JOURNAL, **10**, 193, October 1899.

[2] *Phil. Trans.*, 1833. Plate XIII, Fig. 48.

[3] *Observations of Nebulae and Clusters of Stars*, Plate V.

nearly circular ring, not quite regular in outline, pretty sharply defined at the outer edge. The outside dimensions are:[1]

Major axis	42.5"
Minor axis	40.5"
Position angle of major axis	32°

The interior diameter cannot be given accurately, since the ring fades quite gradually toward the center. It is roughly 25".

Lord Rosse's star on the inner edge of the ring is very conspicuous, but his second star does not appear. There is, however, (as I had confidently expected), a nucleus, or star, exactly in the center of the ring. It is very conspicuous on the photograph, but visual examination with the 36-inch refractor showed that it is at the very limit of visibility with that instrument; in fact neither Mr. Aitken nor I could be certain that we saw it. It cannot be brighter therefore, than the sixteenth magnitude. Lord Rosse's star was easily seen at the same time, and its magnitude estimated as 14.5.

From a few places inside the ring, rays of nebulosity, slightly brighter than the background, extend part way toward the center like imperfect spokes. The brightest of these is in position angle 90°, measured from the central star.

The whole neighborhood of the nebula is rich in stars, and on the plate many small stars occur quite close to the ring. The positions of seven of these, referred to the central star, were measured on the negative of August 10, by Mr. Palmer, with the Observatory measuring engine, and checked by my own measurements of the positive enlargement. The magnitudes were determined roughly with the aid of a curve obtained from measurements of the star disks. The results, which are given below, will permit any possible proper motion of the nebula to be detected in the future.

[1] In the second observation of Sir John Herschel's catalogue the estimated diameter is given as 15', perhaps a misprint for 1.5'. This error has found its way into some popular books.

Star	Mag.	Pos. Angle	Dist.
a	17.5	57.6°	51.5"
b	20	91.0	52.1
c	19	149.6	43.3
d	18	216.1	36.9
e	16.5	278.8	24.0
f	14.5	330.0	11.2
g	15.0	348.1	32.2

The star *f* is the one on the ring, described by Lord Rosse. There is no evidence that it is physically connected with the nebula, and its position on the ring is probably accidental. The central star, on the other hand, is undoubtedly a real nucleus.

The spectrum of the nebula has, I think, never been determined. On one night I looked at the nebula through a 45° prism held in front of a 1 inch eyepiece on the Crossley reflector, and saw what appeared to be a monochromatic image of the nebula, corresponding probably to the line λ 5007; but with the low dispersion used I am not certain that I could distinguish between a monochromatic image and the slightly blurred image which would result from a short continuous spectrum. An investigation of the spectrum will be made later, by photography.

LICK OBSERVATORY,
September 1899.

ON THE EFFECT OF PRESSURE UPON THE WAVE-LENGTHS OF THE LINES OF THE HYDROGEN SPECTRUM.[1]

By J. WILSING.

IN my paper on "The Interpretation of the Typical Spectrum of New Stars," published in the *Sitzungsberichte* of the Academy for May 4 of this year,[2] I communicated the results of experiments which very clearly confirmed the effect of the pressure of the vapor on the wave-length of lines of metallic spectra discovered by Humphreys and Mohler. I there availed myself of the disruptive discharge between metallic electrodes in water, which occurred with explosive violence, and caused a rise of several hundred atmospheres in the pressure of the spark. In consequence of this there resulted in case of most lines an increase of wave-length associated with broadening, which amounted in some instances to several hundredths of a tenth-meter. The following experiments now show that also the lines of the hydrogen spectrum similarly undergo an appreciable displacement toward the red on increase of pressure.

I have already indicated in the paper cited that the disturbing broadening and poor definition of the hydrogen lines with increasing pressure occur only when the temperature of the discharge simultaneously increases, and that it might therefore be possible to make exact determinations of the wave-lengths if at the same time provision was made for keeping down the potential by decreasing the distance between the electrodes. I was thus able by measuring spectrograms of the "second" hydrogen spectrum at atmospheric pressure to make it seem likely that appreciable displacements of certain lines do actually occur in the direction indicated, but on account of the slight difference of pressure a final conclusion could not be reached.

[1] *Sitzungsberichte der Akademie der Wissenschaften zu Berlin.* Joint session on July 27, 1899.

[2] This JOURNAL, **10**, 113, 1899.

A very noticeable displacement toward the red of the hydrogen line β with increase of pressure is proven with certainty by the following experiments, to the success of which the assistance of Dr. Eberhard has contributed much.

It was my intention, starting from the same point of view as in the experiments on the change in wave-length of metallic lines, to investigate first the position of the hydrogen lines in the arc spectrum when the carbon poles dipped in water, but the voltage of the small Siemens dynamo of early type at my disposal was not sufficient for the production of an intense hydrogen spectrum. On using the high potential spark between metallic electrodes in water, the intense continuous spectrum overpowered the weak bands of the hydrogen spectrum, while carbon electrodes, which give a less bright continuous spectrum, are rapidly deformed in water, so that a uniform illumination of the slit of the spectrograph could only have been obtained with greater sparking distances than I could employ.

When the discharge took place in air, however, as soon as the carbons were moistened with water, $H\beta$ especially appeared as a broad, faint, bright band, in the middle of which a comparatively fine dark line could be seen. Liveing and Dewar[1] observed the flashing out of the hydrogen lines $H\alpha$ and $H\beta$ when they put a drop of water on the electrodes, but they did not perceive any phenomena of reversal.

With the spectrograph employed for the above mentioned metallic spectra, photographs of the spectrum of the discharge between moistened carbon electrodes and of the spectrum of a hydrogen tube were now made upon the same plate. The displacement of the absorption line $H\beta$ toward the less refrangible end of the spectrum was clearly visible on these plates. The following summary gives in the first column the amounts in revolutions of the micrometer of the displacements measured upon five plates, in the second column their values in wave-lengths, together with weights and remarks:

[1] "Note on the reversal of hydrogen lines; and on the outburst of hydrogen lines when water is dropped into the arc." *Proc. R. S.*, 35, 1883.

Displacement R	$\mu\mu$	Wt.	
+0.096	+0.20	½	$H\beta$ weak
+0.028	+0.06	½	$H\beta$ weak and covered up; absorption line sharper on its edge toward violet
+0.046	+0.10		
+0.049	+0.10		Very good plate
+0.051	+0.11		Good plate

The width of the bright band $H\beta$, diffusely bounded on both sides, amounted to more than $5\mu\mu$, and its position could not be measured with sufficient accuracy on account of its diffuse edges. The mean width of the absorption line was $0.8\mu\mu$. On one plate it was more sharply bounded toward the violet, on the others it was equally diffuse on the two sides. The different photographs necessarily show differences in respect to superposition, sharpness and displacement of the dark lines, since the image of $H\beta$ probably results from the superposition of the radiation of many layers in which pressure and temperature differ considerably, and since further the phenomenon of reversal appears in different degrees of distinctness corresponding to the varying conditions of the discharge. Taking this into consideration we may regard $+0.11\mu\mu$ as the mean amount of the displacement of the $H\beta$ line in the spark spectrum from the corresponding line in the spectrum of the tube under the given conditions.

Of the other hydrogen lines, $H\alpha$ was easily visible in the spark spectrum, $H\gamma$ appeared to be present on some plates as a faint and excessively broadened band, but the strongly developed carbon lines in the neighborhood were disturbing. The bands corresponding to $H\delta$ and $H\epsilon$, which were presumably even more broadened and diffuse than the less refrangible hydrogen lines, were completely concealed by the strong calcium lines at $\lambda 423\mu\mu$, $397\mu\mu$, $393\mu\mu$, and the cyanogen band, whose less refrangible edge has the wave-length $422\mu\mu$.

KGL. ASTROPHYSIKALISCHES OBSERVATORIUM,
Potsdam.

ON THE PRESENCE OF OXYGEN IN THE ATMOSPHERES OF CERTAIN FIXED STARS.[1]

By DAVID GILL.

IN a paper read before the Society on April 8, 1897, and in a subsequent paper,[2] Mr. Frank McClean draws attention to the grouping of lines other than those of helium and hydrogen in the spectra of β Scorpii, β Canis Majoris, β Centauri and β Crucis, suggesting that the close correspondence between the grouping of these extra lines and the known lines of oxygen, points to the probable presence of that gas in the atmosphere of these stars.

In the latter paper he writes: "The most remarkable correspondence is in the case of the large group on either side of $H\delta$. A slight shift of about a tenth-meter is required to bring the groups into identical positions. However, the close similarity of the whole grouping of the two spectra as they appear on the plate, admits of little doubt that the extra lines actually constitute the spectrum of oxygen. If this be established, the spectrum of the first division of helium stars would be due to hydrogen, helium, and oxygen."

In his subsequent work[3] Mr. McClean concludes: "Taking everything into account, the succession of coincidences between the extra lines of β Crucis and the oxygen spectrum can only be accounted for on the basis of the extra lines being in the main actually due to oxygen."

This conclusion does not as yet appear to have been fully accepted by spectroscopists, partly because from the low dispersion used the lines of the groups are not separately shown. It is very generally known that the instrumental equipment of the Royal Observatory at the Cape has recently

[1] *Proc. Roy. Soc.*, **55**, 196, 1899.

[2] *Proc. Roy. Soc.*, **42**, 418, No. 386.

[3] *Spectra of Southern Stars.* (Stanford, London, 1898.)

been enriched by a complete equipment for astrophysical research, the whole being the munificent gift of Mr. McClean, F. R. S.

The slit-spectroscope, for attachment to the photographic refractor, reached the Cape in the middle of January last, and I resolved that its first published work should deal with Mr. McClean's interesting discovery.

As a complete account of the instrument and its Observatory will be subsequently published, it may be sufficient for the present to state that the object-glass of the photographic telescope has an aperture of 24 inches, and focal length of 22 feet 6 inches, its minimum focus being, at present, for rays about midway, between $H\beta$ and $H\gamma$.

The collimator of the spectroscope has an aperture of 2¼ inches, and focal length of 22½ inches, so that a cylinder of parallel rays 2 inches in diameter falls on the prisms, and the latter are of sufficient size to pass the whole of the rays which form the image of the spectrum on the sensitive plate. The instrument is provided with two camera-telescopes of 2¾ inches aperture, one being of about 36 inches focal length, the other of 16 inches.

Only the larger of the two camera-telescopes has been employed in the after-mentioned observations.

There are two cast-iron prism boxes; one of them contains three prisms of about 60° each, which for rays near $H\gamma$ produce a deviation of 180°, so that the camera-telescope becomes parallel in the reverse direction to the slit-telescope. The other prism box contains a single prism of 62°. The prisms in both boxes are fixed, without screw adjustment, in minimum deviation for $H\gamma$. The collimator is in the axis of a solid drawn steel cylinder—the latter attaching by a flange at one end to the butt end of the telescope; a cast-iron plate attaching to the flange on the other end of the cylinder carries either one or other of the two prism boxes.

The slit-slide and the 60° prism for reflecting the comparison spark on the slit (made on the plan of the Lick spectroscope)

are contained in a strong cast-steel box which is permanently attached to one end of the collimator tube. This latter is a very strong solid drawn steel tube, the external surface of which has been turned and finished to a true cylindrical form, and it rests in proper geometrical bearings formed in strong cast-iron diaphragms, which are fitted inside the cylindrical body of the instrument.

A powerful slow motion permits the collimator to be slid along its axis so as to focus the slit upon the image of the star at any required reading of the focusing scale. The object-glass of the collimator is mounted on the end of another steel cylinder which also rests on geometrical bearings inside the outer collimator tube, and it is also provided with a fine slow motion and a focusing scale. Both these scales are illuminated at will by small incandescent lamps, and are read by microscopes which are accessible from the outside.

The whole instrument can be enveloped in felt to prevent any but very slow change of temperature.

The comparison-spark apparatus is arranged with wide angle object-glasses, in such a way that if the image of the spark shines on the slit the object-glass of the collimator must be full of light. Numerous trials in all positions of the instrument have invariably given photographs of the lines of the comparison spectrum of iron rigorously coincident with the corresponding lines of the solar spectrum, the latter being obtained by exposing the slit in diffuse daylight.

The camera end with its focusing and tilting adjustments can be attached to either telescope by a flange with a bayonet joint. The focusing scale is divided to $\frac{1}{10}$ mm, and the amount of tilt of the plate-holder is measured on a graduated arc.

As the large telescope is fitted with an object-glass prism of 24 inches aperture (which, when the slit-spectroscope is in use, is folded back in the manner shown in the frontispiece of Mr. McClean's *Spectra of Southern Stars*), heavy counterpoises are required to balance the tube about the declination axis if the spectroscope is not attached.

In designing the slit-spectroscope I was thus not limited by the necessity for lightness in its construction. The complete instrument weighs 400 pounds, being almost exactly the equivalent of the counterpoises and focusing slide and camera, which are removed for its adaptation. In every detail the fittings of the spectroscope are designed with the necessary geometrical limitations of freedom *and no more*, so that no shake nor variation of adjustment can arise from imperfection of workmanship; that is to say, all the adjustments depend on adequate spring pressure against the *minimum number* of necessary rigid points of support.

The object-glasses of the spectroscope were made by Brashear, and are all excellent. The three dense prisms were also made by Brashear; their definition is very fine, but the glass is rather yellow in color, and produces great absorption of rays more refrangible than $H\gamma$. The single prism by Steinheil gives excellent definition, and the glass is much whiter than in Brashear's prisms. The optical constants of the prisms have not yet been determined.

The mounting was constructed by the Cambridge Scientific Instrument Company, to my designs, in the most careful and satisfactory way. I am greatly indebted to Mr. Horace Darwin for much care in supervision of the work, and for some very ingenious and important improvements in detail which he carried out.

Above all I am indebted to Mr. H. F. Newall, who has taken infinite trouble in making and testing the permanent adjustments of the instrument and in supervising the arrangement of its final details. To him I owe the fact that the instrument arrived at the Cape practically in perfect adjustment and ready for work. After a series of preliminary focusing trials by Newall's method,[1] a number of photographs of star spectra were made with the three-prism box and long telescope. The present paper deals chiefly with the results of measures of a photograph of the spectrum of β Crucis, and of a comparison iron

[1] *M. N.*, 57, 572.

spectrum obtained on March 15. The plate was exposed to the comparison spectrum of iron immediately before and immediately after the exposure for the star spectrum.

Lines of the iron spectrum cover the whole exposed length of the plate from Fe λ4187.99 to 4563.99, the linear interval on the plate between these lines being 70.367 mm.

As a preliminary step, the intervals between successive pairs of iron lines were measured with the micrometer of the old Repsold astro-photographic measuring apparatus.[1]

If Δs is the interval between the two adjoining lines in terms of revolutions of the micrometer screw, λ_1 and λ_2 their respective wave lengths,

$$\lambda_m = \frac{1}{2}(\lambda_1 + \lambda_2) \text{ and } \Delta\lambda = \lambda_1 - \lambda_2.$$

I found to my surprise on computing $\Delta\lambda / \Delta s$ for many different values of λ_m, and plotting these on millimeter paper with $\Delta\lambda / \Delta s$ as ordinate and λ_m as abscissa, that within the limits of error of plotting and observation the resulting curve was practically a straight line; in other words the screw values can be represented by

$$\frac{\Delta\lambda_\lambda}{\Delta s} = n\,, \quad \frac{\Delta\lambda_{\lambda+1}}{\Delta s} = n + d\,, \quad \frac{\Delta\lambda_{\lambda-1}}{\Delta s} = n - d\,,$$

$$\frac{\Delta\lambda_{\lambda+2}}{\Delta s} = n + 2d\,, \quad \frac{\Delta\lambda_{\lambda-2}}{\Delta s} = n - 2d\,,$$

or generally

$$\frac{\Delta\lambda}{\Delta s} = d(\lambda_\lambda - \lambda_0)$$

where

$$\lambda_0 = \lambda_\lambda - n / d\,.$$

Such a law can only be applicable to a limited portion of the whole spectrum, for it is obvious that for no value of λ can $\Delta\lambda / \Delta s$ be really $= 0$.

In order to test over what range of the spectrum this law might be regarded as sufficiently rigorous, four selected iron lines were measured in terms of the millimeter scale which is

[1] Described by Bakhuyzen, *Bulletin du Congrès Astrographique*, I, 169.

attached to the instrument, the division errors of which are known for each 5th millimeter. For convenience the measures are converted into screw revolutions of the micrometer microscope which was used in measuring the spectra, viz.: 1 revolution = 0.5 mm.

		Screw Rev.
$\lambda_1 =$	Fe 4583.99	200.254
$\lambda_2 =$	" 4442.52	159.130
$\lambda_3 =$	" 4282.54	101.802
$\lambda_4 =$	" 4187.99	59.520

Hence

	d.	*o.*
for $\frac{\lambda_1 + \lambda_2}{2}$ we have $\frac{\Delta\lambda\ 4513.26}{\Delta s} = 3.4400$		
	0.0043053	3714.80
for $\frac{\lambda_2 + \lambda_3}{2}$ we have $\frac{\Delta\lambda\ 4362.53}{\Delta s} = 2.7906$		
	0.0043572	3722.08
for $\frac{\lambda_3\ \ \lambda_4}{2}$ we have $\frac{\Delta\lambda\ 4325.57}{\Delta s} = 2.2361$		

Thus far interpolation between adjoining known iron lines the second differences, *d*, are practically constant over the range of spectrum with which we have to deal, and consequently the logs of $(\lambda - \lambda_0)$ vary proportionally to the measured intervals between the lines.

If this were *strictly* true for the whole range of our spectrum one would obtain rigorously accurate interpolation as follows:

Let m_1 and m_3 be the micrometer readings on any two known lines.

Let m_2 be the micrometer reading for an intermediate unknown line.

Let λ_1 and λ_3 be the corresponding wave-lengths of the known lines. Then to find λ_2 corresponding to m_2 we have

$$\Big([\lambda_1 - \lambda_0] - [\lambda_3 - \lambda_0]\Big)\frac{m_3 - m_2}{m_3 - m_1} + [\lambda_3] = [\lambda_2 - \lambda_0] \qquad (1)$$

where the square brackets denote the logs of the included quantities.

That the second differences of $\Delta\lambda / \Delta s$ for successive values of λ are not strictly constant for the whole length of our spectrum is shown by the different values of d and λ_0 obtained above.

If we assume λ_0 a constant $= 3718$ and interpolate the wave lengths of λ_2 and λ_3 by means of our formula (1) we obtain

	Known	Computed	Known – Computed
λ_2	4425.52	4425.36	$+0.16$
λ_3	4282.54	4282.72	-0.18

The differences "known – computed" considerably exceed the probable errors due to the observations and the small inaccuracies of the determinations of the fundamental wave-lengths, thus showing that errors have been introduced by neglect of the small variations in the second differences of the successive values of $\Delta\lambda / \Delta s$.

It is, however, very easy to take account of these variations by computing an auxiliary table for different values of λ_0 with the argument λ_1. Thus on the assumption that λ_0 varies proportionally to λ_1 we have:

Mg. λ.	λ_0
4100	3732
4200	3727
4300	3722
4400	3717
4500	3712
4600	3707

Taking λ_0 from this table with the argument "approximate wave-length of the line, whose definite wave-length is required," we find the intermediate wave-lengths accurately represented.[1]

As well-known iron lines are found within thirty or forty tenth-meters of all the unknown lines, it is always sufficient for the purpose of interpolation by our formula to employ the value of λ_0 from the above table with the argument $\frac{\lambda_1 - \lambda_3}{2}$.

[1] I find that the wave-lengths of solar lines as measured by Campbell (this JOURNAL, October 1898) are also very beautifully represented throughout by this simple method of interpolation.

In this way the wave-lengths of the lines in the spectrum of β Crucis given in the following table (p. 280), have been determined:

Column (1) gives the wave-lengths of all known oxygen lines of intensity 3 or brighter, between λ 4303 and 4575, according to Neovius as well as Trowbridge and Hutchins.[1]

Column (2) gives the wave-lengths of all helium lines according to Runge and Paschen,[2] contained within limits of the spectrum under observation.

Column (3) gives the results of my measurements of the negative of the spectrum of β Crucis, those under head I being my first essays in the measurement of any photographed spectrum, those under head II being the results of my second measurement, including all the lines which could be detected under most careful and repeated scrutiny. Each result in series I depends on two pointings, each in series II on four pointings.

The lines whose wave-lengths are given to two decimal places of the tenth-meter were measured with a magnifying power of fifteen diameters, those given to one decimal place with a power of only three diameters, — the lines of the latter class being very faint and only certainly visible under a very low power. When possible, different iron lines were used in series I and II for determination of the wave-length of the stellar lines.

The observations were not arranged for determination of motion in the line of sight, but the exact coincidence of the star line 4417.06 with the air (oxygen) line, and the general agreement of the stellar hydrogen and helium lines with their known wave-lengths, tend to show that the relative motion of β Crucis to the Earth on February 21, did not exceed $\pm$ 3 kilometers per second. On that date the Earth in its motion round the Sun was moving towards β Crucis with a velocity of 18 kilometers per second, and consequently β Crucis is probably receding from the Sun with a velocity of 18 km $\pm$ 3 km per second.

The whole of the known helium lines within the measured

[1] WATTS' *Index of Spectra*, Appendix E.

[2] This JOURNAL, 3, 10.

(1) Known oxygen lines. Intensity 3 or brighter			(2) Helium (Runge and Paschen, this JOURNAL, 3, 10)	(3) Cape measures of spectrum of β Crucis		(4) Remarks
Neovius	Intensity	Trowbridge and Hutchins		I	II	
					4253.9	Faint, rather broad, edges fairly distinct
					4267.2	Probably carbon, E. and V., 4267.5
					4285.3	Undoubted line, only seen with low power
					4298.0	Too indistinct for measurement, positions estimated only
					4303.0	
					4304.0	
4317.4	(4)	4317.20		4317.36	4317.35	O
4319.9	(4)	4319.50		4319.78	4319.78	O
4325.9	(3)	4325.90		4325.83	4326.0	O
4327.3	(3)	4327.60				
4337.1	(3)			4337.19	4337.0	O slightly more refraction than Fe 4337.22
					4338.8	
				4340.57	4340.63	*Hγ* (4340.634) Rowland
					4342.6	Suspect this faint close double line
4345.8	(6)	4345.52		4345.53	4345.50	O
4347.9	(6)	4347.94 4347.47		4347.33	4347.30	O coincident with 4347.47
4349.4	(8)	4349.30		4349.42	4349.49	O
4351.6	(6)	4351.40		4351.34	4351.39	O
					4361.4 4363.1	Both very fine faint lines very difficult to see
4367.0	(6)	4366.92		4367.01	4366.99	O
4368.3	(3)					
4369.7	(3)	4369.60				
			4388.100 (3)	4388.18	4388.24	He
4396.1	(3*n*)	4396.30			4396.2	O
4415.0	(9)	4515.00		4415.01	4415.09	O
4417.3	(9)	4417.17		4417.06	In coinc. with air line	O
					4431.1	Certainly less refraction than Fe 4430.79
					4433.1	Certainly more refraction than Fe 4433.32
					4435.1	
			4437.718 (1)		4437.9	He faint, well defined edges
4452.7	(3)	4452.40			4452.7	O
4465.4	(4)	4465.40				
4467.8	(4)	4468.04				
4469.6	(3*n*)	4469.50				
			4471.646 (6)	4471.61	4471.56	He
					4481.17	Probably magnesium
					4522.84	Very faint
				4552.80	4552.78	Strong well marked lines, origin unknown
				4567.90	4568.06	
				4574.67	4574.69	

range of spectrum are unquestionably present, as also are all known oxygen lines stronger than intensity 4.

The exceedingly faint lines:

β Crucis 4253.9 may be coincident with Neovius 4254.1 = T. and H. 4253.42, and

β Crucis 4303.0 or 4304.0 may be coincident with Neovius 4304.4 = T. and H. 4303.8

but these coincidences are very doubtful and it is improbable that such very faint lines would be represented, while the neighboring line 4327.3, intensity 3, is wanting on the photograph.

The oxygen lines of intensity 4 which are wanting, viz.: Neovius 4465.4 and 4467.8 are in a portion of the spectrum which is somewhat over-exposed, and this fact probably accounts for the non-appearance on the plate. Possibly also the relative intensities of the oxygen lines at the temperature and pressure of the atmosphere of β Crucis may be different from their relative intensities in the conditions under which Neovius determined the intensities of the air lines (spark spectrum).

There remains, however, not the slightest doubt that all the stronger oxygen lines are present in the spectrum of β Crucis, at least between λ 4250 and 4575, and this fact requires no further laboratory experiments for its establishment. It is almost equally certain that there is no trace of true nitrogen lines in this spectrum.

The only measured lines of β Crucis near known nitrogen lines are:

Neovius	4341.8	Intensity	1*b*	β Crucis	4342.6
"	4523.0	"	1*b*	"	4522.84

but it is improbable, although not impossible, that nitrogen lines of intensity (1) should be present while the strong nitrogen line 4507.7, of intensity 6, is absent.

Besides hydrogen, helium and oxygen, the spectrum of β Crucis shows the probable presence of carbon (4267.2) and magnesium (4481.17). These lines were not included in the first measurements, in the former case, because the line could not be distinctly seen with the higher power, in the latter because I was

doubtful of the existence of the line on account of a slight defect in the film at that point, but other negatives confirm its existence.

The spectra of β Crucis, β and ϵ Canis Majoris, and probably β Centauri are all practically identical. They all contain the three unknown strong lines

4552.79
4567.09
4574.68

besides the probable magnesium line 4481.17, the lines of hydrogen, helium, the stronger oxygen lines, and the probable carbon line 4267.2.

Further investigations on this class of stars will be subsequently communicated; in the meanwhile I forward also for reproduction a contact positive from a negative of the spectrum of ϵ Canis Majoris taken on March 15, with the single prism, in which the slit has been focused for rays of about λ 4080, and with a comparison spectrum from an oxygen tube, a Leyden jar and air space being introduced in the secondary circuit of the Ruhmkorff coil.

This photograph shows the coincidence of stellar lines with the group of three strong oxygen lines, viz.:

Neovius	T. and H.	Intensity
4076 3	4076.19	9
4072.4	4072.34	9
4070.1	4070.24	8

and other neighboring oxygen lines beyond the range of the three-prism train. The lines are all displaced towards the red by motion.

On March 15 the Earth by its motion round the Sun was receding from ϵ Canis Minoris with a velocity of 17km per second, which agrees with the direction of the displacement of oxygen lines. There has not yet been time to determine the constants of the single-prism spectroscope, but this does not affect the question of the identification of the oxygen lines in the spectrum of ϵ Canis Majoris.

The plates of the spectra of β Crucis and ϵ Canis Majoris were exposed and developed by my assistant, Mr. J. Lunt.

Minor Contributions and Notes

CORRECTIONS TO DETERMINATIONS OF ABSOLUTE WAVE-LENGTH.

In so far as the literature of the subject is at present available to me, it appears that two corrections have hitherto been neglected in determinations of the absolute wave-length of lines of the solar spectrum. These corrections are so small as to be of slight importance in many researches upon the spectrum, including the identification of solar and metallic lines, but may rise to significance in investigations of the motion of stars in the line of sight, which are very exacting in regard to the accuracy of the wave-lengths of both metallic and gaseous comparison lines, solar lines occurring in stellar spectra, and lines of normally gaseous elements found in stars.

The first correction is due to the effect of the eccentricity of the Earth's orbit. The ordinary formula for the component of the Earth's velocity along the radius vector of its orbit is

$$v = -\frac{Ve\sin(\Pi - \odot)}{\sqrt{1-e^2}},$$

V being the mean orbital velocity of the Earth and equaling $\frac{2\pi a}{T}$, and Π being the longitude of the Sun when the Earth is at perihelion. The values of v will obviously be at a maximum when the Earth is at quadrature, in April and October, and zero at perihelion and aphelion. Introducing the proper values for a, e and T we at once obtain the maximum value, $v = 0.50$ kilometers per second, which may, of course, be directly obtained from the Nautical Almanac. The effect of this velocity in shifting the lines of the solar spectrum would be as follows :

	C	F	$H\gamma$	K	
	6563	4862	4341	3934	tenth-meters
Shift	0.011	0.008	0.007	0.007	" "

The second correction is the diurnal one, due to the Earth's rotation, habitually applied to the reduction of stellar velocities by all

observers. It is not to be presumed that this has been accidentally overlooked in laboratory determinations of wave-length, but rather that it has been deemed insignificant. At present, however, when the best measures of wave lengths are given to the thousandth of a tenth-meter (although not assumed to be accurate, as absolute measures, to that extent), the correction should be taken into account.

The well-known formula, expressed in kilometers, is

$$v_d = -0.47 \sin t \cos \delta \cos \phi,$$

the letters having their customary significance. The maximum value, for a solar declination of zero, and an hour angle of 90°, will be for the latitude of Baltimore 0.36 km, for that of Potsdam 0.29 km per second; for an hour angle of 45°, the values will be respectively 0.24 and 0.20 km per second. The effect of this shift on the wave-length would be as follows for Baltimore:

	C	F	$H\gamma$	K	
Hour angle 90°	0.008	0.006	0.005	0.005	tenth-meters
45°	0.005	0.004	0.004	0.003	" "

The uncertainty of an absolute wave-length might therefore amount to over one one-hundredth of a tenth-meter, and a velocity depending directly upon it to over one-half a kilometer per second.

This result, however, would certainly be misleading to those unfamiliar with the reduction of velocities in the line of sight, for probably all observers depend upon relative rather than absolute wave-lengths. In fact, in cases where direct displacements of identical lines in the comparison and star spectrum are measured, the effect of even very large errors in the absolute wave-length will be inappreciable. Where indirect comparisons are made, that is, where the sharpest star lines are measured with reference to the best defined neighboring lines of the comparison spectrum — certainly a very convenient method — any inaccuracy of the relative wave-lengths is nevertheless of decided significance. For instance, an error of 0.01 tenth-meter in the relative wave-length of either a comparison line or a solar line identified in the spectrum of Polaris will make an error of 0.7 km per second in the velocity of that star as determined by that line. Fortunately, the best measurements of the wave-lengths of the lines of metallic and gaseous elements, by Hasselberg, and by Kayser and Runge and others, are based upon Rowland's system, so that corrections for differences of system do not arise to any great extent.

However, the need of a more accurate determination of absolute wave-length, and still more of an accuracy to the one thousandth of a tenth-meter in relative wave-lengths in comparison and solar spectra (to which Rowland's admirable table does not profess to attain), has become urgent for line of sight work, in which it is hoped that even fractions of a kilometer may be of actual significance.

EDWIN B. FROST.

POSITION OF NOVA SAGITTARII.[1]

THE method of determining the precise positions of the stars from measures of their photographic images, described in the *Annals*, **26**, 237, is now generally employed here when accurate positions are desired. The positions of the planet Eros on the photographs described in *Circulars* Nos. 36 and 37 are being determined by this method. As an illustration of the accuracy readily attained, measures were made of the New Star in Sagittarius described in *Circular* No. 42. The first pair of plates measured were enlargements of Plate B 21319, taken with the 8-inch Bache telescope on April 29, 1898, with an exposure of ten minutes. The star was then of the eighth magnitude. The second pair of plates were enlargements of Plate C 11827, taken with the 11-inch Draper telescope on March 20, 1899, when the star was of the eleventh magnitude. Six exposures of ten minutes each were given to the last plate, moving it slightly in declination after each. Six adjacent images of the Nova and of each of the comparison stars were thus obtained. A single enlargement, therefore, furnishes six independent determinations of the position of the Nova. It was found more convenient, however, to take the mean of the measures of the six images and treat them like a single observation. Two enlargements were made of each of the two original negatives, taking care to move the reticule about a millimeter in each coördinate after the first enlargement of each plate, and thus eliminate, or at least reduce, errors in ruling. Four independent determinations of position are thus obtained, which are given below. Two positions of this object have been published, derived from visual observations. The first, by Professor O. C. Wendell, is given in *Circular* No. 42; the second, by Professor E. Hartwig, is given in the *A. N.*, **149**, 29. It should be stated that in the first of these determinations the times were noted on

[1] *Harvard College Observatory Circular* No. 46.

a chronometer by a comparatively inexperienced recorder, and in the second case, a single comparison star, $-13^\circ 5194$, was used whose position was taken from *Weisse*, and differs several seconds from the recent observations of its position made here. The photographic determinations depend on the positions of five stars, $-13^\circ 5183$, $-13^\circ 5185$, $-13^\circ 5194$, $-13^\circ 5197$, and $-13^\circ 5200$, measured with the Harvard Meridian Circle.

Wendell	R. A. 1900 = $18^h\ 56^m\ 12.2^s$	Dec. 1900 = $-13^\circ\ 18'\ 16''$
Hartwig	" " = 18 56 12.69	" " = −13 18 22.3
B 21319, I	" " = 18 56 12.82	" " = −13 18 12.6
" II	" " = 18 56 12.90	" " = −13 18 12.8
C 11827, I	" " = 18 56 12.81	" " = −13 18 13.2
" II	" " = 18 56 12.79	" " = −13 18 13.3

The close agreement of the four photographic measures indicates that this method gives results of an accuracy at least equal to the best meridian circle observations. As in the case of other Novae, there is no evidence of proper motion, or change of position as the star fades.

Several measures were made of each image of the Nova. A comparison of them shows that the probable error of a single setting is $\pm 0.13''$, the largest residual being $0.3''$. A similar determination from settings of the comparison stars gave the probable error derived from successive settings $\pm 0.08''$. In three cases only did successive settings differ more than half a second, and these gave the values, $1.0''$, $0.7''$ and $0.6''$.

The results of the first measures of the position of the Nova, as derived from the six images of Plate C 11827, and expressed in seconds of arc, differ from their mean in x by $+0.02''$, $-0.08''$, $-0.38''$, $+0.09''$, $+0.27''$, and $+0.07''$, and in y by $-0.48''$, $-0.32''$, $+0.18''$, $+0.36''$, $-0.08''$, and $+0.26''$. The second measures similarly gave in x, $-0.28''$, $+0.03''$, $-0.04''$, $+0.06''$, $+0.35''$, and $-0.09''$, and in y $-0.45''$, $+0.03''$, $+0.12''$, $+0.19'$, $-0.08''$, and $+0.14''$. A part of these differences is real and is perhaps due to a change in orientation of the plate when it was moved. Thus, the largest value, $-0.48''$, has nearly the same value, $-0.45''$, in the second measure. The designations of the five comparison stars, their assumed right ascensions and declinations for 1890, and their residuals in x and in y, as derived from the two measures of the two plates, are given below:

DM	R. A. 1890	Dec. 1890	x				y			
$-13°5183$	$18^h\ 54^m\ 58.19^s$	$-13°\ 8'36.4$	$+0.1''$	$+0.2''$	$-0.1''$	$-0.3''$	$+0.4''$	$+0.3''$	$+0.2''$	$+0.2''$
$-13°5185$	18 55 1.90	-13 28 31.1	$+0.2$	-0.1	$+0.5$	-0.1	-1.5	-1.2	-0.6	-0.6
$-13°5194$	18 56 1.58	-13 24 37.6	-0.3	$+0.3$	$+1.1$	$+1.3$	$+3.2$	$+2.7$	$+1.6$	$+1.6$
$-13°5197$	18 56 26.58	-13 19 8.0	-1.8	-1.8	-2.0	-1.8	-1.4	-1.1	-1.4	-1.3
$-13°5200$	18 56 41.60	-13 19 25.1	$+1.7$	$+1.4$	$+0.4$	$+0.7$	-0.7	-0.5	$+0.3$	$+0.2$

EDWARD C. PICKERING.

September 23, 1899.

THE YERKES OBSERVATORY OF THE UNIVERSITY OF CHICAGO.

BULLETIN NO. 12.

CARBON IN THE CHROMOSPHERE.

THE green fluting of carbon, which terminates at λ 5165.3, was first found in the spectrum of the chromosphere at the Yerkes Observatory in September 1897. A large image of the Sun and excellent atmospheric conditions are required to render the bright lines of the fluting visible, as the layer of carbon (or hydrocarbon) vapor to which it is due is probably less than a second of arc in thickness, and lies in immediate contact with the photosphere. The forty-inch telescope, which gives a focal image of the Sun seven inches in diameter, was used for the observations, in conjunction with a large solar spectroscope containing a five-inch Rowland grating with 20,000 lines to the inch. The bright lines of the fluting were identified with the faint dark carbon lines in this part of the solar spectrum, by placing the slit on the image of the photosphere, where the dark lines were visible, and then quickly moving it to the chromosphere, where the dark lines were seen to be replaced by bright ones. At this time an unsuccessful search was made for the yellow and blue flutings of carbon.

In August 1899, after the solar spectroscope had been reconstructed and the 20,000-line grating replaced by a four-inch Rowland grating with 14,438 lines to the inch, the green fluting was better seen than before. The lines were carefully reidentified with those of car-

bon, and had a specially constructed micrometer been available at the time, it would have been possible to measure many of them. As the atmospheric conditions were very fine another search was made for the yellow fluting, which terminates at $\lambda 5635.4$. With perfect adjustment of the apparatus, and careful setting for tangency, the fluting was soon found. It can be observed, however, only with the largest instruments used under the best conditions. The blue fluting, which terminates at $\lambda 4737.2$, could not be seen.

Mr. W. S. Adams, Fellow in Astronomy, assisted in the work and confirmed the observations of the yellow fluting, which was also seen by Professor Frost, and has since been observed on other occasions by the writer. In this fluting less than a dozen lines could be separately distinguished. The green fluting contains so many bright lines that it is impossible to count them.

SOME NEW FORMS OF SPECTROHELIOGRAPHS.

Of the various forms of spectroheliographs described in my previous papers, the simplest and best is undoubtedly that in which the instrument is moved as a whole at right angles to the axis of the telescope, the solar image and photographic plate remaining stationary. It is not always possible, however, to employ a spectroheliograph of this form. With the forty-inch telescope, for example, the motion of the very heavy spectroheliograph required could not be effected without jarring the instrument. For this reason it has been decided to cause the solar image to move across the first slit by means of the slow motion declination motor.[1] The first and second slits are fixed with reference to each other, and the photographic plate is moved across the second slit by means of a screw driven by the same motor, which is mounted on the tube of the forty-inch telescope. A wide range of exposures can be secured by means of a system of change gears. This spectroheliograph, which has an aperture of six and one fourth inches, is now nearly ready for trial.

Two other forms of spectroheliographs may occasionally prove useful. In both the first slit and the axis of the collimator are supposed to remain coincident with the optical axis of the telescope. In the first form a photographic doublet, of large field, is so placed that the solar image, at the principal focal plane of the telescope, and the first slit

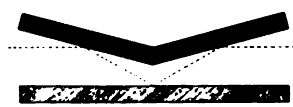
[1] The plan of moving the solar image in declination was first suggested by Mr. W. H. Maw several years ago.

are in its conjugate foci. If the lens is then moved at right angles to its optical axis the image of the Sun will move across the first slit. The carriage which bears the photographic plate across the second slit is geared to the screw that moves the doublet in such a manner as to give a photograph of the Sun which is not distorted by the relative motion of plate and solar image.

The other form of spectroheliograph is similar, but the photographic doublet is replaced by a right angle prism, with hypothenuse parallel to the axis of the telescope, or a combination of three mirrors,[1] arranged as shown in the figure.

When this device is used it is placed immediately in front of the first slit, which lies in the principal focal plane of the telescope. It is connected with the plate-carriage, and both are moved at right angles to the axis of the collimator.

Both of these instruments have obvious defects, such as liability to distortion of the photograph, due to the limited field of the doublet; diffuse light from lens and mirror surfaces; possible distortion of the prism or mirrors, due to heating; and the necessity of using large lenses and mirrors when a large solar image is to be photographed. If the details of the design are carefully worked out, however, these defects can be reduced to a minimum or perhaps altogether obviated. For example, the effects of heating and diffuse light can be greatly decreased by the use of moving screens, so arranged as to cover all parts of the solar image except the particular region which is being photographed at a given moment.



It will be seen that both instruments are better adapted for laboratory use, in conjunction with a heliostat, than for attachment to an equatorial. In the laboratory the considerable distance which must separate the doublet from the solar image and slit, in case a large image is desired, is not very objectionable. Either device affords a simple means of transforming a large fixed spectroscope, of almost any type, into a spectroheliograph. GEORGE E. HALE.

October 21, 1899.

[1] Such combinations of mirrors, as well as right angle prisms, have long been used for the purpose of rotating the solar image in spectroscopic observations. It is obvious that rotation of the image (and photographic plate) may also be employed in spectroheliographic work, if it is intended to photograph only the limb of the Sun, and not the central part of the disk.

RESULTS OF AN EXAMINATION OF SPECTROGRAMS OF α ORIONIS OBTAINED DURING THE RECENT IRREGULAR MINIMUM.

DURING the irregular minimum of α Orionis at the past opposition it became a matter of interest to determine whether a change in its spectrum had taken place. The most convenient and accurate method for making such a determination is to compare spectrograms taken with the same instrument in the same adjustment. For this suggestion, as well as for the necessary earlier photographs for making the comparison, I am indebted to Professor Keeler.

The photographs loaned me by Professor Keeler, together with the necessary data, are tabulated below with the prefix *K;* those taken by myself with the prefix *H.* All these photographs were taken with the Thaw spectroscope[1] having a prism-train of three dense flint 60° prisms, and collimator and camera each of 16 inches focal length. When set on λ 5708 the dispersion was just sufficient to divide the double line at λ 5340.6. The photographs were taken on Cramer "Isochromatic Instantaneous" plates. Within the range (by which is meant the limits within which the lines were sharp and within which the photographs were compared) of *H* I (λ 5130–λ 5370), 63 lines were measured, and on *H* VI, 73 lines.

No.	Date	Setting	Range	Quality of negative
K I	December 5, 1894	*b*	λ 5130–λ 5370	Fine
K II	December 9, 1894	*b*	λ 5130–λ 5370	Fine
K III	October 17, 1894	5352	λ 5328–λ 5655	Fine
H I	December 24, 1898	*b*	λ 5130–λ 5370	Fine
H II	February 27, 1899	*b*	λ 5160–λ 5370	Good
H III	March 20, 1899	*b*	λ 5215–λ 5370	Fair
H IV	April 3, 1899	5708	λ 5405–λ 5655	Fair
H V	April 4, 1899	5708	λ 5328–λ 5655	Good
H VI	April 5, 1899	5708	λ 5328–λ 5655	Fine

Comparisons of the spectrograms taken with the prism-train set on (*b*) were made by placing *K* I on *H* I under the comparator with the film sides together and lines of the same wave-length adjacent, and observing lines of the same wave-length separately to note any change in their relative intensities, or the absence of any lines on the one

[1] *A. and A. P.*, 12, 40.

photograph shown on the other. A like comparison was made with *K* II. In the same way *H* II and *H* III were each compared with *K* I and *K* II.

The comparisons revealed no changes either in the number or relative intensities of the lines. When placed close together under the comparator they appeared like a single spectrum.

Owing to a considerable difference in the dispersion with the prism-train set on λ 5352 and λ 5708 a modification of the above method was used. The wave-lengths of the stellar lines of each of the photographs *K* III and *H* VI were obtained by means of standard spectrograms of the sky, and the intensity of each line, as well as the ratio of its intensity to that of the solar line of the same wave-length was also noted. These data were then tabulated and the tables compared. No change in the relative intensities of the lines not traceable to difference in development or irregularities in the grain of the plate could be detected, nor could any diffeience in the number of the lines be found, except what appeared to be a few exceedingly faint lines, visible only under the best illumination with the microscope, which, however, were always so irregular in appearance as to make it more probable that they were due wholly to the grain in the photographic plate.

The wave-lengths of each photograph having been determined, the lines on each photograph were marked off into groups. They were then placed, film sides together and like groups adjacent, on the comparator, and the relative intensities of the lines and their number compared, with the result that no change could be detected, confirming the conclusion drawn from the tables.

Having found no change in the spectrogram, *H* VI, *H* IV, and *H* V were compared with it. This comparison showed in all respects an identity in the appearance of the three photographs.

The above six photographs therefore revealed no change in the spectrum of α Orionis within the limits which have been stated.

HENRY HARRER.

ALLEGHENY OBSERVATORY,
September 16, 1899.

RECOMMENDATIONS OF THE BOARD OF VISITORS TO THE UNITED STATES NAVAL OBSERVATORY.

IN the report of the proceedings of the Third Conference of Astronomers and Astrophysicists published in the last (October) number of this JOURNAL, reference was made to the appointment of a Board of Visitors to the United States Naval Observatory. The Board presented its report to the Secretary of the Navy on October 2, together with the following letter of transmittal, which contains an outline of the report. The first subdivision, consisting of the recommendations of the Board, is given herewith. The entire report will soon be published by the Navy Department.

Washington, D. C., October 2, 1899.

Hon. John D. Long, Secretary of the Navy.

SIR: In compliance with the request contained in your letter of June 30, 1899, the undersigned have acted as a Board of Visitors to the United States Naval Observatory in Washington and now submit their report, including subdivisions as follows:

I. Recommendations of the Board of Visitors.
II. Circumstances leading to the appointment of the Board of Visitors.
III. Cost of the Observatory.
IV. Comparison with other observatories.
V. Present condition and methods of observatory work and the delay in printing its results.
VI. Historical sketch of the Observatory.
VII. Minutes of the Proceedings of the Board of Visitors.
VIII. Appendix.

The several portions of the report were put in form by the astronomers who are members of the Board. The recommendations are made unanimously.

Very respectfully,

WM. E. CHANDLER.
A. G. DAYTON.
EDWARD C. PICKERING.
GEO. C. COMSTOCK.
GEORGE E. HALE.

RECOMMENDATIONS OF THE BOARD OF VISITORS.

In accordance with the instructions contained in the following letter all the members of the Board of Visitors to the United States Naval Observatory therein named met at the Observatory in Washington on Friday, June 30, 1899, and organized by the selection of William E. Chandler as chairman and George C. Comstock as secretary.

NAVY DEPARTMENT,

Washington, June 30, 1899.

GENTLEMEN:

In accordance with previous correspondence and oral conversations, you are hereby requested to act as a Board of Visitors at the United States Naval Observatory in Washington, convening there today, and to proceed to examine into the condition of that institution and to report to me your conclusions and recommendations.

Very respectfully,

JOHN D. LONG,

Secretary.

Hon. William E. Chandler.
Hon. Alston G. Dayton
Professor Edward C. Pickering.
Professor George C. Comstock.
Professor George E. Hale.

Captain Charles H. Davis, U. S. N., Superintendent of the Naval Observatory, presented to the Board an informal statement of circumstances leading to the appointment of the Board of Visitors and submitted correspondence relating thereto (Appendix, Exhibit A) and to a proposed reorganization of the Observatory (Appendix, Exhibit B). He also placed before the Board a list of Professors of Mathematics upon the active list of the Navy (Appendix, Exhibit C) from which corps the staff of the Observatory is largely drawn, and a list of all persons performing duty at the Observatory with their respective ranks (Appendix, Exhibit D).

At the request of Messrs. Chandler and Dayton there was submitted to the Board by its other members the correspondence conducted by them as a committee of the Second Annual Conference of Astronomers and Astrophysicists for the purpose of obtaining the views of American astronomers and physicists upon the organization and work of the Naval Observatory. Mr. Pickering submitted to the Board a statement regarding correspondence on the same subject conducted by a committee of the American Association for the Advancement of Science.

In view of the facts brought before the Board at its several sessions and after the best consideration which it has been able to give to the subject, the Board of Visitors reports and recommends as follows:

The Naval Observatory, which was originally established as a scientific bureau, auxiliary to the needs of the naval service, has become through half a century of growth and through the expenditure of large sums of money, as authorized by law, an astronomical observatory of

the first rank in respect of buildings, instruments and equipment. But by far the larger and more valuable part of its equipment has little or no reference to any direct requirement of the naval service and its existence can be justified only on the ground that Congress has intended to establish and maintain a national astronomical observatory. Under these changed circumstances its continued connection with the Navy Department has seemed to many of those whose views have been submitted to the Board of Visitors, illogical and undesirable. In view, however, of the absence of a national university, a department of science and industries, or other department or bureau of the government especially suited to the conduct of such scientific work, and in view of the diversity of opinion among American astronomers upon the question to which existing department the Observatory could be wisely transferred, we believe it to be inexpedient for us at the present time to further consider the subject of such transfer.

With reference to the organization of the Observatory under naval administration, the Board of Visitors disapproves of those parts of the "Proposed Organization of Naval Observatory" (Appendix, Exhibit B), submitted under date of September 7, 1897, by "F. E. Chadwick, Chief of Bureau of Equipment and C. H. Davis, Superintendent U. S. Naval Observatory," which require the establishment of a formal observatory council with nominal functions and which by omission practically abolish the office of Astronomical Director. We are by no means objecting to the assembling in conference of the astronomers engaged in observatory work, but the proposed transfer of duties and responsibilities from a single director to a committee of five appears to us a step in the wrong direction; and when, as under the proposed scheme, an absolute power of veto upon all action by the council is lodged in the hands of one of its members, the usefulness of the body seems to approach the vanishing point. In the history of observatories we have been unable to find a case of successful administration without a competent astronomer in immediate supervision of the work, and we believe that the ideal conditions for the successful administration of an astronomical observatory are most nearly realized when a professional astronomer is made the responsible director of the work. This system, which is adopted in every great national observatory, the Board of Visitors believes to be the one best suited to secure the astronomical efficiency of the Naval Observatory.

If the Naval Observatory as a shore-station charged with the performance of certain functions assumed to have a relation to the navy is to continue under the command of a line officer, we recommend that the astronomical staff of the Naval Observatory shall consist of an Astronomical Director, four astronomers, three assistant astronomers and such computers and other minor officers as may be provided by law. The Astronomical Director and astronomers, whether Professors of Mathematics or taken from civil life, and the assistant astronomers, should be appointed by the President, by and with the advice and consent of the Senate, to hold their offices until their successors are appointed.

The Nautical Almanac office, which was formerly a distinct bureau, is now administered by departmental regulations as a part of the Naval Observatory, and it appears from the evidence submitted to the Board of Visitors that the successful administration of the Observatory is much impeded by reason of imposing upon its Astronomical Director the duties of Director of the Nautical Almanac. Each of these offices furnishes abundant employment for the entire time of an able astronomer, and we therefore recommend that there shall be a Director of the Nautical Almanac appointed by the President, by and with the advice and consent of the Senate, to hold office until his successor is appointed.

We also recommend that provision be made for the continuation of the admirable series of memoirs published under the title "Professional Papers of the American Ephemeris and Nautical Almanac."

A criticism, frequently and forcibly urged against the administration of the Naval Observatory, not limited to the present time but covering almost the entire period of its existence, is that its astronomical work has not been prosecuted with that vigor and continuity of purpose which should be shown in a national observatory. The possibility of conducting well planned researches with unvarying regularity over long series of years should constitute the great advantage of a national observatory, an advantage which is not fully realized in the history of the Naval Observatory, where each principal astronomer seems to have been left to choose his own line of work and to alter it from time to time or abandon it. This is perhaps inevitable in a system which places at the head of an observatory an officer who is not a technical expert in astronomical work; and therefore in order to secure continuity in the prosecution of work well chosen and coördinated with that of other observatories, and also to obtain for the

Observatory and the Department advice and criticism which shall be both disinterested and responsible, we recommend the establishment of a permanent Board of Visitors substantially as follows:

There shall be appointed by the President, from persons not officers of the United States, a Board of nine Visitors to the Naval Observatory, six to be astronomers of high professional standing, and three to be eminent citizens of the United States. Appointments to this Board shall be made for periods of three years, but provision shall be made by initial appointments for shorter terms so that two astronomers and one member of the Board not an astronomer shall retire in each year. Members of this Board shall serve without compensation, but the Secretary of the Navy shall pay the actual expenses necessarily incurred by members of the Board in the discharge of such duties as are assigned them by the Secretary of the Navy, or are otherwise imposed upon them. The Board of Visitors shall make an annual visitation to the Naval Observatory at a date to be determined by the Secretary of the Navy and may make such other visitations, not exceeding two in number annually, by the full Board or by a duly appointed committee, as may be deemed needful or expedient by a majority of the Board.

The Board of Visitors shall report to the Secretary of the Navy at least once in each year the result of its examinations of the Naval Observatory as respects the condition of buildings, instruments and apparatus, and the efficiency with which its scientific work is prosecuted. The Board of Visitors shall prepare and submit to the Secretary of the Navy regulations prescribing the scope of the astronomical and other researches of the Naval Observatory and the duties of its staff with reference thereto. When appointments or details are to be made to the office of Astronomical Director, Director of the Nautical Almanac, astronomer or assistant astronomer, the Board of Visitors may recommend to the Secretary of the Navy suitable persons to fill such offices, but such recommendations shall be determined only by a majority vote of the members present at a regularly called meeting of the Board held in the city of Washington.

Special attention is at this point called to the fact that the appointment of a Board of Visitors to the Naval Observatory was recommended by Secretary Tracy in 1891, has been repeatedly urged by superintendents of the Observatory, and is requested by F. E. Chadwick, Chief of the Bureau of Equipment, and C. H. Davis, Superintenden

U. S. Naval Observatory, in the "Proposed Organization of Naval Observatory," dated September 7, 1897 (Appendix, Exhibit B). The duties of the Board, as defined by these naval officers, would be in part as follows: "It lays down the general course of policy to be pursued for the coming year, including printing and publication of observations; fixes the estimates for the astronomical departments; nominates to fill vacancies in the astronomical staff (either by appointment or promotion); recommends as to repairs and acquisitions of new instruments."

If a permanent Board of Visitors as above recommended is established as a part of the administration of the Naval Observatory, it is evident that to it should be committed those questions of policy to be pursued in the conduct of the Observatory which are contained in the memorandum (Appendix, Exhibit B) submitted to the present Board by the Secretary of the Navy under date of June 28, 1899. We therefore abstain from specific recommendations upon these subjects, many of which, indeed, call for a more prolonged and minute study of the situation than the members of the present Board have been able to give to it.

We heartily endorse the recommendation contained in your report as Secretary of the Navy for the year 1897, that "the statute authorizing the appointment of Professors of Mathematics be so amended that without disturbing those who now hold office, which would be unjust to them, no further appointments shall be made" to the staff of the Naval Observatory (Appendix, Exhibit L). In addition to the reasons for this action which are urged by you in that report, we submit for your consideration that the conditions under which astronomical work is done are so different from those which obtain in the naval service that a fixed tenure of office with the certainty of a retiring pension in no way dependent upon the zeal or efficiency with which service has been rendered may easily produce diminished diligence and a purely perfunctory discharge of duties. A more serious evil of the existing system of naval commissions for astronomers, and one which has been forcibly exemplified within the past decade, is the compulsory retirement at the age of sixty-two of astronomers who are then in the maturity of their powers and who under civilian appointments would continue to render to the Observatory a service of undiminished efficiency which they now transfer to other institutions. The reasons which impel the retirement of a naval officer from active service upon attaining a fixed age have no application in the case of an astronomer, and

he should be placed upon the same footing with other officers of the government performing strictly civilian duties.

If astronomers are appointed to the Naval Observatory from civil life to succeed retiring Professors of Mathematics the salaries provided should be sufficient, as recommended by you in that report for 1897, "to make up for the refusal to them of the privilege of retirement and also to secure men of high scientific attainments adequate to the demands of one of the most capable observatories in the world." To secure the services of the ablest astronomers the salaries provided should be slightly larger than those paid in the higher class of university observatories and account should be taken of the fact that university vacations are much longer than leaves of absence from the public service. The Board of Visitors recommends the following as a schedule of salaries which could be expected to attract astronomers of the class desired:

Astronomical Director - - -	$6000
Director of Nautical Almanac -	5000
First Astronomer - - - -	4000
Second Astronomer - - -	3600
Third Astronomer - - - -	3200
Fourth Astronomer - - -	2800
First Assistant Astronomer - -	2400
Second Assistant Astronomer -	2200
Third Assistant Astronomer - -	2000

The experience of every great observatory shows that the efficiency of its staff is materially increased by the provision of quarters near the observing rooms for those persons who are engaged in work by night, and we recommend that there should be quarters provided upon the Observatory grounds for all members of the astronomical staff regularly assigned to night work.

In concluding its recommendations the Board of Visitors wishes earnestly to urge upon your consideration the necessity of making a success of the movement which you have begun in order to improve the condition of the Naval Observatory and to make its administration satisfactory to the great body of the astronomers of the country and to the public.

Some of our recommendations, if they meet your approval, can be carried into effect by departmental action, but the changes which we regard as vital can only be obtained through legislation by Congress.

If such legislation is withheld, the continuance of present conditions is sure to result in a renewed, persistent, and possibly acrimonious demand for the removal of the Observatory from naval control. If, however, the legislation is enacted, and the improved system is given a fair trial, unquestionably much improvement will result, and it is not improbable that the Observatory will attain and hold that high standing in the scientific world which should be required of such an institution.

To help bring about such a desirable consummation we have complied with your request, although not made in pursuance of any law, that we should visit and investigate the Observatory, and we have recommended specific measures which we hope will lead to those reforms in administration which are imperatively necessary if the Observatory is to receive and retain the confidence and support of the astronomers and scientists of the world.

ERRATA.

Vol. IX, No. 3, page 160, line 3, for 3 hours and 15 hours, read 6 hours and 18 hours.

Vol. X, No. 1, page 4, line 5, λ for T Ceti, for 335° 15.0′, read 355° 15.0′.

Vol. X, No. 1, page 107, last line, for [3.5392] read [3.5392_n].

NOTICE.

The scope of the ASTROPHYSICAL JOURNAL includes all investigations of radiant energy, whether conducted in the observatory or in the laboratory. The subjects to which special attention will be given are photographic and visual observations of the heavenly bodies (other than those pertaining to "astronomy of position"); spectroscopic, photometric, bolometric and radiometric work of all kinds; descriptions of instruments and apparatus used in such investigations; and theoretical papers bearing on any of these subjects.

In the department of *Minor Contributions and Notes* subjects may be discussed which belong to other closely related fields of investigation.

Articles written in any language will be accepted for publication, but unless a wish to the contrary is expressed by the author, they will be translated into English. Tables of wave-lengths will be printed with the short wave-lengths at the top, and maps of spectra with the red end on the right, unless the author requests that the reverse procedure be followed. If a request is sent *with the manuscript* one hundred reprint copies of each paper, bound in covers, will be furnished free of charge to the author. Additional copies may be obtained at cost price. No reprints can be sent unless a request for them is received before the JOURNAL goes to press.

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THE

ASTROPHYSICAL JOURNAL

AN INTERNATIONAL REVIEW OF SPECTROSCOPY
AND ASTRONOMICAL PHYSICS

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ROBERT WILHELM BUNSEN.

By HENRY CREW.

THE disappearance of a great leader of men marks, for all of us, a time of reckoning. For we naturally pause to consider what results of the man's work will live after him, what his sources of inspiration were, where lay his power, and even the most optimistic of us will ask, though in no ungracious spirit, whether another leader can be found. In the case of Robert W. Bunsen, who died at Heidelberg on the 16th of August, 1899, this reckoning is one of unusual interest to all students of science.

Each of the physical and natural sciences will probably always be likened to an unfinished structure. But in the case of chemistry we have good reason to think that the structure has at least reached the point where its foundations have been completed. If these foundations were begun by Lavoisier and Dalton, then Bunsen may be thought of as the last of that remarkable group of men, including Wöhler, Liebig, and Dumas, by whom this substructure was completed.

Bunsen was born at Göttingen on the 13th of March, 1811. Here he spent his early boyhood and received his university training, taking his doctor's degree at the age of twenty.

His father was a member of the theological faculty of this place. His education was continued at Paris, Berlin, and Vienna. At the age of twenty-two he began with a privat-docentship at Göttingen that marvelous career of teacher and investigator destined to extend over more than half a century, and to make his name beloved by his own students and a household word for all others. The years between 1836 and 1851 were spent principally in Cassel and Marburg; at the former place he succeeded Wöhler, at the latter he was succeeded by Kolbe. The laboratory which Bunsen started at Marburg in 1840 and that of Liebig at Giessen were among the very first of all those established for the instruction of students. The years from 1851 to 1899 were spent at Heidelberg.

A brief statement of only the more important of Bunsen's contributions to chemistry and related physical sciences would be something like the following:

1. He discovered the first member of a series of organo-metallic compounds, known as the *cacodyls*. It was the investigation of these poisonous, explosive, and ill-smelling compounds that cost him the sight of one eye. The work is reprinted in Ostwald's *Wiss. Klassiker*, No. 27; where its important bearing on the doctrine of radicals is set forth by the editor.

2. By the invention of the Bunsen battery the expensive platinum plate of the Grove cell was replaced by a cheap carbon rod. It is no easy matter for those of us who live in "the electrical age" to realize what a powerful aid this cell was to the teacher and to the investigator of those early days when dynamos were unknown and a current of more than a few amperes was rare and expensive.

3. The Bunsen burner is now in use everywhere from the kitchen to the research laboratory. There seems to be no doubt but that Bunsen was led to this invention purely by his thorough understanding of the principles of combustion. For his original description see *Pogg. Ann.*, **100**, 1856.

4. In 1847, after a trip to Iceland, he gave the only satisfactory explanation ever offered for the phenomenon of the

geyser. By lowering thermometers into the geyser-tube immediately before eruption he discovered the key to the whole situation, namely, that at no point in this column does the water reach its boiling point. When any considerable portion of the column of water is slightly elevated its pressure is correspondingly lowered; its boiling point is thus reduced to a temperature lower than the temperature of the water; the result is sudden evolution of steam and eruption. Much has been said against Bunsen's theory of geysers, but experiment shows beyond all question that the causes assigned by him are sufficient, if not necessary. (*Pogg. Ann.*, **72**, 1847.)

5. The chemistry of blast-furnace gases engaged his early attention and led practically to the creation of that branch of chemistry known as gas-analysis. These general results are embodied in his *Gasometrische Methode*, which appeared first in 1860, with a second edition in 1877. His work along this line is marked above all else by its precision.

6. The chemical action of light he studied in conjunction with Sir Henry Roscoe. This subject, like all others, was taken up in a careful, quantitative manner. Among other interesting results was an accurate measure of the sky-radiation in various latitudes extending from Egypt to Iceland, and for all hours of the day, from morning till evening. This research is well summarized in Watt's *Dictionary of Chemistry*.

7. His ice calorimeter is well known. English readers will find his original description translated in the *Phil. Mag.*, **41**, 161–182, 1871.

8. But the work for which Bunsen will, possibly, be longest remembered is that which he did with Kirchhoff in establishing the branch science of spectrum analysis.

The invention of the spectroscope considered as an instrument of research must undoubtedly be referred to Fraunhofer; and the physical basis upon which spectra are to be interpreted was first distinctly enunciated by Kirchhoff. What then were the contributions of Bunsen to this science?

Ostwald in an editorial note to his reprint of the joint papers

of Kirchhoff and Bunsen (*Wiss. Klassiker*, No. 72, p. 71) gives the following account as coming from the latter author who was then, 1895, still living:

"Bunsen had for sometime been engaged in the study of flames as colored by various salts and had used these colors as a means of analysis.

"For the purpose of distinguishing between flames having apparently the same color, he interposed colored glasses and colored solutions—and with considerable success.

"It was during a conversation on this subject that Kirchhoff suggested to him that the different colors in the flame might be more completely separated by means of a prism; later Bunsen was aided by Kirchhoff in carrying out this idea."

This led to their two joint papers, published in *Pogg. Ann.* in 1860 and 1861, where the question is first raised as to whether each element has a fixed and characteristic spectrum of its own, which is independent of the kind of flame in which it is brought to incandescence, and independent of the compound of which the element may be a component.

After examining the spectra of a large number of compounds in which one and the same element appears as a factor, and after employing a variety of heat-sources ranging from the hot oxy-hydrogen flame to the cold flame of sulphur, they conclude that the positions, at least, of the spectral lines, for any of the alkali metals, are not affected by the remainder of the compound in which the metal appears, or by any range of temperature, or by any of the various chemical processes going on in the flame employed.

Upon this experimental foundation, they established the prism as an instrument whose delicacy, for qualitative analysis, is unsurpassed; and in their second paper applied it to the discovery of two new elements, caesium and rubidium.

The peculiar merit of Bunsen in the field of spectroscopy is then that he perfected a simple method which is capable of detecting the presence of an element by the use of quantities vastly smaller than are required by any other known method.

It opened to chemists, therefore, an entirely new field of investigation and one which, as the sequel has proved, is not confined to our own planet or even to the solar system.

Colored flames had been previously studied by Talbot, Herschel, Swan and Miller, but the work of Kirchhoff and Bunsen was needed to convince chemists that the prism was an instrument both reliable and sensitive.

In this connection, it is interesting to note that it is precisely the deviations from the general principle above mentioned, viz., constancy of position for each line, which form the most interesting subjects of present spectroscopic investigations. Among these may be mentioned the difference between the spectra of various salts of any one metal as determined by Mitscherlich, the shift of position due to motion in line of sight, the shift due to pressure at the luminous source.

Since 1889, Bunsen had not been engaged in active teaching. As a matter of course, learned societies sought to honor themselves by heaping upon him memberships, degrees and orders. These, however, ought hardly to be mentioned on the same page with the genuine admiration which his marvelous skill of hand, his wonderful clearness of thought and his marked originality called forth from the scholars and students of two continents. And even this praise can hardly have a place beside the loving esteem which his simple-mindedness and his warmheartedness gained for him among all his personal friends.

THE WAVE-LENGTH OF THE CORONA LINE.

By C. A. Young.

Professor Campbell's note in the October number of this Journal seems to require a word from me, as having been the original author of the apparent misidentification of the corona line with the "1474" line (λ 5317) of the cromosphere spectrum. I am obliged to say frankly, although with some natural regret, that I see no reasonable ground on which to contest the conclusion of Sir Norman Lockyer and Professor Campbell, that the wave-length of the corona line is 5303, and that the line is not identical with the conspicuous chromosphere line at 5317 as I have hitherto supposed. The spectrum photographs which Professor Campbell has kindly sent me appear to be conclusive on this point; and I presume that those of the English spectroscopist are no less so, although I have not yet seen any detailed exhibition of his data such as would enable one to examine them critically.

The explanation given by Professor Campbell to account for the original mistake is doubtless correct, and if the totality had lasted a few seconds longer at the eclipses of 1870 and 1878 I should have detected the error myself. I had planned to set the micrometer wires on the corona line just before the end of totality, and to watch the reappearance of the chromosphere spectrum, followed by the reëstablisment of the ordinary dark line spectrum. Had I succeeded in so doing, the non-coincidence of the corona line with its much more brilliant chromospheric neighbor could not have possibly escaped notice. But on both occasions totality ended prematurely, while I was still searching for other coronal lines in the violet region of the spectrum.

If the weather is favorable next May the eclipse observer will need at the proper moment only a glance to verify or disprove the identity of the corona line with the old "1474."

It is, I think, worth noting that at λ 5303 there is no line of the solar spectrum, either dark or chromospheric; nor, so far as I know at present, is there at that point any line belonging to the spectrum of any terrestrial element yet investigated. The new determination of wave-length frees "Coronium" from all appearance of puzzling affinities or "entangling alliances" with other bodies less exceptional in their character.

PRINCETON, N. J.
Nov. 4, 1899.

DENSITY OF CLOSE DOUBLE STARS.

By Alexander Roberts.

In connection with the theory of the relation of the temperature and density of stars, it is important to put in evidence what is known of the size, mass, or density of celestial gaseous bodies.

The need for such evidence will be manifest when we consider that there is no final certification of the truth of any law, even if we admit its theoretical soundness, so long as it lacks the complete verification that follows from a comparison of theory with the facts deduced from observation.

As regards mass we, of course, can only ascertain this in the case of double stars whose parallax and orbital elements are known. And to determine the density necessitates further a knowledge of the size of the component stars of the system, and this can only be arrived at indirectly by an examination of the light variation of those binary stars whose orbits lie in the same plane as the line of sight.

In the present paper I would seek to deal with the densities of four Algol stars, the light variations of which have been under observation at Lovedale for several years.

The four stars are:

X	Carinae (*Ch.* 3055)	R.A.	$8^h\ 29^m\ 8^s$	Dec. $-58^\circ\ 53.2'$ (1900)
S	Velorum (*Ch.* 3416)	R.A.	9 29 27	Dec. -44 45.9
—	Centauri (*L.* 5861)	R.A.	14 9 55	Dec. -57 23.3
R S	Sagittarii (*Ch.* 6546)	R.A.	18 10 59	Dec. -34 8.5

Before dealing with the stars severally it is necessary to indicate the general expressions for the density of any system.

Let us employ the following designations:

Periodic time (in years) - - - - -	t
Semi-axis major - - - - - -	r
Eccentricity of orbit - - - - -	e
Diameter of component (1) - - -	pr
Diameter of component (2) - - - -	qr

Mass of component (1) - - - - m_1
Mass of component (2) - - - - - m_2

Then

$$\text{Density of (1)} = \frac{(0.0092)^3}{p^3\, t^2}\left(\frac{m_1}{m_1 + m_2}\right);$$

$$\text{Density of (2)} = \frac{(0.0092)^3}{q^3\, t^2}\left(\frac{m_2}{m_1 + m_2}\right).$$

Putting

$$\frac{m_2}{m_1} = k \qquad \frac{m_1}{m_2} = k_1$$

the expressions become,

$$\text{Density of (1)} = \frac{(0.0092)^3}{p^3\, t^2}\left(\frac{1}{1 + k}\right); \qquad (1)$$

$$\text{Density of (2)} = \frac{(0.0092)^3}{q^3\, t^2}\left(\frac{1}{1 + k_1}\right). \qquad (2)$$

Now $\frac{1}{1+k}$ and $\frac{1}{1+k_1}$ must always be less than unity, indeed only one of the two expressions can ever approach unity, for as k or $\frac{m_2}{m_1}$ approaches zero, k_1 or $\frac{m_1}{m_2}$ must approach infinity.

We have practically, therefore, a limit in one direction to the density of any system, in the expressions

$$\frac{(0.0092)^3}{p^3\, t^2} \text{ and } \frac{(0.0092)^3}{q^3\, t^2}. \qquad (3) \text{ and } (4)$$

The densities of any two stars of a binary system may be very much less; but never can be greater than, or even equal to the values yielded by these expressions.

The quantity t is simply the light period of the variable expressed in years, and p and q are quantities, which, in certain cases, can be determined from an examination of the light variation at minimum phase.

It is evident that in the case where the masses of the two components are nearly equal the densities of either star will be half that yielded by the expressions (3) and (4).

X CARINAE; CH. 3055.

The light variation of this star is extremely regular; the similarity between the increasing and decreasing phases, and the

regularity of the successive minima indicate a more or less circular orbit.

The elements of variation are:

Total period - - - -	13 hours
Constant period - - -	6½ "
Descending period - - -	3¼ "
Ascending period - - -	3¼ "

Considering the several minima to be due to the revolution around one another of two stars of equal brightness and equal size, the orbital period of this close binary will be 26 hours.

These data yield the following values:

t - - - -	0.0030^{y}
p - - -	0.71
q - - -	0.71

Substituting in equations (1) and (2),

$$\text{Density of (1)} = 0.25\left(\frac{1}{1+k}\right);$$

$$\text{Density of (2)} = 0.25\left(\frac{1}{1+k_1}\right).$$

If we take both stars to be of equal mass then the density of either component is equal to

$$0.125$$

or one eighth of the Sun's density.

We have assumed a central eclipse. On any other assumption the density will be less than that just obtained.

S VELORUM; CH. 3416.

I have already in *Astronomical Journal*, No. 327, indicated the nature and theory of the variation of this remarkable Algol star.

Briefly stated the elements of variation are:

Total period - - - -	5^d	22^h	24^m	23^s
Descending period - -		4	10	0
Constant period at min. - -		6	30	0
Ascending period - - -		4	10	0

The normal magnitude of S Velorum is 7.8, and the magnitude at minimum 9.3. During the constant magnitude, 7.8,

we have the combined light of both stars; during the constant period when the magnitude is 9.3 we have only the light of the fainter, but larger companion. That is, the smaller star is four times brighter than the larger.

Assuming a circular orbit for S Velorum we have the following values for a determination of the density:

t - - -	0.01624^{y}
p (brighter star) -	0.17
q (fainter star) - -	0.46.

From which we obtain

$$\text{Density of brighter star} \ldots 0.61 \left(\frac{1}{1+k}\right);$$

$$\text{Density of fainter star} \ldots 0.03 \left(\frac{1}{1+k_1}\right);$$

or maximum limit of density,

Bright star - - -	0.61
Faint star - - -	0.03

The remarkable circumstance about this result is the density of the fainter but larger star. In the case of the smaller star contraction into very much less bulk has evolved four times more light than that emitted by the larger star.

It may be objected to this result that it depends upon an assumed circular orbit.

Considering the orbit as elliptical, and

Eccentricity - - - - - - -	e
Angular distance between line of sight and line of apsides - - - - - -	θ

Then

$$\text{Density of bright star} = 0.61 \left(\frac{\sqrt{1-e^2}}{1+e\cos\theta}\right)^3;$$

$$\text{Density of faint star} = 0.31 \left(\frac{\sqrt{1-e^2}}{1+e\cos\theta}\right)^3.$$

The relation between the diameters of the stars and the axis-major of the orbit precludes a higher eccentricity than 0.5, for any greater value would mean collision at periastron passage.

Also it is evident that the greatest value of the density is obtained when $\theta = 180°$.

Considering now the value of the eccentricity to be the greatest possible, 0.5 ; and eclipse to take place at apastron, then the greatest possible values of the densities of the two stars become

Bright star - - -	3.17
Faint star - - - -	0.16

That both these values are far too great is certain. They assume a high eccentricity. The regularity of the light curve is directly against this assumption. They assume a value of the mass of either star infinitesimally small.

But with every possible combination of mass, position in orbit and eccentricity, the value of the density of the faint star is still exceedingly small.

— CENTAURI ; LAC. 5861.

The light variation of this star is extremely regular, being capable of representation under a simple trigonometrical form.

Its light changes are accomplished in $7^h\ 16^m$. Considering the variation to be due to the revolution of two stars of equal brightness, around one another, this means an orbital period of $14^h\ 32^m$, in which time two maxima and two minima take place.

From the absence of any stationary period at either the maximum or minimum epoch, it would appear that the two stars revolve around one another almost in contact.

Although it is impossible to grant under these conditions a spheroidal shape to either star, still the departure from this figure will not be so great as to vitiate the following results.

[illegible] - - - -	[illegible]
[illegible] - - - - -	1.0
[illegible] - - - -	1.0

from which we obtain as the density of the two components of — Centauri

First star . . . [illegible] $\left(\frac{[illegible]}{1 - [illegible]}\right)$.

Second star . [illegible] $\left(\frac{[illegible]}{1 - [illegible]}\right)$.

Assuming the masses of the two stars to be equal we have as the maximum possible value of density of either star,

$$0.14.$$

This result is based on the assumption of a central eclipse; any other assumption will only diminish this quantity.

R S SAGITTARII; CH. 6546.

A full investigation of the orbital elements of this remarkable Algol variable is given in *A. J.*, No. 373.

More extended observations have only slightly modified the conclusions reached in that paper.

The form of the light curve is extremely regular, the elements of variation being

Full period - - - - -	$2^d\ 9^h\ 58.6^m$
Duration of Chief Min. - - -	10 40.
Duration of Secondary Min. -	7 0.

Secondary minimum takes place almost midway between two chief minima.

From these data we obtain the following values:

Periodic time - - - - -	0.0066^y
Eccentricity - - - - - -	0.25
p - - - - - - - -	0.48
q - - - - - - - -	0.44

The larger of the two stars is twice brighter than its smaller companion.

The values just given yield the following densities:

$$\text{Bright star} \ldots 0.16\left(\frac{1}{1+k}\right);$$

$$\text{Faint star} \ldots 0.21\left(\frac{1}{1+k_1}\right).$$

As already indicated $\frac{1}{1+k}$ and $\frac{1}{1+k_1}$ must under any relative condition of mass be less than unity, so that 0.16 and 0.21 represent the greatest possible densities of the two stars which make up R S Sagittarii.

In the case of the masses of the two stars being nearly equal, these values will be reduced one half.

GENERAL RESULTS.

It will be sufficiently apparent that a definite determination of the density of any one of the stars discussed in this paper is an impossibility in the present state of our knowledge; we must content ourselves with being able to indicate a limit in one direction to the values sought. Precise values cannot be secured so long as the inclination of the orbit and the relative masses of the two components remain unknown.

Gathering the results together we find for the densities

X Carinae	0.25	and	0.25
S Velorum	0.61	"	0.03
— Centauri	0.27	"	0.27
R S Sagittarii	0.21	"	0.16

These are absolute maximum values of the density of the stars dealt with, assuming in the case of S Velorum a circular orbit, an assumption in keeping with the facts of variation.

We may push the investigation one step further and deduce the mean average densities of all the stars, and in this case it will be sufficient to consider k and k_1 equal; their inequality will have very little effect on the average of all the values.

The average density of the four systems dealt with in the foregoing paper is

$$0.13.$$

That is, the average density of a close Algol variable is only one eighth that of the Sun.

This result is sufficiently striking, but it is in no way new. The low density of close double stars has been indicated by more than one astronomer. Any value which the preceding investigation has does not depend upon its novelty but on its relation to the laws connecting stellar temperatures and densities. And it is as a contribution to any discussion which may arise as to the validity or application of such laws that the investigation is offered.

LOVEDALE, S. Africa
April 1899.

THE DENSITIES OF THE VARIABLE STARS OF THE ALGOL TYPE.

By HENRY NORRIS RUSSELL.

IT is possible in the case of an Algol-star, assuming the eclipse theory of its variation, and a circular orbit, to deduce a limiting value for its mean density from its period and the duration of its light-variation, any uncertainty as to the result being due to the uncertainty in the ratio between these two quantities, which is, as a rule, not yet very accurately determined,

Let r be the radius of the bright component, r' that of the dark, and a the radius of the orbit. Let t be the period, and d the duration of the light-variation, that is, of the eclipse, and let m be the total mass of the system.

The mean density ρ' of the system is the quotient of its mass by its total volume; that is

$$\rho = \frac{m}{\frac{4}{3}\pi(r^3 + r'^3)}. \qquad (1)$$

Now

$$r^3 + r'^3 \geqq \frac{1}{4}(r + r')^3, \qquad (2)$$

the sign of equality holding only when $r = r'$.

When the eclipse begins or ends, the disks of the two stars are apparently tangent, and the distance of the projections of their centers upon a plane perpendicular to the line of sight is $r + r'$. Between the beginning and middle of the eclipse the stars move through an arc of their orbit equal to $\frac{\pi d}{t}$, and the relative displacement of their projections is $a \sin \frac{\pi d}{t}$. But this displacement is equal to the projection of the distance of their centers at first contact upon the line of relative motion. Therefore

$$r + r' \geqq a \sin \frac{\pi d}{t}, \qquad (3)$$

the sign of equality holding only when the transit is central.

We have also

$$m = K\frac{a^3}{t^2} \tag{4}$$

where k is a constant depending on the system of units used.

Combining (1), (2), (3) and (4) we have

$$\rho \leq \frac{3K}{\pi t^2 \sin^3 \frac{\pi d}{t}} \tag{5}$$

where the sign of equality is to be used only in the case of a central eclipse of equal stars. This formula gives a superior limit for the mean density of the system independent of its dimensions.

It remains to determine the constant $\frac{3K}{\pi}$. The period of a particle revolving close to the Earth's surface is 1.411 hours. That of two Earths revolving just out of contact would be twice this. The eclipse of one of these bodies by the other would occupy half the period.

The sign of equality may in this case be used in formula (5) and, taking the Earth's density at 5.53, we obtain $\frac{3K}{\pi} = 44.1$ where the unit of time is one hour, and the unit of density that of water.

The results of the application of the formula (5) to the seventeen known Algol-variables are given in the following table :

No. in Chandler's Catalogue	Name	t in hours	d in hours	Limit of density
320	U Cephei	59.82	10.	0.098
1090	β Persei	68.81	9.15	0.139
1411	λ Tauri	94.87	10.	0.142
2610	R Canis Maj.	27.26	5.	0.366
3055	X Carinae[1]	13.00	6.5	0.261
3109	S Cancri	227.63	21.5	0.035
3416	S Velorum	142.40	15.2	0.061
5374	δ Librae	55.85	12.	0.058
5484	U Coronae	82.85	9.75	0.137
5949	R Arae	106.21	10.3	0.145
6189	U Ophiuchi	20.13	5.1	0.298
6442	Z Herculis[2] *	95.82	5.3	0.728
6546	RS Sagittarii[3] *	57.98	10.4	0.086
7399	W Delphini[4]	115.35	10.	0.170
7488	Y Cygni *	71.90	8.	0.212
	DM.+12° 3557[5]	21.35	5.	0.320
	B.D.+45° 3062[6]	109.75	13.	0.076

The data are those of Chandler's *Third Catalogue of Variable Stars*, except as noted.

It is possible that the gradual change of brightness near the beg nning and end of the light oscillation is produced by some other cause than the interposition of an opaque body. If this is true, the tabulated values of d are greater than the true intervals between the contacts at eclipse, and the calculated limits of density are consequently too small. In the case of Algol, Vogel[7] concludes that the real duration of eclipse is about 6½ hours, and so finds a density about twice as great as the limit found in this article upon the assumption of an eclipse lasting 9.15 hours.

Since it is improbable that the orbits of these systems are actually circular, the numerical values in the table are only rough approximations to the truth. An eclipse near periastron would occupy less time than if the orbit were circular, and the calculated limit of density would be too great. The reverse

* Secondary minima.

[1] Roberts, *A. J.*, 383.

[2] Dunèr, this Journal, **1**, 285.

[3] Roberts, *A. J.*, 373.

[4] *Harvard Observatory Circular* No. 5.

[5] Sawyer, *A. J.*, 450.

[6] Ceraski, *A. N.*, 3572.

[7] *A. N.*, 2947.

would be the case at apastron, since the computed limit of density varies as the cube of the relative velocities of the stars.

Since in an elliptical orbit the velocity is greater than in a circular orbit of the same period for more than half its circumference, and since the velocity at periastron is increased above the normal in a greater ratio than it is diminished at apastron, the mean value of the limit of density computed by formula (5) for a considerable number of stars would be increased by the effects of eccentricity above its true value.

The stars Z Herculis, Y Cygni and R S Sagittarii have secondary minima. By taking the mean of the durations of the two minima—as has been done in the table—the eccentricity is nearly eliminated, and since Dunèr[1] and Roberts[2] have shown that in these systems the components are almost equal in size, and the transits almost central, the limits of density given above are probably close to the true values.

Notwithstanding the causes of uncertainty, it is evident that the Algol-variables as a class are much less dense than the Sun, probably less than one fourth as dense. If these stars consist of a nucleus and an extensive luminous atmosphere, the nuclei may, of course, be much denser.

PRINCETON UNIVERSITY,
October 9, 1899.

[1] This JOURNAL, 1, 285. *A. J.*, 265.

[2] *A. J.*, 373.

NOTE ON THE SPECTRUM OF P CYGNI.

By A. Bélopolsky.

I have very recently obtained several spectrograms of the well-known star P Cygni (mag. 5.0) with our two-prism spectrograph, to which a camera of only 250 mm focus was attached. The spectrum extends from F to $H\gamma$, and the plates show the comparison lines of *H*, *Fe*, *Sn*, and *N* (air lines). As is well known, the lines in the spectrum of this star are both dark and bright; this is particularly the case on my plates for the *He* line at λ 4472 and for the hydrogen lines. The bright lines are about in coincidence with the artificial lines, while the dark lines are sharp (particularly λ 4472) and strongly displaced toward the violet end of the spectrum.

But my attention was especially struck by the fact that a dark and bright line correspond in the star's spectrum to each of the comparison air lines of the group from λ 460 $\mu\mu$ to 465 $\mu\mu$, with possibly one exception. The bright lines are faint and not measurable, but they nearly coincide in position with the artificial lines. The dark lines are displaced toward the violet. I give in the following table the wave-lengths reduced by Hartmann's formula of the lines measured on the plate of September 21 and 22, 1899. Column *B* gives the bright lines, *D* the dark lines, *C* the artificial comparison lines, and *S* the wave-lengths, according to Rowland, Runge, and Neovius.[1]

A repetition of the measurements of the differences between the comparison lines and the dark lines of the group λ 4607 to 4651 gave these results:

λ	Difference	
	Revs.	Tenth-meters
4607	—0.185	—1.56
4622	—0.145	—1.22
4631	—0.188	—1.58
4642	—0.175	—1.48
4651	—0.133	—1.12

[1] "Om skiljandet af Kväpvets och syvets linier." Stockholm, 1891.

P CYGNI. SEPTEMBER 21 AND 22, 1899.

B	*D*	*C*	*S*	*El*	Remarks
		4308.0	4308.0	*Fe*	
		4326.0	4326.0	*Fe*	
4340.8	4338.1	4340.6	4340.7	*Hγ*	Very bright
4345.7	4344.8		4345.4	*O?*	Very fine line
		4383.8	4383.7	*Fe*	
	4386.7		4388.1	*He*	
4396.1					
		4404.9	4404.9	*Fe*	
4420.1					
4472.1	4469.6		4471.8	*He*	Brightest line of all
		4525.0		*Sn*	
	4551.7		4552.6	*N?*	
	4561.7				
4601.9		4601.5	4601.3	*N*	
	4606.2	4607.2	4607.2	*N*	
		4613.9	4614.2	*N*	
	4620.3	4621.5	4622.0	*N*	
4631.5	4629.2	4630.8	4630.9	*N*	Strongest of the air lines
	4640.8	4642.7	{ 4640.5 4643.4	*N*	Comparison line double
	4648.3	4649.2	4651.0	*N*	O : λ=4649.2 ; Fe : λ=4647.6
4713.5	4711.0		4713.3	*He*	
4861.6	4858.2	4861.5	4861.5	*Hβ*	Very bright

The corresponding differences for the hydrogen lines appear somewhat greater: for F — 3.4, and for $H\gamma$ 2.7 tenth-meters. This is certainly to be explained by silver precipitation of the bright lines overlying the dark lines at their edges, so that the observer makes his settings on the edge of the bright line, and not on the dark line.

No noteworthy changes from the above plate occur on the seven spectrograms of the star which I obtained between September 21 and October 1.

PULKOWA,
October 1899.

APPARATUS AND METHOD FOR THE PHOTOGRAPHIC MEASUREMENT OF THE BRIGHTNESS OF SURFACES.[1]

By J. HARTMANN.

ON a photographic plate which has been properly exposed and developed, the blackening of the different portions of the picture corresponds directly to the photographic brightness of those points of the object photographed (the phenomenon of solarization being left out of account), and we may therefore conclude as to the brightness of the object from the degree of blackening. In order to carry out such photographic determinations of brightness in a precise manner the following points must be considered. We shall here always understand the brightness of an object to be the sum of those rays of light effective on the plate employed, which, after leaving the object and passing through the intervening media, actually reach the sensitive film. In each case it will be necessary to determine by special investigation to what extent the rays emitted by the object, for which the plate in question is sensitive, are lost by absorption in the air, as well as in the lenses and other optical parts of the photographic apparatus, which will depend upon the arrangement of the experiment. This definition of the photographic brightness corresponds exactly to that of the optical brightness as measured with ordinary photometers. For also in the case of optical measurements the intensity is determined of only those rays which actually pass from the source to the eye of the observer, and for which this eye is also sensitive. We should therefore expect differences in the determination of intensity between the eyes of different observers, as well as between different kinds of plates, particularly in the case of differently colored surfaces, and only measurements with the spectral photometer, whether they are made visually or photographically, are independent of this subjective definition of brightness.

[1] *Zeitschrift für Instrumentenkunde*, **19**, April 1899.

The first difference in principle between the photographic and the optical method arises from the property of the photographic plate that its blackening does not depend solely upon the intensity of the light falling upon it, but also in an equal degree upon the duration of exposure, upon the sensitiveness of the film, and upon the mode of development. It is true that within certain limits the correctness of the Bunsen-Roscoe law, according to which the blackening is proportional to the product of the intensity and the duration of exposure, has been also proven for bromide of silver gelatine plates. A law true only for special cases, the validity of which must first be proven by photometric measurements, cannot, however, be made the basis of such measurements, and for this reason photometric methods should be discarded which employ the different exposures of different parts of the photographic plate as a measure of brightness. For the determination of *light intensities*, use should not be made of rotating disks, with sectors of different sizes cut out of them, which are very suitable for sensitometer comparisons, in which the question is the determination of those *exposures* which produce equal blackening on different plates.

The only principle which can be made the basis of a photographic-photometric method, free from unnecessary assumptions, is this: Two sources of light are photographically equally bright when they produce equal blackening on one and the same plate with equal exposures.

We assume here only that every plate has the same sensitiveness over its whole surface, and that the development, and other treatment of the plate, have been precisely the same for all different points. If we should not make these two assumptions, the photometric utilization of photographic plates would be entirely impossible.

If any source L is to be compared with a standard lamp N, a portion of the plate is illuminated by L for a definite time from an accurately measured distance; at a neighboring point on the plate the scale is produced, the separate fields of which are obtained by precisely the same exposure at different distances

from the standard lamp. It therefore only remains to determine to which field of the scale the blackening produced by the source *L* corresponds, in order to calculate accurately, according to the fundamental law of photometry, the brightness of *L* expressed in units of the standard lamp. In exactly the same way the separate parts of a picture taken with any kind of photographic apparatus can be measured photometrically by a normal scale produced on the same plate, but it is to be remarked here that the brightness of the picture thus found is not to be immediately considered as the brightness of the object, as mentioned above.

The process here given makes it possible to measure photometrically whatever can be obtained photographically; in particular, objects can be rapidly measured in this way which have hitherto escaped direct photometric observation on account of their faintness, their rapid changes, or other reasons. I will only recall how laborious any accurate comparison of the brightness of the different formations of the Moon's surface has been hitherto, on account of the constantly changing illumination, while on a photograph of the Moon, which can be obtained in a second, the instantaneous distribution of light can be preserved and afterwards measured with great accuracy, at as many points on the image as may be desired. Similarly the brightness of comets and nebulae is easily determined in this way.

The observation proper in this photometric method lies in the comparison of the picture with the fields of the scale which are found on the same plate. In sensitometer measurements such a comparison has to be made, but then the two exposed places are on two different plates. Hitherto this observation has commonly been obtained by placing the scale near that portion of the plate where the blackening is to be measured (by cutting the plate if necessary), and then estimating to which step of the scale the blackening corresponds. This process, however, is neither very reliable nor always applicable. If the place which is to be measured lies between regions of different degrees of blackening, or if the plate cannot be cut, then the scale cannot

be brought sufficiently near, so that the estimation becomes very uncertain. Even when it is possible to bring into contact the two fields which are to be compared the process is not wholly

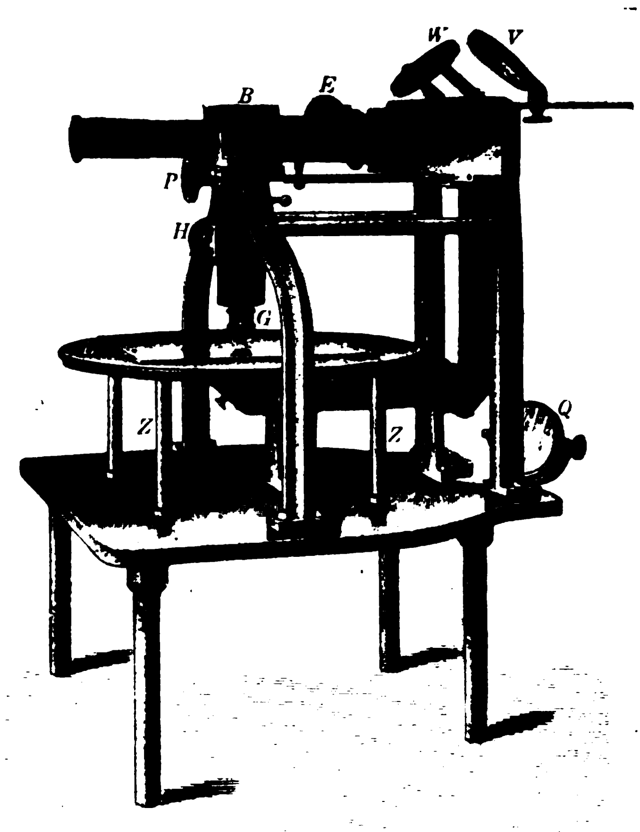


FIG. 1.

free from objection, since a peculiar optical phenomenon complicates the precise estimate of the blackening. On all scales which consist of a series of fields of different blackening, in close succession, each field will appear to be shaded off, and is the darker the nearer it comes to the neighboring brighter field, although

the blackening within each field is known to be perfectly uniform from the mode of their production. The boundary of this neighboring field may be recognized as a line of especial darkness. It follows from this that in judging the brightness of surfaces the eye is in a high degree influenced by the brightness of the adjoining surfaces. In the exact measurement of surface brightness it is therefore essential that the area to be observed is entirely isolated from its surroundings.

These were the guiding principles in the construction of the apparatus described below, which may be designated as a microphotometer, since it is a combination of microscope and photometer. The apparatus was built for the Astrophysical Observatory by O. Toepfer, of Potsdam, in an admirable manner, and is intended to be used for the photographic measurement of the surface brightness of celestial bodies and their spectra.

Fig. 1 gives a perspective view of the instrument, and Fig. 2 a vertical section. A round table of 25 cm diameter is supported by four posts *Z*, parallel to a horizontal base plate *U U'*. Its surface *L*, is covered by a dull black plate of ebonite, and it is sufficiently large so that the edges of large plates can be brought over the aperture in the middle of the plate without the necessity of fastening the plate. Vertically above the aperture is situated the objective of a broken microscope *A B G*, the optical parts of which are so calculated that the sharp image of the granulations of the plate can be thrown in the middle of the base of the rectangular reflecting prism *B*, on adjusting the objective by the rack and pinion *H*. The magnifying power is 12.

The prism has the arrangement proposed by Lummer and Brodhun,[1] which may be seen more clearly in Fig. 3. To the base *a b* of the prism *B* a second precisely similar prism *C* is cemented in such a way that reflection can only occur at the small portion *g h*, while all about this portion the rays pass unhindered from one prism to the other. This is accomplished either by making a slight depression in the middle of the base of the prism *C* before cementing or, more simply, by first silvering the portion

[1] *Zeitschrift für Instrumentenkunde*, 9, 41, 1889.

selected on the base of *B* and then cementing the prisms. It is possible to look directly through the cube thus produced in the direction *A B D* (Figs. 1 and 2), while the rays coming from *G* are seen in the small mirror in the middle. The form and size

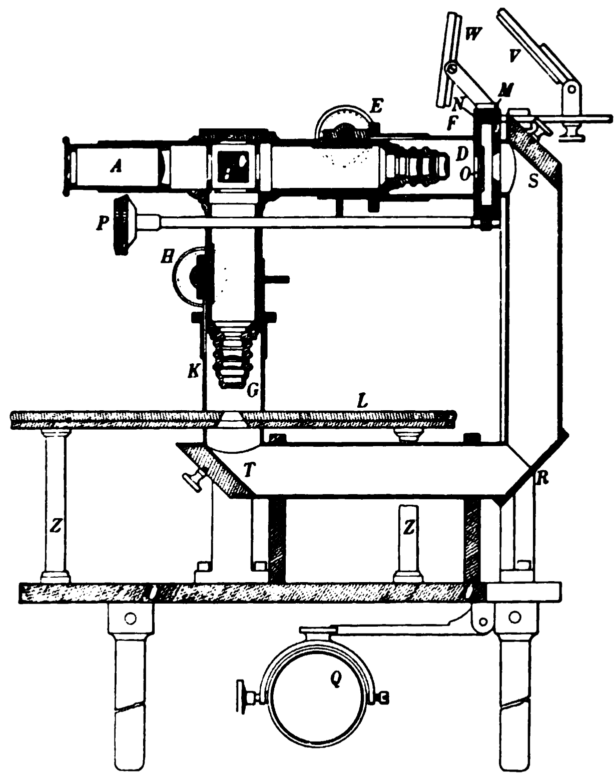


FIG. 2.

of this mirror will be governed by the nature of the object to be measured, and various combinations of prisms can be furnished with the apparatus. If circular regions are to be observed on the plate under investigation, the mirror will have the form of an ellipse, the major axis of which, lying in the direction of *g h*, is in the ratio of 2 : 1 to the minor axis; the sides of the

rectangular aperture should be given the same ratio if squares are to be measured on the plate. The apparatus furnished the Astrophysical Observatory permits the measurement of the blackening of regions of precisely circular shape and having a diameter of 0.12mm.

A second microscope objective precisely similar to *G* is placed at *D* and can be so focused by the pinion *E*, on the film of the plate situated at *O* (Fig. 2), that this image also falls at the middle of the double prism. On looking into the eyepiece *A* there will be seen in the middle of the field of view a small portion of the plate lying on the table *L*, while the remainder of the field of view is filled by the plate *O*.

The plate *O* is the measuring part proper, and may be designated as a photometer wedge produced photographically. It is 90 mm long and 20 mm wide, and in the direction of its length the film is given a blackening which increases as uniformly as possible. The construction of this wedge, to be made on the same kind of plate as the plate to be measured, if possible, can be accomplished in different ways, as by illumination behind a rotating disk, from which portions bounded by spirals have been cut out; or in a region of penumbra; or by the uniform withdrawal of the cover from in front of the plate during its exposure; or by copying another similar wedge. The law of the increase of the blackening in the wedge is practically a matter of indifference, but there must be no irregular jumps.

The wedge is fastened in the slide *N* (Fig. 2), which can be moved in the frame *M*, perpendicularly to the axis of the microscope by the pinion *P*. The position of the slide in the frame can be read on a millimeter graduation with vernier, which is illuminated by the two mirrors *V* and *W* and is situated at the distance of distinct vision for the observer at the eyepiece. The wedge is fastened in the slide by means of two stops perpendicular to each other and by springs. It can be reversed at will, which is important for the elimination of certain errors, or can be exchanged for another wedge. The film side of the plate is of course always toward the microscope objective.

In the construction of the apparatus particular attention was paid to the equal illumination of the two plates. Measurements can be made either by daylight or by artificial illumination. If diffuse light of the sky is employed, or, for very dense blackening, direct sunlight, the rays are thrown perpendicularly on the ground-glass plate *R*, by the mirror *Q*,[1] which may be turned at will, and pass from *R* after emerging in both directions at an angle of 45°, through closed tubes to the plane mirrors *S* and *T* which then give vertical illumination to the two photographic plates. In place of the mirror *Q* a lamp may be placed so as to throw its light perpendicularly on the plate *R*; this form of illumination has proved most satisfactory in my measurements. Variations in the brightness of the source of light have no effect whatever on the measurements, but nevertheless precautions should be taken that the direction from which the plate *R* is illuminated is not changed during a series of measurements—which may easily occur with the employment of direct sunshine.

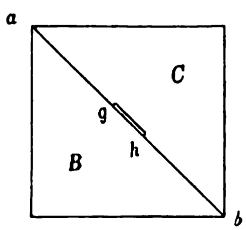


Fig. 3.

The mirrors *S* and *T* can be readily removed from the tubes for cleaning. In order to avoid all diffuse light the sliding tubes *F* and *K* are attached around the objective ends of the microscope. *F* is brought into contact with the frame *M*, and *K* is moved to within a few tenths of a millimeter from the plate to be measured.

As appears from the above description, the path of light from the ground-glass disk *R* to the eyepiece is completely enclosed and precisely similar for the two microscopes. In spite of this equality of the two optical systems, the apparatus is not intended to serve for the direct comparison of two blackened portions, but the two portions to be compared are successively brought under the microscope objective *G*, on the horizontal table, and

[1] The mirror then takes the position shown in Fig. 1. When lamplight is used the arm carrying the mirror is turned about the joint at *U'*, so that *Q* comes into position under the base plate, as given in Fig. 2.

compared with the wedge. In this way the result is attained that the two surfaces to be compared are observed under perfectly identical conditions.

The measurements with this apparatus are accordingly made in the following manner: after an appropriate wedge has been placed in the slide and the illumination regulated so that the ground-glass disk *R* is perpendicularly illuminated, the observer first focuses the eyepiece sharply on the edge of the small mirror in the double prism, draws the tube *F* out to the frame *M*, and focuses on the grain of the wedge with the pinion *E*. The plate to be measured is then laid upon the table, and the observer focuses upon it with the pinion *H* and slides the tube *K* close up to the plate. The measurements can then begin. The plate is then moved upon the table in such a way that the first field of the scale on the plate appears in the small mirror, and the wedge is moved by turning the head *P* until this place in the middle of the prism has exactly the same appearance as its surroundings. The equalization of the two surfaces can be accurately accomplished and sharply determined, since the dividing line disappears entirely at the correct setting. The setting of the scale is then read off and the next field of the scale is taken up. A series of measurements thus obtained then furnishes a table of the settings of the wedge which correspond to the several fields of the scale and accordingly to the known brightnesses of this plate. After this the picture taken on the plate is measured in the same way and the readings of the wedge thus obtained serve for interpolating from the table the brightness corresponding to the degree of blackening in units of the lamp employed.

I have only to refer to another attachment intended both for rapidly finding the place to be observed in the picture and for making a preliminary test of the plate. If the area of blackening which is to be measured is very small and situated in an irregular area of different brightness, it would be difficult to set the desired portion of the plate exactly at the reflecting surface in the middle of the prism, since all of the surrounding

parts of the plate cannot be seen in the eyepiece, so that the orientation is entirely lost. In order to render possible a rapid and exact setting of the desired object in this case, the double prism *B C* can be removed by pressure on a button, and in its place a simple reflecting prism, similar to *B*, can be brought up, which permits the greater part of the plate to be seen. A curve drawn on the base of this prism and corresponding to the outline of the reflecting surface in the double prism then bounds that part of the plate in the field of view which alone can be observed after replacing the double prism. The simple prism and double prism are attached beside each other in a slide which can be moved back and forth between stops, so that a change of prisms can be made in less than a second. In Fig. 1 the slide may be seen pushed out to its right hand stop.

In the form thus described the apparatus serves for the direct comparison of the density of the silver precipitation, since the magnification employed clearly shows the grain of the more sensitive bromide of silver plates. If the grain of the two plates to be compared is very different, however, as may be the case in sensitometer measurements, it is better to compare the transparency of the two plates. For this purpose we may either screw on in place of the objectives *D* and *G*, two equal diaphragms furnished with the apparatus, which may be brought close to the film, as is sufficient in many cases, or we may disturb the focus by the pinions *E H* until the grain disappears. Measurement with a sharp focus is to be preferred in general, since it is here easy to recognize with the use of the small prism, whether the area focused upon is uniformly blackened or whether its transparency is affected by specks or holes in the film.

Measurements can be made with extreme rapidity and ease with this apparatus, and I beg to give an example of their accuracy.

Each of three observers made ten settings on one and the same portion of a scale and obtained the following readings of the setting of the wedge.

Observers:	Müller	Kempf	Hartmann
	37.8 mm	37.6 mm	37.7 mm
	37.7	37.5	37.7
	37.9	37.4	37.4
	37.4	37.4	37.7
	37.4	37.4	37.8
	37.4	37.6	37.6
	37.4	37.5	37.7
	37.8	37.6	37.6
	37.6	37.6	37.7
	37.7	37.8	37.7
Mean	37.61	37.54	37.66

In order to determine to what range of brightness the displacement of the wedge by 1 mm corresponds at the portion here used, the two neighboring fields of the scale, the coefficients of absorption of which had been previously determined, were also compared, with the following result:

Setting of wedge	Log I	C
40.50 mm	—0.534	
37.66	—0.632	—0.035
35.10	—0.728	—0.038

Log I is the logarithm of the quantity of light transmitted by the particular portion of the scale, the quantity of incident light being called unity. C is the range of log I for displacement of the wedge by 1 mm. If we place $C = -0.0365$ as a mean value, for the portion of the wedge used above, 1 mm of the wedge will here correspond to a range of brightness of 8.77 per cent., or, expressed astronomically, to 0.091 magnitudes.

The differences between the above results of the three observers are accordingly:

M.—K.	+ 0.07 mm	0.61%	0.0064 magnitudes
K.—H.	− 0.12	1.05	0.0109
H.—M.	+ 0.05	0.44	0.0046

The largest difference occurring between two settings by a single observer was:

for M.	0.5 mm	4.4%	0.046 magnitudes
K.	0.4	3.5	0.036
H.	0.4	3.5	0.036

while for the probable error of a single setting of the wedge the following figures are obtained:

for M.	0.133 mm	1.17%	0.012 magnitudes
K.	0.085	0.75	0.008
H.	0.072	0.63	0.007

Further measures will be elsewhere published.

KGL. ASTROPHYSIKALISCHES OBSERVATORIUM,
Potsdam.

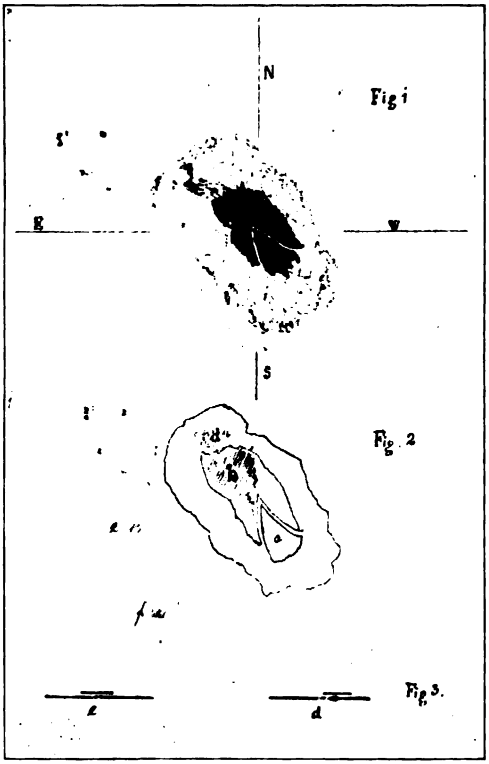
THE GREAT SUN-SPOT OF SEPTEMBER 1898.

By J. FÉNYI, S. J.

THIS phenomenon, remarkable enough on account of its enormous size, receives a special significance from the very unusual aurora which accompanied it. Even at Kalocsa, in latitude $+46° 32'$ and longitude $18° 58'$ east of Greenwich, the aurora was observed for about a quarter of an hour at 9^h Gr. M. T. on September 9. An aurora is here an almost unheard of occurrence, one like this not having been seen in this section, even at a solar maximum, since 1872. It therefore does not seem superfluous to communicate the observations of the spot made at Kalocsa.

I made drawings of the spot on twelve days of its transit. The accompanying drawing was at 10 A.M. Gr. M. T. on September 5, with the helioscope attached to the 4½-inch telescope. From the whole number of observations with projection apparatus I determined the heliographic longitude as 241.7°, and the latitude as —12.5°. According to these determinations the spot passed around the east limb of the disk on September 2 at 8^h A. M.; when I made the regular observations of spots with the projection apparatus at $1^h\ 45^m$ the spot was not yet visible, but as I showed the Sun to a visitor at $3^h\ 45^m$ the strong dark line on the eastern edge at once attracted attention. According to computation the spot was then at a distance of 3.3″ from the edge of the disk, without regarding refraction. I at once applied the spectroscope and now found the place, which at the regular observations at $1^h\ 30^m$ P. M. showed nothing unusual, to be already in an eruptive state. The eruptive line at $\lambda 6677$ was visible for an extent of 11°, and from position angle 119° 30′ to 121° 0′ the chromosphere appeared extremely bright and covered with the familiar flames. This point, however, did not lie over the nucleus of the spot indicated by the dark strip, although very near to it, being about one half degree nearer to

the equator, as I was able to observe directly in the spectroscope with certainty. At the nearest place to the nucleus there appeared a significant displacement of the spectral line toward the red. This was measured at $4^h\ 55^m$ and indicated a recession of 470 km per second. While I was showing all this to my well-



informed visitor, a protuberance rose up at 118°, which was at first also visible in the line λ6677. It was inclined toward the equator at an angle of 60°, thus directed away from the spot region, and at $5^h\ 3^m$ had already attained a height of 128″. Further observations were impossible. At no time, either on September 2 at 2 P. M., or at three observations on the 3d, could I see a prominence over the spot region,—a fact which has

already been reported by others. I find in this only a confirmation of the known fact that quiescent prominences do not occur over areas of spots—which are the scene of transitory eruptive phenomena.

As to the further development of the spot I would only remark that the nucleus made a rotation of about 60° in the positive direction from the 5th to the 12th of September, and that the group did not develop strongly until the 9th. The observations of September 5 seem to me to be of especial significance. Between 11 and 12 and again at 1, I let the spot run over the slit. The *Ha* line was very bright over a portion of the nucleus and out beyond it. I could readily recognize the parts of the nucleus, and sketch the form of the red mass. Fig. 2 is intended to represent the outline of the spot and the parts of the nucleus. The reversed portion is indicated by the shading. This appeared at the same time to be brown in the helioscope, in contrast to the bluish-black color which spot nuclei usually have in my refractor. This reversal of the *Ha* could also be seen beyond the spot at a distance of a few degrees, particularly in the places marked *e* and *f*. The location of the latter were not accurately fixed by measures, but hastily estimated in the field of view by the eye. These two points *e* and *f*, as well as *d*, near the nucleus, exhibited the rare phenomenon of a considerable and regular displacement of the dark *Ha* line on the Sun's disk. The peculiarity of this disturbance is best shown by reproducing the observed forms themselves, as they are given in Fig. 3, with the letters in question designated. The shaded portions denote the bright reversal which occurred simultaneously. If we should seek an explanation of these bright reversals, probably the simplest would be the assumption at these points of violent metallic eruptions, which are known to be extremely luminous, and thus could overpower the photosphere in monochromatic light. At these bright places in fact the dark line was not merely enlarged and broken, but the place was brighter than the surrounding spectrum, especially over the nucleus. We are thus also obliged to transfer the origin of eruption within the

spot to the very bright bridge there, for the reversal did not extend beyond the limits of the spot.

The very remarkable displacement of the dark *Ha* line and its doubling on the solar disk is confirmed by the measurement near *d*, where it corresponded to a recession of 108 km. These peculiar appearances were seen on this day only, from 10^h to 12^h, nothing of the sort being visible on the days following. I find it expressly noted in the observing book that no unusual phenomena and no reversals were visible at three times of observation on September 6th ; on the afternoon of the 9th, the day of the aurora, no reversals were seen; and similarly on the 12th and 14th. The spot disappeared on the 15th, and on the particular point on the Sun's limb could be seen only the bright flames and stripes which as a rule characterize the departure of a region of spots.

The simultaneous occurrence of such extraordinary phenomena of a similar nature at a considerable distance from the spot area justifies the conclusion that the cause and source of these powerful disturbances, as well as the common rotation of the group of spots, are not to be sought in a comparatively shallow layer of the surface, but rather in decided depths of the body of the Sun.

In the course of the year 1899 I have otherwise observed such a reversal on the disk only on March 12 and 14, and then less strongly marked. It was situated over the last spot of the large group then visible. A great magnetic disturbance was also reported at that time on March 14. A close relation between these two phenomena is indicated by the fact that according to the magnetic observations at Stonyhurst only two very large magnetic disturbances have occurred during this year, and that both fell at the time when I observed these extraordinary phenomena over the spots — each time following a few days thereafter. This is also confirmed by the magnetic observations at Pola, which registered at these times the only great disturbances in declination.

HAYNALD OBSERVATORIUM.
June 1899.

A SPECTROSCOPE OF FIXED DEVIATION.[1]

By Ph. Pellin and André Broca.

It is well known that when a ray of light has traversed a prism at minimum deviation its direction, after reflection on a mirror attached to the prism, is independent of its color. This is readily understood when it is remembered that the direction of a ray which has traversed a prism at minimum deviation is the same as if it were reflected from its base. If it is then made to undergo reflection the result will be the same as if the ray were reflected on two mirrors making an angle with each other. The final ray will make with the incident ray an angle double that between the two mirrors.

This principle has frequently been employed in bolometric researches where a definite dispersion and a fixed receiving apparatus were required. To realize these conditions it is only necessary to rotate a system composed of a prism and an attached mirror in order to cause the dispersed radiations, at minimum deviation, to pass in succession over the bolometer.

In this form the apparatus is not suitable for practical use in a spectroscope, for silvered glass mirrors give a bad image and metallic mirrors are soon unfit for use. It is therefore necessary to devise a prism giving total reflection on one of its interior surfaces in such a manner as to realize a fixed deviation without the use of a perishable mirror. It must be shown at the outset that under these conditions the property of fixed deviation for a ray twice refracted at the desired angle is preserved.

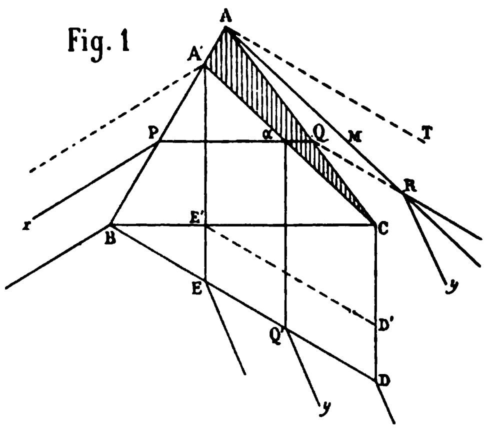
Imagine a prism of 60°, ABC, and a mirror M (Fig. 1). Consider the ray $xPQRy$, and let us endeavor to replace the external reflection M by a total reflection. The emergent ray must take the direction Ry; since we may replace QR by its symmetrical with respect to M, and since we wish to preserve

[1] *Séances de la Société Française de Physique*, No. 1, 1899.

the same angle of refraction Q, we must take as the surface of emergence a surface symmetrical to $A\ C$ with respect to the mirror M. We must therefore take a face parallel to $A\ T$. Under these conditions it is evident that the ray which is twice refracted at the same angle at P and Q' will have precisely the same properties as if it had been refracted at P and Q in the prism $B\ A\ C$.

Let us now consider a 60° prism to be cut by a plane $A'\ C$, such that the angle $B\ C\ A'$ shall be 45°, which gives $B\ A'\ C = 75°$;

Fig. 1



let us join to it a rectangular prism $B\ C\ D$, having acute angles of 30° and 60°, and we shall obtain a prism having an internal reflection and a fixed deviation of 90° which is equivalent to a prism of 60°. It is moreover evident, since the angles $D\ Q'\ y$, $x\ P\ B$ are equal, that the faces $A'B$, $B\ D$, must be at right angles.

In the actual construction the faces may be limited to the parts actually used. If we assume an aperture $B\ A'$, we shall obtain a useful prism by joining to $B\ A'\ C$ the prism $E'\ C\ D'$. But this requires the use of cement, which should be avoided if possible. We are thus led to the solution which consists in cutting a single block $B\ A\ C\ D$. This requires a slight increase in the thickness of glass to be traversed, but it avoids the necessity of cementing the surfaces, which is far more important.

Such a prism renders possible the construction of a very

convenient spectroscope. The collimator and observing telescope may be fixed at an angle of 90°. When the prism is rotated the spectral lines will always be at the position of minimum deviation at the moment of crossing the reticule in the telescope. They will thus always be of maximum sharpness.

The operation of setting for minimum deviation of a line is accomplished by a single motion and the fixed position of the telescope simplifies the construction and the use of the instrument. It is only necessary to have a good achromatic objective in order to be able to focus the photographic camera, for example, on the D line, and then, by simply rotating the prism, to bring a given region of the ultra-violet at minimum deviation to the point previously occupied by the D line.

It is evident that the prism just described causes the ray which has traversed it at minimum deviation to turn through an angle of 90° to the right. We will call this prism right-handed. If the same prism worked in the opposite direction, receiving the light on its large face and at a suitable angle, it would be left-handed. Or such a prism may be considered to be reversed top for bottom, in which case the new prism is right-handed when it receives the light on its large face and left-handed in the inverse direction.

The condition of fixed deviation may be met with the greatest simplicity in the construction of a spectroscope having several prisms. It is evident that if we wish to obtain dispersion we must join in pairs prisms which are opposite in character.

Each of these prisms will produce a deviation of 90°, either in one direction or the other, depending upon whether it is made to work in this or that direction. Let us consider the possible solutions for various numbers of prisms.

For a one prism spectroscope either position of the prism may be employed. With two prisms we may obtain either direct vision with displaced path (Fig. 2), or parallelism of the collimator and telescope, as shown in Figs. 3 and 4. With this arrangement the observer is distant only some 20 cm from the slit and source of light. When a telescope of longer focus than

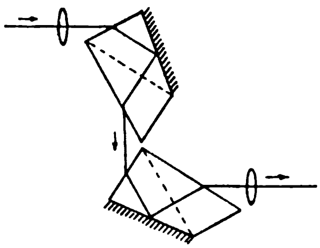


Fig. 2

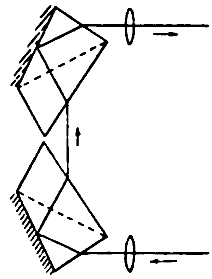


Fig. 3

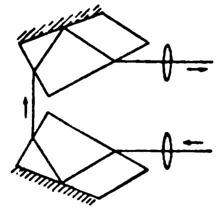


Fig. 4

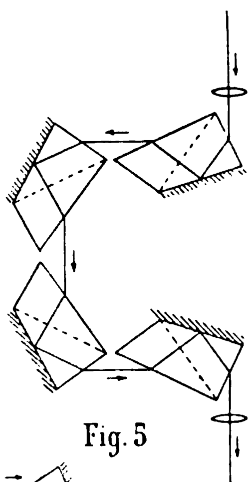


Fig. 5

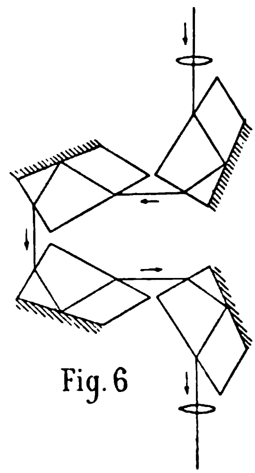


Fig. 6

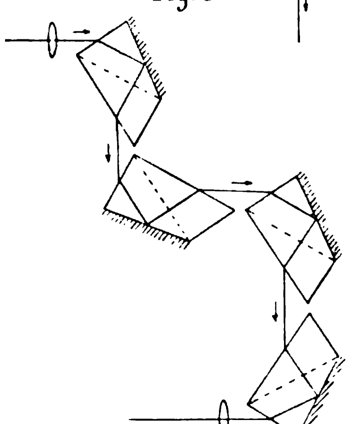


Fig. 7

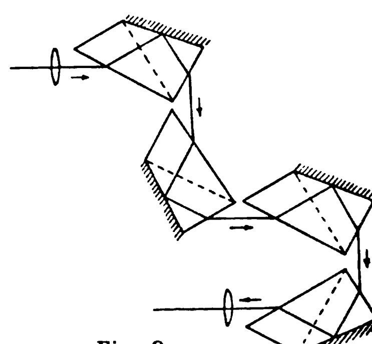


Fig. 8

the collimator is employed, the light source and slit may be placed at any desired distance. In both cases the motions of the prisms are effected by causing them to turn about fixed centers lying upon a perpendicular to the direction of the telescope, by means of two rods moved by a single fixed screw.

If three prisms are used, the collimator and telescope will lie at right angles, either to the right or left. The three prism spectroscope may be of the form (2), (3) or (4). Form (4) is most convenient.

Finally, four prisms may be used, in which case we may have either direct vision (5), or an arrangement like that shown in Fig. 3, with the slit close at hand. For each of these forms there are two solutions, shown in (5), (6), (7), (8).

In all of these solutions, in order to keep the prisms in the required position, it is only necessary to arrange them in such a manner that alternate prisms will rotate in opposite directions. This is easily accomplished by means of a tangent screw having right and left threads, or, in the case where the slit and eyepiece closely adjoin, two right and two left threads. It is evident that the faces of the prisms are always completely utilized.

It is further evident that by setting the telescope in position determined in advance by the aid of a scale, it is possible to observe with $(p-1)$ prisms, if that of order p is removed.

In this instrument a spectral line is defined by the position of the prisms at which it crosses the reticle. The angle through which the spectrum is moved by a rotation a of the prisms is $8a$. The amount of this rotation may be determined with a precision equal to the sensitiveness by means of the image of a micrometer reflected once on each of the refracting prisms. We have realized this by means of total reflection prisms placed above each of the first three refracting prisms. A double total reflection prism brings down the ray so as to utilize as a reflecting surface the last refracting face of the last prism. Thus the observing telescope may be used to measure the rotation. If, moreover, there is any error in the motion of the prism, the readings will be affected in exactly the same way as the displacement of the

spectrum, and a line will always be defined by that division of the micrometer with which it coincides at the desired point.

In arrangements (5) or (7) we may have a motion of the micrometer in the direction of the spectrum, if the rectangular prism giving double total reflection is fixed. If it partakes of the motion of the fourth dispersing prism the motion of the micrometer will be in the opposite direction to that of the spectrum. The pointing of the micrometer wire will not be affected by this; but the system having direct motion is preferable.

Of these instruments we have constructed type (5), which performs perfectly. With the instrument it is possible to make a general examination of a region of the spectrum with a single prism and then to study it in detail with the four prisms. In order to realize the dispersion of eight prisms it is only necessary to employ the principle of auto-collimation, by placing a mirror in the position previously occupied by the collimator.

These instruments seem to be well suited for the use of chemists, as they will permit the use of greater dispersions than are now employed, on account of the possibility of determining positions with a precision of the same order as the dispersion.

RESEARCHES ON THE ARC-SPECTRA OF THE METALS.

V. SPECTRUM OF VANADIUM.[1]

By B. HASSELBERG.

INTRODUCTION.

IN my previous memoirs under the above title I have sought to bring to a temporary conclusion the spectroscopy of the metals nearest to iron. In connection with the spectroscopic researches published in the meantime by Kayser and Runge, and by Rowland, these will satisfy the most pressing necessity in this department of spectroscopy. But more than this they will not do. Only the first step has so far been taken toward the final goal, the complete knowledge of the spectra of the metals, since in spite of all our efforts surely no one of us can cherish the hope of having disentangled more than the principal features of these complicated phenomena. Many peculiarities of a doubtful character in the preceding results will first find their true explanation when similar investigations shall have been carried out for the whole series of the so-called rare metals, which are often widely disseminated, although in limited quantities. An important part of this problem is indeed accomplished in the recent large catalogue of solar spectrum wave-lengths by Rowland; in view of the fact, however, that many of these metals are only meagerly represented in the general solar spectrum, it will not appear unreasonable to undertake this investigation again and to continue it as far as possible.

I was led to give my attention first to vanadium among these metals by the circumstance that Baron Nordenskiold had received a good-sized piece of this metal from Moissan, who had prepared it in the electric furnace, and which Baron Nordenskiold had

[1] *Kongl. Svenska Vetenskaps-Akademiens Handlingar*, Bandet 32, No. 2. Stockholm, 1899.

kindly placed at my disposal. The investigation of the arc-spectrum had already progressed so far that all the measures and reductions were almost ready, when on a comparison of the catalogue of wave-lengths with the titanium spectrum, previously obtained with the use of a Norwegian rutile, several fainter lines in the latter were found, which undoubtedly coincide with the strongest vanadium lines, and hence led to the unexpected discovery of the presence of vanadium in this mineral. This led to the investigation of a whole series of varieties of rutile of different origin, and established the fact elsewhere more fully discussed[1] that vanadium is a common constituent of almost all kinds of rutile. But this also delayed the conclusion of the investigation proper on the vanadium spectrum.

In the meantime a list of lines in the arc-spectrum of this metal, by Rowland and Harrison, appeared in the April number of this JOURNAL, 1898. At first my intention was to at once suppress my own investigations as under the circumstances superfluous, but I later decided otherwise, partly because my measures were almost entirely completed at the appearance of Rowland's catalogue, and partly also because of the fact that my plates contained a considerable number of lines not occurring with Rowland, for the exclusion of which as foreign impurities from my list I did not have, at least provisionally, sufficient justification, while on the other hand several of Rowland's lines were missing on my plates. As I shall show below, I have been able to identify several of these latter lines as easily recognizable impurities, but in general a closer examination of these conditions is difficult, as Rowland gives only the bare list of wave-lengths without any statement whatever as to the elimination of foreign lines. Finally, it seems to me that a comparison of the

[1] This JOURNAL, **6**, 1897. According to a note in this JOURNAL (**6**, 157) Rowland had already observed the presence of vanadium in rutile. However, nothing appears to have been published about it before the appearance of my communications. Furthermore, the occurrence of vanadium in one kind of rutile, that of St. Yrieix, was, as I have recently found, discovered long before us by Sainte-Claire Deville. For this, as well as the wide dissemination of the metal in different minerals, see *Annales de Chimie et de Physique*, 3, 61, 342, 1861 and W. F. Hillebrand in *Am. Jour. Sci.*, **6**, 209, 1898.

determinations of the lines common to us both, which naturally constitute the great majority, is of no slight interest. In this comparison I have rounded off Rowland's wave-lengths which were expressed to the third place of the Ångström unit to two places, because my value obtained with a plane grating cannot profess a greater accuracy than about two units of this place. On the assumption that the third decimal has a real significance in Rowland's values, which seems to me somewhat doubtful, in view of the diffuseness of the edges often occurring in metallic lines, and their considerable breadth, the greater part of the differences appearing in our measures would fall to me. Nevertheless, even with this limitation, our agreement is throughout very satisfactory, as will be seen from the tables of wave-lengths below.

THE MEASURING APPARATUS.

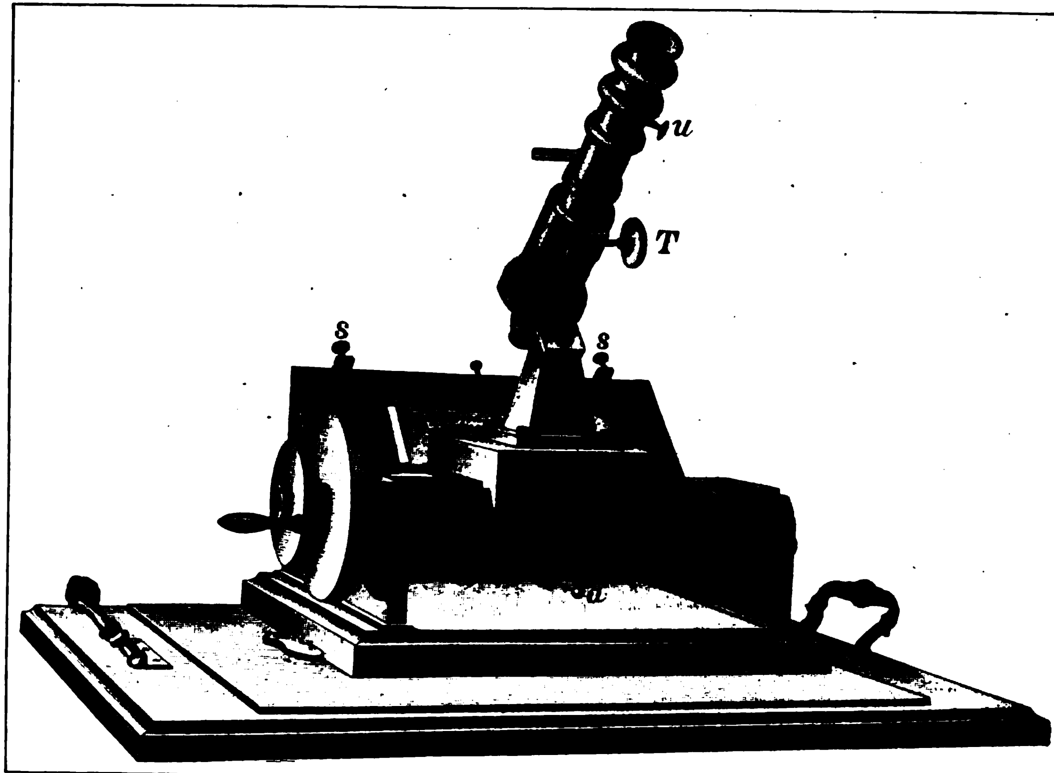
In my previous investigations of metallic spectra I employed an old dividing engine by Perreaux for measuring the negatives. There is nothing to be said against this in respect to the precision obtainable with it, but there is in respect to the convenience of measurement. In this latter respect the vertical position of the microscope wearies the eye very quickly and the unnecessary length of the screw greatly impedes its continuous use. To escape these difficulties I have had a special and very compact measuring machine constructed for the measurement of my spectrum plates, the arrangement of which may be readily seen from the figure (Plate VIII). Two steel rails, the upper one of which is a knife edge and the lower plane, are attached by two brass side-pieces to a strong frame, on which the carriage *S*, with the microscope obliquely attached to it, is moved along by the screw of about 20 cm length bearing in the end-pieces. The end-pieces are prolonged backwards and cut out in such a way that a brass plate with a rectangular aperture can be attached as a support for the photographic plate in such a position that the plane of the latter comes into a position parallel to the screw and perpendicular to the optical axis of the microscope. In order to place the length of the spectrum parallel to the screw the plate is first attached to a strong,

plane-parallel glass disk, which rests upon the brass plate and can be adjusted by means of the two screws *s s* and the spring pressing it against them. In this way the image can be so adjusted that on moving the screw the intersection of the cross hairs of the microscope runs parallel to the edge of the spectrum. One of the threads of the microscope is set parallel to the spectral line by turning the ocular end and clamping by the screw *u*. After the eyepiece has been sharply focused on the cross hairs the final setting on the spectral line is accomplished by moving the whole microscope with the pinion *T*. A small mirror, movable in all directions and serving for the illumination of the plate, is attached to the carriage, and finally by simply releasing the nut from the screw at *a* the carriage can be moved rapidly.

I have employed Bessel's method for the determination of the periodic and progressive errors of the screw. For this purpose the microscope was replaced by another of much higher magnification taken from our Repsold comparator, and then a thread interval in the eyepiece micrometer of the latter of nearly 0.5 revolutions of the screw of the apparatus was measured by successively bringing the two threads into coincidence with a fine line drawn on a cover glass. Starting from each tenth of a revolution the measures were each repeated thrice, and continued through ten successive turns in the middle part of the screw. In this way the following values were obtained for the interval measured:

Turn	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
120—21	0.5138r	0.5132	0.5098	0.5088	0.5067	0.5087	0.5083	0.5115	0.5128	0.5137
21—22	37	22	93	85	65	83	87	105	33	35
22—23	25	35	97	87	72	77	72	095	18	30
23—24	32	32	103	90	77	83	87	090	19	30
24—25	35	23	102	78	80	83	92	113	12	30
25—26	33	33	112	100	75	97	88	095	17	40
26—27	22	25	114	82	73	94	80	084	13	32
27—28	30	22	84	100	80	85	79	079	11	32
28—29	37	36	105	100	80	98	90	111	26	30
29—30	38	31	117	97	88	91	94	097	03	23
Mean	0.51327	0.51291	0.51025	0.50907	0.50757	0.50878	0.50852	0.50984	0.51180	0.51319

PLATE VIII.



Professor Hasselberg's Measuring Machine.

Whence is derived the following correction formula:

$$\Delta u = + 0.001365 \text{r. } \cos u - 0.000248 \sin u + 0.000466 \cos 2u - 0.002927 \sin 2u$$

which, employed at the beginning and end of each measure, leads to the following values of the interval:

Beginning at 0.0	$I = 0.5105$
0.1	.5110
0.2	.5106
0.3	.5104
0.4	.5101
0.5	.5115
0.6	.5104
0.7	.5102
0.8	.5105
0.9	.5107
	0.51059

The comparison is therefore entirely satisfactory. It follows from the table of corrections calculated with the aid of this formula, that the greatest correction for periodic error affecting an interval measured between lines with this portion of the screw does not numerically exceed 0.0065 revs., which introduces a correction of at most 0.008 tenth-meters in difference of wave-length in the spectrum of the third order of my spectrograph, where a revolution of the screw corresponds to about 1.3 tenth-meters. Since in the measurement of my plates I have not been able to carry the precision further than would correspond to a probable error of $\pm$ 0.02 tenth-meters in the wave-length to be determined, the periodic errors of the screw will be of no significance in the present measures, and it may accordingly be regarded as free from error in this respect.

Nevertheless, in order to check the validity of this assumption and at the same time to obtain a somewhat more reliable opinion as to the construction than was possible from the above investigations of its middle part only, I have carried out similar measures at two other points of the screw, each at a distance of forty revolutions from the middle toward the end. These

measures of the interval were here repeated in each group through only five successive turns, and led to the following values:

	To right of middle	To left of middle
Beginning at 0.0	I = 0.50736 revs.	I = 0.51078 revs.
0.1	.50236	.51092
0.2	.50036	.51082
0.3	.50184	.51094
0.4	.50542	.51174
0.5	.51246	.51248
0.6	.51752	.51220
0.7	.52032	.51132
0.8	.51838	.51074
0.9	.51390	.51048
Mean	0.50999	0.51124

On comparing these figures with those obtained for the central part it appears that the errors decrease from one end of the screw to the other, and that they must be appreciably greater for the turns toward the right than for the rest. The mean values of the measured intervals, viz.:

To right of middle - - -	I = 0.50999 revs.
At middle - - - -	0.51052
To left of middle - - -	0.51124

show further that the terms gradually decrease in pitch from right to left. In judging of the screw a more suspicious circumstance is that the phase of the errors changes from one part to another, the position of the maxima and minima of the measured interval not corresponding to the same divisions of the micrometer head in the three series of measures. It becomes therefore impossible to combine all the measures into one system in order to derive a table of errors valid for the whole screw; and considered rigorously it will be necessary either to draw up different tables of errors for the different parts, or to eliminate the periodic errors by repetitions of the measures from several starting points of the head. For this reason, therefore, the screw cannot be considered as of the first class; but since, as already remarked, the errors are too slight to appreciably affect

the measures of the spectral plates, I have for the present decided not to procure a new screw. As to the cause of this peculiar behavior, moreover, it is now hardly possible to speak definitely, but the suspicion does not seem impossible that the screw has been subject to a torsion, either during its construction or as an elastic reaction after tempering.

In order to obtain an idea as to the progressive errors, an interval of the above mentioned thin glass plate of 5 mm, or about 10 revolutions, was measured with the 150 central sections, which were almost exclusively used in the measures of spectra. These measures yielded the following:

Beginning at 0 revs.	$\Delta =$ 10.0224r.	10.0226 r.
10	.0235	
20	.0227	
30	.0229	
40	.0226	
50	.0247	10.0245
60	.0235	
70	.0242	
80	.0248	
90	.0254	
100	.0245	10.0258
110	.0261	
120	.0251	
130	.0263	
140	.0269	

which lead to the following corrections:

$f_{0} =$ 0.0000 revs.	$f_{80} = +$ 0.0085 revs.
$f_{10} = +$ .0020	$f_{90} = +$ .0081
$f_{20} = +$ .0029	$f_{100} = +$ .0071
$f_{30} = +$ .0046	$f_{110} = +$ .0070
$f_{40} = +$ .0059	$f_{120} = +$ .0053
$f_{50} = +$ .0077	$f_{130} = +$ .0046
$f_{60} = +$ .0074	$f_{140} = +$ .0027
$f_{70} = +$ .0083	$f_{150} = +$ .0002

We see that the pitch of the screw gradually decreases a little from right to left, but at the same time that the corrections of these intervals for progressive errors may be neglected as

entirely vanishing, since the distances measured between lines on the plates commonly amount to only a very few turns. For this reason I have also considered it unnecessary to attempt a sharper determination of the progressive errors by the measurement of numerous intervals on the glass plate.

It appears, first, that numerous exact coincidences occur here, in which the lines in the spectrum of vanadium are very intense but those of titanium weak, and consequently they are without any doubt to be assigned to the first metal. I had no reason, as will be readily admitted, to suspect the presence of this impurity in the Norwegian rutile used by me in obtaining the spectrum of titanium, since at the time of my investigation of titanium the vanadium spectrum was unknown to me, and since, moreover, there was no mention in so comprehensive a work as Dana's *System of Mineralogy*, of the discovery of the metal in a French variety of rutile almost forty years before by Saint Claire Deville. The experience thus gained shows that this variety of rutile also contains vanadium, and that, accordingly, the following lines must be removed from my catalogue of titanium lines, as they belong to vanadium:

λ	i	λ	i	λ	i
3636.05	1	4095.65	1	4384.85	2
3688.19	1	4099.94	1.2	4390.11	2
3692.35	2	4105.31	1.2	4400.74	1.2
3704.84	1	4109.92	1.2	4407.85	1.2
3818.37	2	4115.32	2	4408.39	1.2
3823.03	1	4116.64	1.2	4408.70	1.2
3840.90	2	4128.20	2	4441.86	1.2
3855.99	1.2	4134.60	1.2	4444.41	1.2
4090.73	1	4353.01	1	4594.28	1
4092.83	1.2	4379.40	2		

If we omit further those pairs in the above list whose components are separate and therefore independent of each other, there will remain, as common to vanadium and titanium, the lines:

V		Ti	
λ	i	λ	i
4111.93	4	4111.91	2.3
4799.94	2	4799.95	2.3

The following are to be designated as not certainly separate or possibly due to a third metal:

V λ	i	Ti λ	i
3844.88	1.2	3844.87	2
3836.20	2—	3836.22	1.2
4515.75	1.2	4515.76	1.2
4798.12	1.2	4798.13	2
4819.22	1.2	4819.20	1.2

In only the second and last pairs does the coincidence appear to be exact. Of the remainder, the line 3836.22 also occurs as a faint line in chromium, and in the investigation of titanium was designated as a possible impurity of these two metals. Since the vanadium line 3836.20 appears separated from the chromium line, as will be shown immediately below, it is probable that this is also independent of the titanium line.

ELIMINATION OF FOREIGN LINES.

Vanadium and iron.—The extraordinary abundance of lines in the spectrum of vanadium, as well as that of iron, led to the anticipation that the number of the actual, or very close, coincidences would be very considerable. Although in the first examination of the vanadium spectrum the simultaneous comparison with plates of iron permitted the separation of all those lines which, judging by the relations of intensity, could be assigned to the latter metal, nevertheless the subsequent comparison with the tables of wave-lengths of Kayser and Runge yielded an unexpected number of approximate coincidences, for which the lines differed in wave-length by less than 0.1 tenth-meter in wave-length. In order to be able to judge as to these pairs of lines with certainty I made as usual double exposures of the two spectra on the same plate, and accurately investigated the position of the components of each pair under the microscope. The following table shows the results of these observations; in this the wave-lengths of Kayser and Runge have been reduced to the later standard of Rowland by the addition of the correction + 0.07 tenth-meter, and for

convenience of comparison their intensities have been reduced to my scale as well as possible.

V		Fe		Remarks
λ	i	λ	i	
3476.15	2	3476.24	1	Fe lacking
83.91	2.3	83.98	1	
89.63	1	89.56	1	Separated. Fe trace
3493.34	1	3493.44	1	Separated. Fe trace
3504.57	1.2	3504.59	1	Fe lacking
20.18	2	20.21	1	Fe trace. Coin. ?
24.38	2+	24.41	2	Probably separated. $\lambda V < \lambda Fe$
29.90	2—	29.97	3	Widely separated
43.68	1.2	43.60	1	Fe lacking
53.43	3	53.36	1	
69.11	1.2	69.16	2	Separated. $\lambda V < \lambda Fe$
75.26	1	75.29	1	Scarcely separable
78.01	1+	78.10	2	Coin. λFe correct? A common impurity?
83.84	1+	83.81	1	Fe line doubtful
92.71	1+	92.68	1	Separated. $\lambda V > \lambda Fe$
3593.48	2	3593.53	1	Coin.
3605.75	1.2	3605.69	3	Separated. $\lambda V \lambda > Fe$
11.00	1.2	10.93	1	Separated
14.39	1.2	14.33	1	Fe lacking
16.91	1+	16.83	1	Separated. Trace of Fe
20.53	1.2	20.44	1	Fe lacking
21.37	2.3	21.31	1	
38.57	1	38.51	3	Separated. $\lambda V > \lambda Fe$
45.77	1.2	45.70	1	Separated
63.73	2.3	63.67	1	Fe lacking
65.30	2	65.40	1	
65.88	1	65.97	1	Separated. $\lambda V < \lambda Fe$
83.27	3	83.25	1	Fe lacking
86.40	2	86.47	1	A strong Fe line coincides with V
3687.61	2.3	3687.65	4	Probably separated
3703.71	3.4	3703.75	3	Not separable. V line reversed
08.88	1.2	08.79	1	Separated
22.76	2	22.76	4	Separated. $\lambda V > \lambda Fe$
40.38	1.2	40.29	1	Fe lacking
46.02	2	46.02	4	Separated. $\lambda V < \lambda Fe$
56.18	1	56.24	1	Separated. $\lambda V < \lambda Fe$
61.61	1+	61.59	1	Separated. $\lambda V > \lambda Fe$
77.63	1	77.63	2	Separated. $\lambda V > \lambda Fe$
78.83	2.3	78.89	1	Probably $\lambda V < \lambda Fe$
93.76	2	93.67	1	Separated
3795.12	3.4	3795.20	5	Separated. $\lambda V < \lambda Fe$
3806.93	2	3806.91	3.4	Separated. $\lambda V < \lambda Fe$
17.98	1.2	17.91	1	Fe lacking
28.66	3.4	28.72	1	
39.53	2	39.45	4	Widely separated
40.56	2.3	40.65	5	Hard to separate. $\lambda V < \lambda Fe$. λFe too large, should be 0.59
55.50	3	55.52	1	Perhaps separated
56.00	4	56.07	1	Separated. $\lambda V < \lambda Fe$
76.21	2.3	76.21	3	Separated. $\lambda V > \lambda Fe$. $\Delta\lambda = 0.05$
86.36	1	86.45	3.4	Separated. V ?
93.03	3	93.07	1	Separated. $\lambda V < \lambda Fe$
94.19	2	94.16	2	Separated. $\lambda V > \lambda Fe$

V		Fe		Remarks
λ	i	λ	i	
3898.16	3	3898.12	3.4	Coin.? V line diffuse. V?
3906.89	2	3906.91	2	Coin.
10.01	3	10.02	3	Separated. $\lambda V > \lambda Fe$
10.95	2	11.02	2	Separated. $\lambda V < \lambda Fe$
14.45	2	14.42	1	Separated. $\lambda V > \lambda Fe$
25.36	2+	25.38	1	Fe lacking
41.40	1.2	41.47	2	Widely separated
73.79	2	73.82	3	Separated. $\lambda V < \lambda Fe$
97.30	1.2	97.32	1	Fe lacking
3998.87	3	3998.83	1	
4009.94	1	4009.87	3.4	Widely separated
11.45	1+	11.56	1	Separated
15.50	1	15.47	1	
20.69	1	20.61	1	Fe lacking
23.50	2	23.58	1	
31.37	1.2	31.40	1	Fe trace
32.62	1.2	32.61	1	Fe trace
35.77	2	35.83	2	Widely separated
51.48	2.3	51.47	1	Fe lacking
52.60	1	52.63	1	Probably separated. $\lambda V < \lambda Fe$
60.97	1+	60.95	1	Fe lacking
92.09	1.2	92.18	1	Fe lacking
92.55	2	92.50	3	Separated. $\lambda V > \lambda Fe$
4099.93	3.4	4099.94	2	Fe lacking
4105.32	3	4105.35	2	
07.61	1.2	07.65	5	Perhaps separated. $\lambda V < \lambda Fe$
08.36	2	08.30	1	Fe lacking
09.94	3.4	09.95	5	Coin. Belongs to V | Fe
11.93	4	11.92	2	Coin. Fe only a trace. The line belongs to Ti with K-R
12.47	1.2	12.54	2	Probably separated. $\lambda V < \lambda Fe$
13.66	2.3	13.59	1	
15.32	3.4	15.41	2	Fe lacking
16.85	1.2	16.93	1	
18.73	2	18.69	6	Separated. $\lambda V > \lambda Fe$
19.58	2	19.52	2	Separated. $\lambda V > \lambda Fe$
20.70	2	20.66	1	Separated. $\lambda V > \lambda Fe$
29.00	2	28.98	1	Fe lacking
32.14	3.4	32.22	6	Separated. $\lambda V < \lambda Fe$
34.60	3.4	34.57	2	Separated? Fe faint
41.96	1.2	42.01	2	Separated. $\lambda V < \lambda Fe$
42.75	1.2	42.81	2	Fe trace. Coin.?
52.81	2	52.85	1	Fe lacking
53.49	1.2	53.54	1	Fe trace; separated. $\lambda V < \lambda Fe$
60.57	1+	60.66	1	
77.25	2—	77.23	1	Fe lacking
79.57	2.3	79.53	1	
91.71	2.3	91.79	1	
97.45	1	97.40	1	Coin.? Fe extremely faint
4198.78	2—	4198.82	2	Separated. $\lambda V < \lambda Fe$
4210.55	1	4210.55	5	Separated and $\lambda V > \lambda Fe$. The Fe line has $\lambda = 10.49$.
24.30	2	24.34	3.4	
29.87	2	29.93	1	Fe lacking

V λ	V i	Fe λ	Fe i	Remarks
4232.62	3	4232.64	1	Fe lacking
39.12	1.2	39.07	5	Widely separated. Fe line given too intense
40.53	2+	40.56	2	Separated. λ V < λ Fe
59.46	2	59.45	1	Fe lacking
69.92	2	69.95	1	Coin. ?
79.12	1.2	79.07	1	Fe lacking
84.19	3	84.26	1	Fe lacking
86.57	2	86.64	1	Fe lacking
96.28	2.3	96.20	1	Fe lacking
4298.17	2.3	4298.22	3	Separated. λ V < λ Fe
4368.76	1.2	4368.74	1	Fe lacking
69.25	1+	69.25	1	Fe lacking
79.38	4.5	79.43	2	Fe lacking
80.69	1.2	80.67	1	Fe lacking
84.37	1	84.45	1	Fe lacking
84.87	4.5	84.89	2	Fe trace
90.13	4.5	90.17	2	Fe lacking
91.84	1.2	91.75	1	Fe lacking
4395.40	4.5	4395.46	1.2	Fe trace
4400.74	4	4400.79	1	Fe lacking
06.80	4.5	06.81	1	Fe lacking
07.85	4.5	07.87	3.4	Separated. λ V < λ Fe
08.67	4.5	08.61	3.4	Separated. λ V > λ Fe
12.30	2	12.22	2	Fe lacking
16.63	3	16.63	1	Fe lacking
23.32 23.41	2	23.36	1	Fe line appears to lie in the middle
24.10	1.2	24.08	1	Fe lacking
25.86	2	25.86	1	Fe lacking
26.17	3	26.15	1	Fe lacking
38.02	3.4	37.95	1	Fe lacking
41.88	3.4	41.87	1	Fe lacking
52.19	4	52.29	1	Fe lacking
57.65	3.4	57.75	1	Separated
59.93	4	59.95	1	Fe lacking
60.47	4.5	60.55	1	Fe trace. Separated
74.21	3	74.20	1	Fe lacking
74.84	3.4	74.94	1	Fe lacking
90.95	2.3	90.95	2	Coin. Fe line faint
91.66	1	91.60	1	Fe lacking
4496.26	3	4496.27	2	Coin. Fe line faint
4509.49	2	4509.48	1	Fe lacking
14.36	2.3	14.36	2	Coin. ? Perhaps λ V < λ Fe
20.33	2	20.42	1	Fe trace. Separated
25.34	2+	25.34	4	Coin. ? Perhaps λ V < λ Fe
29.80	2.3	29.82	3	Separated. λ V < λ Fe. **Fe line inconspicuous.**
37.88	2	37.81	1	Fe trace. Coin. ?
79.38	2.3	79.37	1	Fe lacking
83.96	2	84.00	2	Fe trace. Probably λ Fe > λ V
86.54	4.5	86.53	1	Fe lacking
4594.27	4.5	4594.32	1	Fe lacking
4606.33	2.3	4606.41	1	Fe lacking

V λ	i	Fe λ	i	Remarks
4618.96	1	4618.95	2	Separated. λV > λFe. Fe line faint
26.67	2+	26.72	1	Fe lacking. The line 4626.72 belongs to Mn
4630.23	1	4630.29	3	Separated; λV < λFe; $\Delta\lambda = 0.07$.
4705.26	2	4705.17	2	Widely separated
07.62	2	07.52	5	Widely separated
14.28	2.3	14.38	1	Fe lacking
30.57	2	30.48	1	
31.74	1.2	31.67	1	
37.91	1+	37.82	1	
4757.68	2.3	4757.77	2	Widely separated
4827.62	3.4	4827.64	1	Fe lacking
59.33	1.2	59.27	1	Fe lacking
4871.46	2—	4871.50	4.5	Coin. ?
5157.27	1—	5157.75	1	Fe lacking
70.15	1+	70.15	1	
92.22	1	92.17	1	
93.18	2	93.17	1	
5395.58	2—	5395.66	2	Widely separated
5415.51	2.3	5415.50	6	Probably separated
88.18	2+	88.11	2	Separated. Fe line V. V.
5490.22	1.2	5490.17	1	Fe lacking
5585.00	1.2	5585.07	1.2	Separated. λV < λFe
5592.67	3	5592.71	1	Fe lacking
5605.20	2.3	5605.19	1	
24.80	2.3	24.77	4.5	Coin.
5627.86	3.4	5627.79	1.2	Fe trace, doubtful
5707.26	3.4	5707.22	1.2	Coin. ? Fe-line faint
27.25	4	27.27	1	Fe lacking
27.90	2.3	27.93	1	
5761.70	1.2	5761.77	1	

As in previous cases, I have also here been unable to find on my plates a considerable number of the faintest iron lines occurring with Kayser and Runge. If we add to these the pairs which could be separated on the double exposure with greater or less certainty, there remain in the above list only an unimportant number of pairs as to which any uncertainty still continues. Of these the following coincidences are specially pointed out, which, to judge by Kayser and Runge's intensities, must be regarded as belonging to both metals:

V λ	i	Fe λ	i
3578.01	1.2	3578.10	2
3593.48	2	3593.53	1
3703.71	3.4	3703.75	3
3898.16	3	3898.12	3.4

V λ	i	Fe λ	
3906.89	2	3906.91	2
4109.94	3.4	4109.95	5
4111.93	4	4111.92	2
4490.95	2.3	4490.95	2
4496.26	3	4496.27	2
4514.36	2.3	4514.36	2
4525.34	2	4525.34	4
5624.80	2.3	5624.77	4.5

It should be remarked, however, that in case of some of these iron lines the intensity on my plate seems much less than given here, so that it is very probable that these are impurities in the iron spectrum of Kayser and Runge. Such a line is 4111.92, which also occurs with high intensity in case of titanium, and therefore is without doubt to be ascribed to an impurity of this metal in Kayser and Runge's list. There is probably an error of 0.1 tenth-meter in the case of the last pair, which could not be separated on the double exposure.

VANADIUM AND TITANIUM.

The comparison of the list of wave-lengths of these two metals, in conjunction with a close investigation of the double exposures, has led to the results in the following table:

V λ	i	Ti λ	i	Remarks
3493.34	1	3493.44	1	Widely separated
3614.39	1.2	3614.35	2	Coin. Belongs to Ti
36.09	2	36.05	1	Probable coin. Belongs to V
44.88	1.2	44.87	2	Coin.
63.73	2.3	63.82	1	Separated
88.22	2.3	88.19	1	Coin. Belong to V
3692.37	3	3692.35	2	Coin. Belong to V
3704.85	3	3704.84	1	Coin. Belong to V
08.88	1.2	08.83	1.2	Separated
22.76	2	22.70	2.3	Probably separated. λ V > λ Ti
66.58	1.2	66.60	1	Separated. λ V < λ Ti
82.29	1	82.26	1.2	Separated. λ V < λ Ti
3798.40	1.2	3798.47	1.2	Widely separated
3818.37	3	3818.38	2	Coin. Belongs to V

V		Ti		Remarks
λ	i	λ	i	
3822.14	2.3	3822.16	2.3	Separated. λ V < λ Ti
23.00	2+	23.03	1	Coin. Belongs to V
33.36	1+	33.33	2	Separated. λ V > λ Ti
36.20	2—	36.22	1.2	Perhaps separated
40.56	2.3	40.48	1	Separated ?
40.88	3	40.90	2	Coin. Belongs to V
56.00	4	55.99	1.2	Coin. Belongs to V
3888.23	1+	3888.20	2	Separated. λ V < λ Ti
3998.87	3	3998.77	4	Widely separated
4013.67	1	4013.72	2.3	Separated. λ V < λ Ti
15.50	1	15.56	2	Separated. λ V < λ Ti
90.70	3	90.73	1	
92.83	3	92.83	1.2	
95.64	3	95.65	1	
4099.93	3.4	4099.94	1.2	Belong to V. 1191 also Ti
4105.32	3	4105.31	1.2	
09.94	3.4	09.92	1.2	
11.93	4	11.91	2.3	
15.32	3.4	15.32	2	
16.65	3	16.64	1.2	Coin. Belongs to V
23.35	2	23.42	2	Separated
23.65	3	23.68	2.3	Appear separated. λ V < λ Ti
28.25	3.4	28.20	2	Coin. Belongs to V
31.32	1	31.38	1.2	Separated. λ V < λ Ti
34.60	3.4	34.60	1.2	Coin. Belongs to V
59.85	2.3	59.79	2.3	Separated. λ V > λ Ti
69.42	1.2	69.46	2	Separated. λ V < λ Ti
4183.46	1+	4183.45	1.2	Separated. λ V < λ Ti
4227.90	2	4227.80	2	Separated
4315.00	1.2	4314.95	3.4	Separated. λ V > λ Ti
53.02	3.4	53.01	1	
79.38	4.5	79.40	2	Coin. Belong to V
84.87	4.5	84.85	2	
90.13	4.5	90.11	2	
4394.01	1.2	4394.04	3	Separated. λ V < λ T
4400.74	4	4400.74	1.2	
07.85	4.5	07.85	1.2	Coin. Belong to V
08.35	4	08.39	1.2	
08.67	4.5	08.70	1.2	
16.63	3	16.70	2	Separated. λ V < λ Ti
26.17	3	26.24	2.3	Separated. λ V < λ Ti
41.88	3.4	41.86	1.2	Coin. Belongs to V
44.40	3.4	44.41	1.2	Coin. Belongs to V
57.65	3.4	57.59	3.4	Separated
95.16	1.2	95.19	3	Separated. λ V < λ Ti
96.26	3	96.33	3	Widely separated
4515.75	1.2	4415.76	1.2	Perhaps separated
49.82	3	49.79	3	Separated. λ V < λ Ti
4594.27	4.5	4594.28	1	Coin. Belongs to V
4684.64	2	4684.68	1.2	Separated. λ V < λ Ti
4731.42	1.2	4731.33	2.3	Widely separated
98.12	1.2	98.13	2	Coin.
4799.94	2+	4799.95	2.3	Coin.
4819.22	1.2	4819.20	1.2	Perhaps separated

VANADIUM AND CHROMIUM.

The majority of the approximate coincidences in the spectra of vanadium and chromium are due to independent lines, as shown in this table:

V λ	i	Cr λ	i	Remarks
3574.92	1	3574.93	2.3	Probably separated. λ V < λ Cr
3649.14	2	3649.12	3	Coin. Belongs to V | Cr
80.26	3	80.34	1	Widely separated
87.61	2.3	87.65	2.3	Probably separated. λ V < λ Cr
88.22	2.3	88.24	1	Coin.
3696.00	3	3696.02	1	Coin.
3790.62	2	3790.61	2	Coin.
3806.93	2	3806.97	2	Separated. λ V < λ Cr
17.98	1.2	17.97	1.2	Coin.
20.10	2+	20.11	1	Coin.
21.63	2	21.71	1.2	Widely separated
22.14	2.3	22.22	1	Separated
36.20	2	36.22	2	Separated. λ V < λ Cr
49.48	2	49.48	2.3	Perhaps λ V < λ Cr
52.27	1	52.33	2	Separated
3894.19	2	3894.20	3	Coin. Belongs to V | Cr
3963.77	2	3963.82	3.4	Separated. λ V < λ Cr
3992.95	3	3992.95	2.3	Slightly separated. λ V < λ Cr
4051.48	2.3	4051.47	1.2	Coin. Belongs to V | Cr
4067.90	1.2	4067.94	1	Cr line lacking
4120.70	2	4120.78	2	Widely separated
52.81	1.2	52.89	1.2	Widely separated
4197.45	1	4197.38	2	Separated. λ V > λ Cr
4200.35	2	4200.27	2	Widely separated
04.67	1	04.61	2	Separated. λ V > λ Cr
16.52	1	16.50	2	Perhaps λ V < λ Cr
34.70	2.3	34.64	1.2	Probably λ V > λ Cr. $\Delta\lambda > 0.06$
39.12	1.2	39.08	2.3	Coin.
55.60	1.2	55.65	2	Separated. λ V < λ Cr
62.32	2	62.27	1.2	Separated. λ V > λ Cr
4297.86	2.3	4297.91	2.3	Separated. λ V < λ Cr
4312.56	1+	4312.65	1.2	Widely separated
73.40	2	73.41	3	Separated. λ V < λ Cr. $\Delta\lambda > 0.01$
75.47	2	75.52	2.3	Perhaps λ V < λ Cr. $\Delta\lambda < 0.05$
80.69	2	80.73	1	Cr line hardly visible
4391.84	1.2	4391.90	2.3	Widely separated
4423.41	2	4423.46	1.2	Separated. λ V < λ Cr
28.67	3	28.71	2	Coin. Belongs to V | Cr
59.93	4	59.95	2+	Coin. Belongs to V | Cr
97.03	2+	4497.02	3.4	Coin. Belongs to V | Cr
4531.01	2+	4530.92	4	Widely separated
45.60	3.4	45.51	2.3	Widely separated
4583.96	2	4584.02	1.2	Cr line lacking
4600.34	1.2	4600.25	2.3	Separated. λ V > λ Cr
48.08	1	48.00	1.2	V line lacking
49.08	1.2	49.04	2.3	Probably separated. λ V > λ Cr
54.84	1.2	54.90	2.3	Widely separated

V λ	i	Cr λ	i	Remarks
4666.33	2+	4666.35	2.3	Separated. λ V<λ Cr. Δλ>0.02
69.50	1+	69.50	2.3	Probably separated. λ V<λ Cr
4681.07	1.2	4681.01	2	Separated. λ V >λ Cr. Δλ<0.06
4706.34	2.3	4706.25	1.2	Widely separated
17.85	2.3	17.87	1.2	Separated. λ V<λ Cr
54.13	2.3	54.10	1	Separated. λ V>λ Cr
57.49	1.2	57.55	2	Separated
4766.80	2.3	4766.77	2	Coin.
4831.80	3.4	4831.79	1.2	Coin. Belongs to V
4851.65	4	4851.65	1.2	Cr lne extremely faint. Coin.?
5139.74	2—	5139.82	2.3	
5192.22	1	5192.17	2.3	
5225.97	1.2	5225.98	1.2	

The following pairs, strong in both spectra, show exact coincidence, and therefore may be provisionally assigned to the two metals:

V		Cr	
3649.14	2	3649.12	3
3790.62	2	3790.61	2
3894.19	2	3894.20	3
4051.48	2.3	4051.47	1.2
4428.67	3	4428.71	2+
4459.93	4	4459.95	2+
4497.03	2+	4497.02	3.4
4766.80	2.3	4766.77	2

In addition to these coincidences a few occur from which we might infer an impurity of the chromium by vanadium, on account of the very slight intensity of the component ascribed to chromium; this seems to me hardly probable, however, since the most characteristic groups of vanadium are not found in chromium. Such lines are:

V		Cr	
3688.22	2.3	3688.24	1
3696.00	3	3696.02	1
3820.10	2.3	3820.11	1
4380.69	2	4380.73	1
4831.80	3.4	4831.79	1.2

VANADIUM AND MANGANESE.

In the comparison of these two spectra I have found only a small number of lines, with nearly identical wave-lengths.

These were, almost without exception, separated on the double exposures.

V λ	i	Mn λ	i	Remarks
3578.01	1+	3577.99	3.4	Coin. Belongs to V | Mn. Other intense Mn lines are entirely lacking in V spectrum
3680.26	3	3680.32	1	Separated. λ V < λ Mn
3719.06	1+	3719.04	2.3	Coin.? Belongs to Mn
3806.93	2	3806.84	4.5	Widely separated
3886.36	1	3886.42	2.3	Separated
3997.30	1.2	3997.34	1	Separated. λ V < λ Mn
4052.60	1	4052.62	2	Coin. A common impurity?
4090.70	3	4090.73	1+	Probably separated. λ V < λ Mn
4123.35	2—	4121.41	1.2	Separated. λ V < λ Mn
23.65	3	23.68	1.2	Separated. λ V < λ Mn
31.32	1	31.26	2.3	Separated. λ V > λ Mn
4149.02	1.2	4148.94	2.3	Widely separated
4261.37	2—	4261.45	1.2	Widely separated
4284.19	3	4284.22	2.3	Appear separated. λV < λMn
4408.35	4	4408.28	1.2	Widely separated
57.65	3.4	57.71	3	Widely separated
4460.47	4.5	4460.55	2.3	Widely separated
4626.67	2+	4626.74	2	Separated

VANADIUM AND COBALT.

The approximate coincidences are here also found to consist of independent lines, as shown in the following table:

V λ	i	Co λ	i	Remarks
3489.63	1	3489.54	4	Widely separated
3520.18	2	3520.20	3	Separated. λ V < λ Co
3529.90	2—	3529.92	4.5	Separated. λ V λ < Co
3708.88	1.2	3708.96	2.3	Widely separated
40.38	1.2	40.31	2	Widely separated
3777.63	1	3777.65	2	Separated. λ V < λ Co
3820.10	2+	3820.02	2	Widely separated
32.97	1+	33.02	1.2	Separated. λ V < λ Co
70.72	1+	70.65	2	Separated. λ V > λ Co
3894.19	2	3894.21	5	Separated. λ V < λ Co
3910.01	3	3910.08	3.4	Widely separated
25.36	2+	25.32	1.2	Separated. λ V > λ Co
75.51	1	75.48	1.2	Separated. λ V > λ Co
3979.59	2—	3979.65	3	Widely separated
4023.50	2	4023.54	1.2	Coin. Foreign line?
35.77	2	35.73	3.4	Separated
4092.55	2	4092.55	4	Perhaps separated, and λ V > λ Co
4104.55	2	4104.57	2	Separated. λ V < λ Co
4104.92	2	4104.89	2	Separated. λ V > λ Co

V λ	i	Co λ	i	Remarks
4270.49	2	4270.58	1.2	Widely separated.
4298.17	2.3	4298.14	1.2	Separated. λ V > λ Co
4320.46	1+	4320.53	1.2	Separated
4379.38	4.5	4379.37	1.2	Probably separated, and λ V λ < Co
4416.63	3	4416.63	1.2	Impossible to separate. Perhaps λ V < λ Co
36.31	3.4	36.37	1.2	Separated. λ V < λ Co
38.02	3.4	38.05	1	Only a trace of Co
4467.04	2	4467.04	3.4	Slightly separated. λ V < λ Co
4514.36	2.3	4514.33	2.3	Separated. λ V > λ Co
28.20	2.3	28.12	2.3	Separated
49.82	3	49.80	4	Slightly separated, but λ V < λ Co
4588.94	1+	4588.85	2	Separated. λ V > λ Co
4607.40	1.2	4607.46	2	Co line hardly visible
14.08	1	14.18	2	V line lacking
24.62	2+	24.70	1.2	Separated
4640.92	2	4640.99	1.2	Separated. λ V < λ Co
4721.70	2.3	4721.61	1.2	Separated
37.91	1+	37.95	2.3	Separated. V line faint. λ V < λ Co
42.79	2	42.76	1	The Co is lacking on the comparison plate
4793.10	2	4793.03	4	Separated. λ V > λ Co

VANADIUM AND NICKEL.

A comparison of vanadium and nickel yielded only three cases of coincidence in which the contamination of the first metal by the latter will be suspected, viz., at 3486.05, 3609.45, 3793.76. I have, however, provisionally retained the lines in question with vanadium, since a number of other strong nickel lines are entirely lacking in the vanadium spectrum.

λ	i	Ni λ	i	Remarks
3486.05	1	3486.04	2.3	Perhaps separated
3533.85	3	3533.89	1	Separated. Ni line broad and diffuse. i > 1
3609.45	1.2	3609.44	2.3	Perhaps separated, and λ V < λ Ni
3644.05	1+	3644.13	1	Widely separated
3715.62	2	3715.61	1.2	Coin. Ni line faint
3793.76	1.2	3793.75	3	Coin.
3832.97	1+	3833.02	2	Separated. λ V < λ Ni
3912.36	2.3	3912.44	1.2	Widely separated. V line sharp. Ni diffuse
3913.03	2—	3913.12	2	Widely separated
3973.79	2—	3973.70	4	Widely separated
4143.02	1+	4143.12	1	
4307.33	2.3	4307.40	1.2	Widely separated
4356.10	2.3	4356.07	2	Separated. λ V > λ Ni
4423.32	2	4423.24	1.2	Widely separated
4462.56	3.4	4462.59	4	Separated. λ V < λ Ni
4606.33	2.3	4606.37	2.3	Separated. λ V < λ Ni
4786.70	3	4786.66	3	Somewhat separated

(*To be continued.*)

REVIEWS.

Untersuchungen über die Spectra von 528 Sternen. H. C. VOGEL und J. WILSING. *Publicationen des Astrophysikalischen Observatorium zu Potsdam.* No. 39. Bd. XII, Stuck 1. Pp. 73. Engelmann, Leipzig, 1899.

WITH the systematic mode of presentation and fullness of detail that has made the Potsdam Publications the models of their kind, this work gives the results of the investigations of the photographic spectra of the brighter northern stars belonging to spectral Class I. The photographs were made by Professor Wilsing, with a small one-prism spectrograph attached to the 13-inch astrographic refractor, chiefly during the years 1894 to 1897; the examination of the negatives with a magnifying glass and the classification of the spectra was the work of the two authors together; while the measurement of the spectra and the comparison with the results of other observers was made by Professor Vogel.

In the introduction a rather unfavorable opinion is expressed of the sufficiency of the photographic method, as contrasted with visual observations, for distinguishing between spectra of Types II and III, and for differentiating between Types IIIa and IIIb. While hardly agreeing with this opinion, in view of the recent great improvements in sensitizing photographic plates, and of the actual results obtained at the Yerkes Observatory in photographing spectra of the two latter types (this JOURNAL, August 1899), we may heartily assent to the statement of the advantage of the photographic plates for studying spectra of Type I.

Upon the discovery of helium and the accurate determination of its spectrum by Runge and others, during the progress of this work, Professor Vogel applied the new results in extending, and making more precise, his well-known classification of stellar spectra. This is given in some detail, and although his paper on the subject has already appeared in this JOURNAL (2, 333, December 1895), it may be well to

recall the new features of the system. Class Ia now receives three subdivisions, essentially as follows:

Ia1. Spectra in which only the broad and strongly developed hydrogen lines appear.

Ia2. Those containing other metallic lines, especially of calcium, magnesium, and sodium, but no lines of helium. K is sharply bounded, but does not reach the breadth of the hydrogen lines. Lines of other metals are faint.

Ia3. K is of nearly equal intensity to the hydrogen lines; is rarely sharply bounded; or may be broader and stronger than the hydrogen lines. Helium lines cannot be seen, but numerous and strong lines of different metals, particularly of iron, appear. This subclass constitutes the transition to Class II, in which the hydrogen lines are no longer dominant, and only the calcium lines H and K are striking on account of their extraordinary breadth and diffuseness.

Ib. Spectra characterized by the presence of the helium lines, in addition to conspicuous hydrogen lines, and more or less lines of calcium, magnesium, sodium, and iron.

Class Ic is now also subdivided, into

Ic1. Spectra with bright hydrogen lines.

Ic2. Those having bright lines of helium, calcium, magnesium, and other metals in addition to those of hydrogen.

As good examples of the different subclasses we may cite the following stars:

Ia1. λ Bootis; ϕ Draconis; β Cygni, *seq.*; λ Cygni.

Ia2. α Cygni; β Aurigae; β Equulei.

Ia3. ζ Leonis; τ Cygni; β Cassiopeiae; δ Geminorum; α Canis Minoris.

Ib. γ Orionis; γ Pegasi; η Lyrae.

Ic1. Pleione (one component).

Ic2. β Lyrae (one component).

It is distinctly stated that the discrimination between Ia1 and Ib, and Ia1 and Ia2 offers the most difficulty, and that with strong dispersion, and good conditions generally, these spectra would presumably show the additional lines characteristic of subclasses Ib and Ia2, thus leaving Ia1 without any representatives, so that it could be dropped from the classification.

It is also pointed out that the new classification is adapted as closely as possible to the original form, so that the new first type stars with

bright lines, which modern views assign to the earliest stage of development, are retained under Ic rather than made to precede Ia. This is certainly a logical procedure, and reduces the confusion which might otherwise ensue. The reviewer adheres to the opinion that all schemes of stellar classification are at present provisional, and that it will be many years before a radically new and improved order of succession can be established. Such a classification ought to be based on the results obtained with the highest instrumental power attainable, including all the measurable radiations received from the bodies — electrical (if any), thermal, actinic, visual. Meanwhile the reviewer regards Vogel's classification with its present extension as the best system for practical use.

The spectra themselves, on which this research was based, extended from about λ3800 to λ4500 with almost uniform sharpness, due to the fact that the collimator and camera lenses were achromatized in precisely the same manner as the photographic refractor to which the spectrograph was attached.

The range from λ3700 to λ4480 was 10 mm; the most convenient breadth for measuring was 0.2 mm; the average exposure was from 12 to 15 minutes. The slit was accurately set in the focus of the objective for $H\delta$ (λ4102). An arrangement was provided for furnishing a comparison spectrum by a vacuum tube, but was seldom used. A comparison spectrum could also be obtained by bringing a star of known spectrum immediately above and below the center of the slit before and after the exposure on the star to be investigated. Guiding was exceedingly accurate and easy with the 9-inch visual telescope of the photographic refractor.

The work falls in three parts: I. The catalogue of the observed spectra; II. Measurements on plates of 130 stars; III. Comparison with the results of other observers.

The catalogue gives the name, magnitude, position for 1900, and date of plate and type of spectrum, together with abbreviated but readily intelligible remarks. One good plate only was taken of a star, unless the object appeared for some reason of especial interest. Among the 528 stars, 66 of Class II and 3 of Class III fell outside of the original scope of the investigation, which was the study of spectra of Class I. Omitting these and the stars whose spectra were too faint for inclusion in the classification, and those which could not be assigned

with certainty to any of the subclasses, the remaining 437 were distributed as follows:

Ia1	44.	Ia2	168.	Ia3	68.	Ib .	100.
Ia1 to Ia2.	9.	Ia2 to Ia3.	16.	Ia3 to IIa.	19.	Ic . .	1.
Ia2 to Ia1.	3.						
Ia2 to Ib . .	5.						

Double spectra: Ia1 and Ic1, 1; Ib and Ic 2, 2; Ib and? Ic1, 1.

The remarks include data as to strength, character, and number of the lines, separately for the hydrogen and for the metallic lines, with estimates of the comparative appearance of H (including $H\epsilon$) and K. Exclamation points attract instant attention to conspicuous examples of types. From this column, and the extended notes accompanying the measures in part II, we extract interesting details of some of the notable spectra.

K is lacking in the spectrum of ϕ Persei, of which towards 90 plates were taken. No lines at all were visible on many plates. The spectrum is considered variable. No mention is made of the detection of bright $H\beta$ and $H\gamma$, originally discovered by Campbell, which are double (as is always the case with γ Cassiopeiae) on a good plate obtained with the 40-inch refractor by the reviewer. Hence the assignment of this star to Class Ia 2?—of course the only legitimate assignment on the basis of the plates and apparatus employed—is incorrect; more powerful dispersion shows it to belong to Class Ic1. A similar remark would apply to ψ Persei, and some other stars of this variety.

The spectrum of γ Cassiopeiae appeared purely continuous on the five plates obtained. It is not surprising that the dark lines in this spectrum were not seen, for it is vexatiously difficult, even with powerful apparatus, to obtain the necessary contrast for revealing them. The objective prism has a singular advantage in this respect, as shown by the results at Harvard and Stonyhurst. However, the presence of the dark lines has been abundantly confirmed by the spectrographs of the Allegheny, Lick, and Yerkes Observatories. In view of the complexity of the spectra of stars of the γ Cassiopeiae type, it will no doubt be necessary in future to discriminate more sharply in regard to the varieties of Class Ic.

We further note that in the star in Cassiopeiae, 37 Hev., K was double on three plates; this is also found to be the case for β Aurigae. It was suspected in the stars ω Ursae Majoris, 68 Ophiuchi and 109

Virginis. Thirteen plates of the last star were obtained, and on several plates K was double, or, at any rate, two distinct lines were seen at the position of K. On some plates of ζ Ursae Majoris the hydrogen lines were double or brightened up in the center.

The spectrum of Algol showed no changes in character having relation to the light period, but in some cases the hydrogen lines were brightened up in the center.

Numerous plates were obtained of α Cygni, as slight changes were suspected in its spectrum.

The micrometric measures of wave-lengths, made on the spectra of 130 of the stars, were reduced graphically from numerous solar and metallic spectral plates. They are given to the tenth-meter, and are based on Rowland's scale. In most cases a single measurement of a spectrum was sufficient to certify to the presence of helium lines, and an agreement within $\pm$ 1.5 tenth-meters was regarded as adequate for identification — an accuracy which all will agree was abundant for the purpose. A valuable accompaniment to the measures is the estimate, on a decimal scale, of the intensities of the lines.

The average number of lines measured was, of course, not large in this class of spectra — perhaps twenty — but in the case of α Cygni as many as fifty-five lines were determined. β Lyrae and P Cygni are both classified as "Ib and Ic 2" and are described at some length. Comment on these extraordinary spectra would lead us at once to transcend the bounds of a review.

A tabular exhibit is made of the helium lines found in stars of Class Ib, with their intensity in the spectrum of each star thus measured; and another table summarizes the lines not due to helium or hydrogen, with the stars in which they are found.

The results obtained are compared with the "Draper Catalogue" and with Miss Maury's "Spectra of the Brighter Stars." After pointing out cases where the "Draper Catalogue" gives discordant type assignments of the same bright star from different plates, and remarking that this is not to be wondered at, in view of the difficulty of securing a proper exposure for plates containing so many stars of different magnitude Professor Vogel comments that "on the average, however, the comparison with our results gives a right good agreement, and full recognition surely cannot be witheld from the great undertaking of the production of a catalogue of the spectra of 10351 stars." While gracefully acknowledging the value of Miss Maury's work he considers

that the classification is carried decidedly too far, and loses clearness. Tabular statements, arranged by types, compare the three catalogues mentioned, and complete the memoir. As a whole this little volume affords an admirable illustration of what valuable work can be done with comparatively small, but finely adapted and adjusted, instruments, in (it should be added) the hands of experts. Those portions of the work, in part referred to above, for which the dispersion was inadequate, can be fully investigated with the splendid equipment now happily available for the future stellar spectroscopic work of the Potsdam Observatory.
E. B. F.

Comparative Photographic Spectra of Stars to the 3½ Magnitude. By FRANK MCCLEAN. *Phil. Trans.*, A. **191**, 127–138, 1898. 17 plates.

Spectra of Southern Stars. By FRANK MCCLEAN. Pp. 16; 12 plates. London: Edward Stanford, 1898.

THE first of these two works is a collection of reproductions of 160 photographs of spectra of northern stars, accompanied by a brief discussion of their classification and distribution. No description is given of the instrument with which the plates were obtained, but from an earlier note by the author in *Monthly Notices* we may infer that an objective-prism of 12 inches aperture and 20° angle, mounted on a refractor of equal aperture and 11 feet 3 inches focal length, was employed.

As the ordinary data given with spectral plates, such as the date and duration of exposure, are also omitted, the intention of the work would seem to be pictorial and qualitative rather than metrical and quantitative. Regarded in this way, it is of decided value, being the only memoir to which one may turn for a photograph of the spectrum of each of the brighter stars. The plates are accompanied by representations of the spectra of several of the elements, for purposes of comparison and identification, and an approximate scale of wave-lengths is given.

The author divides the celestial sphere into eight zones of equal area referred to the galactic poles, and briefly examines the distribution of the different types of spectra in these zones. He subdivides Secchi's first type into three "divisions," and makes divisions IV, V and VI

correspond to Secchi's types II, III and IV. A tabular statement is given of the arrangement of the stars on the plates which follow, together with the classification of each star according to Secchi, Vogel, Pickering, and Lockyer.

In an examination of the distribution of stellar spectra the arrangement of the successive spectra according to location in the galactic areas, and then according to type, is no doubt necessary, but for convenience of reference, an arrangement by types alone would be more convenient.

The plates themselves, covering the region from about λ 3850 to λ 4900, give evidence of the great efficiency of the objective-prism, although it is probable that the reproductions hardly do justice to the originals. There is clearly much room for improvement in guiding the telescope during the exposures, as the star lines are greatly inclined or curved on some of the spectra, thus requiring considerable ingenuity in fitting them together, one below the other, so that the scale of wave-lengths could apply to all the spectra on a sheet. The plates in the second memoir, which is an extension of the first to the southern skies, show a very great improvement in this regard. The spectral lines are here always nearly vertical, and the appearance is much improved.

The second series of plates was made by the author during a visit to the Cape between May and October 1897, the objective-prism used in England for the first series being attached to the astrographic telescope of the Cape Observatory. The examination of the distribution of the different types is made in the same manner as in the first series, but the classification assigned by the other observers is necessarily omitted for these southern stars.

The chief interest in this memoir, aside from the general value of an accessible reproduction of the spectra of all the brighter stars, lies in the author's discovery of the presence of oxygen in the spectrum of β Crucis. The wave-lengths of the lines in the spectrum were measured and reduced by a cubical interpolation formula, and a table gives a comparison of these with the lines of helium, hydrogen and oxygen. Any lingering doubt as to the certainty of the coincidences with the oxygen spectrum is dispelled by the careful measurements made by Dr. Gill on spectra obtained with a large spectrograph and published in the last number of this JOURNAL.

Another table gives a list of the lines visible in the spectrum of γ Argus, over fifty in number. The author identifies four of these

with lines of the series discovered by Pickering in the spectrum of ζ Puppis. As two of these coincide with helium lines, which are shown by the author to be abundant in the spectrum, this identification perhaps required confirmation by measures on a plate taken with a spectrograph and having a suitable comparison spectrum.

E. B. F.

Annales de l'Observatoire d'Astronomie Physique de Paris. Tome premier. Paris, Gauthier-Villars et Fils. 4to pp. 122, plates XXI.

THE establishment of a national astrophysical observatory by the French government was a natural sequel to the able work of M. Janssen at the total solar eclipse of 1868. The discovery in that year of a method by which the spectrum of the chromosphere could be observed in full sunlight created the greatest enthusiasm, and the spectroscopic study of the Sun was undertaken in many European and American observatories. In Italy, thanks to the labors of Secchi, Tacchini, and Lorenzoni, there was established the *Società degli Spettroscopisti Italiani,* which gave every encouragement to astrophysical research. In England, Huggins made important contributions to our knowledge of the Sun, and did much to develop a method the potentiality of which he had long before foreseen. Lockyer, whose successful attempts to apply the spectroscopic method are also well known, sought in the study of solar phenomena data to support his hypothesis of the dissociation of the elements. In the United States Young entered upon the well-planned and well-executed series of observations which has added so much to our knowledge of the spectrum of the chromosphere, and Langley undertook his important solar investigations at the Allegheny Observatory. In Germany, Zöllner and Vogel recorded a great variety of interesting phenomena, and contributed to the improvement of the solar spectroscope. In France, however, as M. Janssen tells us in the historical sketch which introduces the present volume, the instrumental equipment required for solar research was lacking. A government project to establish a new observatory for M. Janssen might have been carried into immediate effect had not the Franco-Prussian War suddenly directed the attention of every Frenchman to the problem of the national defense. The outcome of the war, and the conditions which it entailed, were unfavorable to the development of the plan to establish

a national astrophysical observatory. Nevertheless the project was not forgotten. In July 1874 it was discussed in the National Assembly, and a few weeks later the Academy of Sciences was requested to report as to the advisability of founding an institution exclusively devoted to astrophysical research. The report of the special committee of the Academy, which was presented by M. Faye, may be found in the present volume of the *Annales*. After describing the first step in astronomical physics, which is credited to Arago, the report outlines the rise and progress of spectroscopy, refers to the discovery of the gaseous nature of the prominences by Janssen and Rayet at the eclipse of 1868, dwells on the nature and importance of astrophysical research, and concludes with a strong recommendation for the establishment of an astrophysical observatory in the immediate vicinity of Paris.

As a result of the recommendations of this report the government voted an annual grant of fifty thousand francs, which was devoted to the establishment and maintenance of a small observatory provisionally erected on the boulevard Ornano. The park surrounding the old chateau on the terrace at Meudon was selected as the permanent site, and preparations were made to occupy it as soon as it had been evacuated by the French troops, who at that time used the building for barracks. The sum necessary to reconstruct the chateau for observatory purposes and to purchase instruments, which amounted to 1,035,000 francs, was appropriated by the Senate in three equal parts in the years 1879, 1880, and 1881.

M. Janssen's historical sketch is followed by a description of the great dome and the principal instruments of the Observatory. The dome has a diameter of 18.5 m, and can be moved by electricity. The observing slit is provided with an adjustable canvas screen to protect the telescope from the wind. Observations are made from a platform, attached to the dome opposite the slit, which can be raised and lowered by means of an electric motor. The telescope, with visual and photographic objectives by the Henry Brothers, of 0.83 m and 0.62 m aperture respectively, is provided with a mounting by Gautier. The objectives are supported side by side in a tube of rectangular section, divided longitudinally by a sheet metal screen. The focal length of the visual objective is 16.16 m, while that of the photographic objective is 15.90 m. The mounting is provided with the usual adjustments. The telescope stands on a massive pier at a great height above the ground in the reconstructed chateau of Meudon.

The equipment of the Observatory includes the elaborate apparatus employed by M. Janssen in his studies of the absorption spectra of gases at various pressures and temperatures; a reflecting telescope of 1 m aperture and 3 m focus; and the photoheliographs with which have been obtained the remarkable photographs of the Sun described in the remaining portion of the volume.

M. Janssen's memoir on solar photography opens with a brief historical sketch of the work of Draper, de la Rue, Rutherfurd and others. The first astronomical photograph, one of the Moon taken only a few months after the publication of Daguerre's discovery, was due to William Draper. The Sun was first photographed by Fizeau and Foucault in 1845. On the advice of Sir John Herschel solar photography was first taken up systematically at Kew in 1858, where Warren de la Rue succeeded in securing a daily series of photographs having a diameter of 0.10 m, which were well adapted for the statistical study of Sun-spots. On account of the small size of the solar image these plates did not bring out the granulation of the photosphere and the details of the spots. The photographs of Rutherfurd and Vogel were more servicable for this purpose, and showed many important details. But it was reserved for M. Janssen to perfect at Meudon a photographic process capable of revealing the structure of the photosphere as it is rarely seen with powerful telescopes, employed under the best atmospheric conditions. This work was undertaken in 1874, in connection with the experiments on the use of photography made in preparation for the transit of Venus of that year.

The Meudon photoheliograph consists of an objective of 0.135 m aperture, by Prazmowski, mounted at the end of an adjustable wooden tube supported by a carriage on rollers. In the focal plane of the objective is a rapid exposing shutter, consisting of an adjustable slit in a metal plate which can be shot by springs across the axis of collimation. An amplifying lens mounted just beyond the exposing shutter gives an enlarged image of the Sun on a wet collodion plate at the lower end of the camera tube.

With this apparatus photographs of the Sun on a scale of 0.20 m, 0.30 m, 0.50 m and in a few cases 0.70 m to the Sun's diameter have been obtained. The diameter ordinarily employed is 0.30 m, as images of this size have been found to be most generally useful. M. Janssen believes the excellence of the photographs may be ascribed to:

1. The use of objectives made of selected glass, and achromatized

for a well-defined maximum of intensity in the spectrum of transmitted sunlight.

2. The employment of a special collodion having a maximum of sensitiveness at this point in the spectrum, and of exceedingly fine grain.

3. The considerable scale of enlargement.

4. The precautions taken to give a properly timed exposure, uniform for all parts of the image.

The photographs, of which several are reproduced in this volume, bring out in a most satisfactory manner the granulation of the photosphere, and afford material for the study of many important problems. M. Janssen considers the form of the grains to be more or less approximately spherical, while their diameter varies from one fourth to two seconds of arc. Larger masses are frequently met with, but they are shown by the best photographs to be composed of numerous very small granules. The diameters of the grains given by M. Janssen are in good agreement with the values previously obtained visually by Professor Langley with a 13-inch telescope. But unless a telescope of larger aperture than that mentioned in the text (0.135m) was employed, it is difficult to understand how details of the photosphere having a diameter of only one fourth or even one third of a second of arc (p. 105) can be resolved on the photographs. For a circular aperture of this diameter, and light of wave-length 4000 tenth-meters, the theoretical resolving power of such a telescope is 0.75″. The remarkable details shown in Plate X are also apparently beyond the reach of a 5-inch objective. Was a larger objective used, or have unpublished experiments shown that the special photographic process employed is capable of recording only a small point at the center of the bright spot of the diffraction pattern, thus surpassing the eye itself? A fault of the volume under review is here illustrated. There is a noticeable lack of exact data, and one who wishes to make a detailed study of the instruments and methods finds it impossible to do so. Such omissions have been only too common in astrophysical literature, and have doubtless given rise to much of the criticism which has been directed against the subject by those who are accustomed to exact methods and complete discussions of all available data.

M. Janssen's opinion that the general form of the grains is spherical does not exactly coincide with the view of Professor Langley, who believes the grains to be the extremities of filaments, which are

seen at all their length in spot penumbrae. According to M. Janssen, however, the penumbral elements are themselves resolved into grains. In portions of the photosphere, where they are beaten down as if by a storm, the "grains" seem to be elongated, and in many cases they are blurred and indistinct. In fact, M. Janssen finds the whole surface of the photosphere to be dotted over with a network of these blurred regions, to which he has given the name *réseau photosphérique*. This phenomenon is beautifully shown in the remarkably fine plates which accompany the volume. The dimensions of the polygons which form the meshes of the net have been found to vary from 100″ to 120″, at times of maximum size, to 10″ or 12″ at their minimum. Photographs showing the *réseau photosphérique* in various stages of development accompany the volume. Some of these plates are truly extraordinary, and will repay careful study. The faculae are shown resolved into closely grouped bright grains, and the bridges and penumbrae of spots are also resolved into granular elements. In some cases the *réseau photosphérique* is almost altogether absent.

It was asked by some who examined the earlier photographs whether the phenomenon of the *réseau photosphérique*, ascribed by M. Janssen to regions of exceptional disturbance in the solar photosphere, may not be caused by disturbances in our own atmosphere. There has been ample time to submit the question to a rigorous test, and it is certainly to be presumed that such a test has been made. A word from M. Janssen might have set all doubts at rest in the minds of any who still consider the matter open for discussion, but the present volume contains no reference to the subject. The difficulty of obtaining two solar photographs of the requisite excellence within a time so short as to exclude the probability of change in the *réseau* is undoubtedly great, and might possibly be insuperable. But in the last edition of *The Sun* Professor Young states that M. Janssen has obtained such pictures, which leave no room for doubt as to the solar origin of the phenomenon. The publication of these photographs in the present volume would have added to its value, as without such positive evidence students of solar physics sometimes find it difficult to reply effectively to those who are inclined to consider the *réseau* a terrestrial phenomenon. The matter is further obscured by the difficulty of reconciling M. Janssen's explanation of the blurred regions of the *réseau* as due to disturbances in the photosphere with the fact that the grains are clearly and distinctly shown in the bridges and penumbrae

of exceptionally active Sun-spots (Plate XII). The discovery of the *réseau photosphérique* was an advance of so great importance to solar physics that all details of the subject are earnestly desired by workers in this field.

But while it can hardly be gainsaid that the volume might be improved by the addition of certain data, as well as by greater thoroughness of discussion, the reader cannot fail to accord a full measure of praise to the originator of this invaluable method of research. Already it has yielded numerous results of the greatest importance. In the future, particularly when employed in conjunction with other devices to facilitate solar observation, it should contribute still further to our knowledge of the Sun. G. E. H.

The Indian Eclipse, 1898. Edited by E. WALTER MAUNDER. Published by the British Astronomical Association. 8vo, pp. xii+172.

THE British Astronomical Association, which was founded in October 1890, has for its principal object the organization of possessors of small telescopes in the work of astronomical observation. The association sent an expedition of volunteer observers to Norway in 1896, where the sky was unfortunately cloudy. The present volume describes the observations of the Indian eclipse of 1898, made by a large number of the members.

Apart from the amply illustrated accounts of the observers' experiences, many of which are interesting narratives of travel, the book contains much that is valuable to students of solar physics. Mr. Evershed's spectroscopic observations were especially important. His instruments consisted of a prismatic camera of 2¼ inches aperture and 36 inches focus; a slit spectrograph containing two quartz prisms for the spectrum of the corona; and a large slitless spectrograph attached to a 6-inch telescope. The two former instruments were supplied with light by a 4-inch polar heliostat, and a 3¼-inch equatorial telescope, provided with a solar spectroscope, was employed for visual observations of the chromospheric spectrum, and to determine the proper moment to expose for the "flash." It is evident that only a skillful and cool-headed observer could hope to operate all of these instruments successfully, and this Mr. Evershed proved himself to be. Except for a single omission, the program of exposures was carried

out to its conclusion by Mr. Evershed alone. The remarkable photographs of the spectrum of the flash, reproductions of which accompany the report, are evidence that excellent judgment was shown in making the exposures, and that the adjustments of the instruments left nothing to be desired. Twelve successful photographs were secured in all. Of these the most interesting and important are the ones taken with the prismatic camera. These extend from λ 6000 to λ 3350, thus including most of the visual spectrum and the greater part of the ultra-violet besides. On one of the plates 218 lines can be counted between H and λ 3342, the extremity of the spectrum in the ultra-violet. The definition is excellent, and the wave-lengths of all the lines have been determined. Mr. Jewell, of Johns Hopkins University, has found three of the lines to be due to titanium. Most of the others are ascribed to iron, calcium, magnesium, etc. Thirty hydrogen lines are shown, the wave-lengths of which, for the most part, agree very accurately with the wave-lengths calculated from Balmer's formula. Beyond $H\omega$, where the lines are faint, the wave-lengths show a somewhat less satisfactory agreement with the theoretical values. The wave-lengths of a considerable number of lines in the visual spectrum, between H and D, have also been determined. The result obtained in the case of the green coronal line accords with the conclusions of Lockyer and Campbell, to the effect that this line is some fourteen tenth-meters more refrangible than the chromospheric line (1474 K) with which it has hitherto been identified. The reproduction of a photograph of the solar spectrum, which is given for comparison with the chromospheric spectrum, is hardly good enough for the purpose. Mr. Evershed states that, while a large number of conspicuous flash lines do not appear to be represented by dark Fraunhofer lines, the appearance can be ascribed in many cases to differences in intensity, the dark lines which are actually present being very faint.

Mr. Maunder's observations were made with a binocular, one of the eyepieces of which had been fitted with a direct-vision prism. His purpose was to ascertain the distribution of "coronium" in the corona. On account of the excessive brightness of the spectrum observed before totality, his eyes were not in a condition to permit the faint green image to be seen at first. When it became visible it appeared that the gas extended pretty uniformly about the Sun up to a height of about 160,000 miles, but that it exhibits no structure corresponding with that of the visual corona. This result confirms the

early observations of Tennant, who found that the green line suffered no interruption in the rifts of the corona.

The remainder of the book contains a large number of reports, many of which are valuable. Mrs. Maunder, using a Dallmeyer stigmatic lens of $1\frac{1}{2}$ inches aperture and 9 inches focal length, secured photographs of the corona of remarkably great extent. In fact, the plates are said to show longer streamers (extreme length = 13.9 lunar radii) than have ever before been photographed. With a triple-coated plate she also obtained a distinct image of the corona 39 seconds after the end of totality. The great advantage of using triple-coated plates in photographing the corona is illustrated by this work and by the success of long exposures in showing the faint extensions of coronal streamers.

Excellent suggestions for those who intend to observe the eclipse of May 28, 1900, may be found in Mr. Maunder's discussion of the photographic results, and in the summary at the end of the volume. The British Astronomical Association is certainly to be congratulated, both on the success of its Indian eclipse expedition and on its good fortune in having so able an editor to arrange and discuss the results.

G. E. H.

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